Abstract

The analytical method is used to derive the formulae of optical properties for magnetic lens and deflector. From these formulae, the optical properties are computed for both magnetic lens and deflector. The minimum spherical and chromatic aberration coefficients (axial, magnification and rotation chromatic aberration coefficients) are selected from the results of calculation for both magnetic lens and deflector.

The inverse power law model is adopted to obtain the axial field distribution of magnetic lenses. The moving objective lens concept is used in the computation of the deflection field distribution for magnetic deflector.

The first and third orders optical properties for magnetic lens and deflector are studied. Also, the minimum values of optical properties are obtained by changing the order n and the index of the zero corresponding to each value of n.

The computation for magnetic lens and deflector shows that the focal length, spherical and axial chromatic aberration coefficients are proportional with the value of z at the focus. As well, the best magnetic lens and deflector which gives rise to minimum spherical and axial chromatic aberration coefficients are found at n=4. Also, it is noticed that the increasing of the values of the index of the zero for each value of n, for magnetic lenses and deflectors, leads to reduce the spherical and axial chromatic aberration coefficients. Additionally for magnetic lens and deflector, the magnification and rotation chromatic aberration coefficients have constant values, for each value of n, corresponding to each index of the zero.

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Last but not least, my deep and sincere thanks to my family, for their permanent helpful and encouragement.

Bahaa

Appendix

Appendix

The Trajectory Equation of Magnetic Deflectors

From Eq. (2-6) which is given by:

$$D(z)=\frac{-d}{2}B'(z) \tag{1}$$

$B'(z)$ is computed by aiding Eq. (2-3)

Then, Eq. (1) can be written as:

$$D(z)=\frac{n\,d\,B_o}{2\,z^{n+1}} \tag{2}$$

For simplicity, let d=1(mm)

Then, Eq. (2) becomes:

$$D(z)=\frac{n\,B_o}{2\,z^{n+1}} \tag{3}$$

Now, Eq. (3) substituting into Eq. (2-1) to get:

$$r''(z)+\left(\frac{\eta\,(B_o)^2 n^2}{32\,V_r}\frac{1}{z^{2n+2}}\right)r(z)=0 \tag{4}$$

Let $$r(z)=z^{\frac{1}{2}}R(z) \tag{5}$$

Now, change the variable from r(z) to R(z) as:

$$r'(z)=z^{\frac{1}{2}}R'(z)+\frac{1}{2}z^{-\frac{1}{2}}R(z) \tag{6}$$

$$r''(z) = z^{\frac{1}{2}}R''(z) + \frac{1}{2}z^{-\frac{1}{2}}R'(z) + \frac{1}{2}z^{-\frac{1}{2}}R'(z) - \frac{1}{4}z^{-\frac{3}{2}}R(z) \qquad (7)$$

Eq. (5) and Eq. (7) substituted into Eq. (4) to obtain:

$$z^{\frac{1}{2}}R''(z) + z^{-\frac{1}{2}}R'(z) - \frac{1}{4}z^{-\frac{3}{2}}R(z) + \left(\frac{\eta\,(B_o)^2 n^2}{32\,V_r}\frac{1}{z^{2n+2}}\right)z^{\frac{1}{2}}R(z) = 0 \qquad (8)$$

Multiplying Eq. (8) by $z^{-\frac{1}{2}}$ and making some rearranging to get:

$$R''(z) + \frac{R'(z)}{z} + \frac{1}{4}\left(\frac{\eta\,(B_o)^2 n^2}{8\,V_r}\frac{1}{z^{2n+2}} - \frac{1}{z^2}\right)R(z) = 0 \qquad (9)$$

If Eq. (9) is compared with Eq. (36-76) from [Hawkes and Kasper 1989], one can be found that ζ is given by:

$$\zeta = \frac{n\,k}{4}z^{-n} \qquad (10)$$

where k is defined by Eq. (3-7).

After some mathematical substitution and rearranging, Eq. (9) becomes:

$$\frac{d^2R}{d\zeta^2} + \frac{1}{\zeta}\frac{dR}{d\zeta} + \left(1 - \frac{\nu^2}{\zeta^2}\right)R = 0 \qquad (11)$$

with $\nu^2 = \dfrac{1}{4\,n^2}$ (12)

Eq. (11) is Bessel function of first order.

Now from Eq. (5), Eq. (10) and Eq. (12), one can find the trajectory equation for magnetic deflectors as the following:

I-when n=2

This value (n=2) can be substituted in Eq. (12) to find the order of Bessel function as:

$$\nu^2 = \frac{1}{16} \Rightarrow \nu = \frac{1}{4} \tag{13}$$

Also, this value (n=2) substitutes in Eq. (10) to find the argument as:

$$\zeta = \frac{k}{2\,z^2} \tag{14}$$

From Eq. (13) and Eq. (14), one can obtain on:

$$R(z) = J_{\frac{1}{4}}\left(\frac{k}{2\,z^2}\right) \tag{15}$$

Now, Eq. (15) substitutes in Eq. (5) to find:

$$r(z) = z^{\frac{1}{2}}\ J_{\frac{1}{4}}\left(\frac{k}{2\,z^2}\right) \tag{16}$$

II-when n=3

This value (n=3) can be substituted in Eq. (12) to find the order of Bessel function as:

$$\nu^2 = \frac{1}{36} \Rightarrow \nu = \frac{1}{6} \tag{17}$$

Additionally, this value (n=3) inserts in Eq. (10) to find the argument as:

$$\zeta = \frac{3\,k}{4\,z^3} \tag{18}$$

From Eq. (17) and Eq. (18), one can find:

$$R(z) = J_{\frac{1}{6}}\left(\frac{3\,k}{4\,z^3}\right) \tag{19}$$

Now, Eq. (19) substitutes in Eq. (5) to find:

$$r(z) = z^{\frac{1}{2}}\ J_{\frac{1}{6}}\left(\frac{3\,k}{4\,z^3}\right) \tag{20}$$

III-when n=4

This value (n=4) can be substituted in Eq. (12) to find the order of Bessel function as:

$$\nu^2 = \frac{1}{64} \Rightarrow \nu = \frac{1}{8} \tag{21}$$

Moreover, this value (n=4) inserts in Eq. (10) to find the argument as:

$$\zeta = \frac{k}{z^4} \tag{22}$$

From Eq. (21) and Eq. (22), one can find:

$$R(z) = J_{\frac{1}{8}}\left(\frac{k}{z^4}\right) \tag{23}$$

Now, Eq. (23) substitutes in Eq. (5) to find:

$$r(z) = z^{\frac{1}{2}}\ J_{\frac{1}{8}}\left(\frac{k}{z^4}\right) \tag{24}$$

Supervisors Certification

We certify that this thesis entitled **"Computational Study of Magnetic Lens and Deflector with a Field Distribution of The Form $B(z) \propto z^{-n}$ "** prepared by **"Bahaa A. H. Jawad"** under our supervision at 'Al-Nahrain University' as a partial fulfillment of requirements for the Degree of Master of Science in Physics.

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In view of the available recommendations, I forward this thesis for debate by the Examination Committee.

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CHAPTER Five

Conclusions and Recommendations for Future Works

CHAPTER ONE

Introduction

CHAPTER Two

Theoretical Considerations

CHAPTER Three

Mathematical Structure

CHAPTER Four

Results and Discussion

Chapter Five

Conclusions and Recommendations for Future Works

5-1 Conclusions

According to the results obtained in the previous chapter several conclusions can be recorded. Before that, one can be mentioned that the analytical method which applied in the present study gives a good observation to understand the theoretical study of electron optics because the behavior of any one of the parameter can be predicted by the formula which has described.

The most important conclusions are listed as the following:

5-1-1 Magnetic lenses and deflectors

The most important of conclusion are summarized in the following:

1- The results of calculations show that the focal length was found to be proportional to the value of z at the focus for all magnetic lenses and deflectors. Furthermore, the results of calculations for the spherical and axial chromatic aberration coefficients were also proportional to the value of z at the focus (and therefore to the focal length).

2- The results of calculation indicate that the various parameter of interest for each value of n, namely f/z_f , C_s/z_f , C_s/f , C_c/z_f and C_c/f are all have constant values corresponding to each value of index of the zero.

These parameters are all slightly dependent on the position of the focus except for f/z_f of monopole lens and C_c/z_f of each type of lenses and deflectors where C_c/z_f has constant value for each index of the zero. Additionally, the values of these parameters depend on the order n.

3- One can note from the results of calculations that the values of both C_m and C_θ for each magnetic lens and deflector are constant corresponding to each one of index of the zero.

Both coefficients are dependent on the position of the focus z (except for C_m of monopole lens) and the order of multipole (n).

4- The best magnetic lenses and deflectors, which gives rise to minimum spherical aberration coefficient and with all axial, magnification and rotation chromatic aberration coefficients, are clear from the results of the calculations occurs when n = 4 although in the case of magnetic lenses has maximum magnification chromatic aberration.

5-2 Recommendations for Future Works

The following topics are put forward as future investigations:

1- One can chose the type of magnetic deflector whether saddle or toroidal type to use as a source of magnetic deflector in the present work. After that, studying the effect of the coil geometry, which may be represented through the length and the angle of the coil, to find the optimum design that gives rise to minimum aberration coefficients.

2- One can suggest different methods such as numerical and DA methods to study the magnetic deflectors, which are studied in the present work, in order to compare them with the results calculated.

Chapter Four

Results and Discussion

4-1 Introduction

The magnetic flux density distribution for the magnetic lens is calculated by applying Eq. (2-3). The MOL concept, which is shown in section (2-4), has been used to find the deflection field of magnetic deflector.

After that, we are applied the formulae that have been derived in chapter three, of each optical properties; such as trajectory of electron beam, focal length, spherical and chromatic aberration coefficients. These optical properties can be calculated by using MATHCAD 14 package.

The calculations procedure are divided into four steps for each type of magnetic system (lens and deflector): the first; calculating the magnetic field, second; calculating the trajectory of electron beam, third; calculating the focal length and fourth; calculating the spherical and chromatic aberration coefficients.

4-2 Magnetic Lenses

4-2-1 Magnetic field distribution by using inverse power law model

The axial flux density distributions of lens B(z) are computed using Eq. (2-3) for each value of n at constant value of (a). Figure (4-1) shows the distributions of the axial flux density for the magnetic lenses which fields in the form $B(z) \propto z^{-n}$ for different values of n (n= 2, 3 and 4).

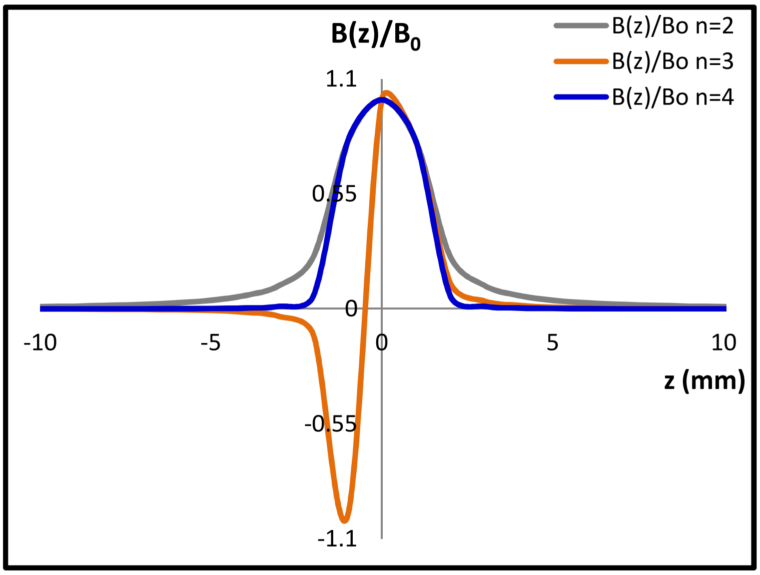


Figure (4-1): The axial flux density distributions of magnetic lenses which fields in the form $B(z) \propto z^{-n}$ for n=2, 3 and 4 at constant value of (a).

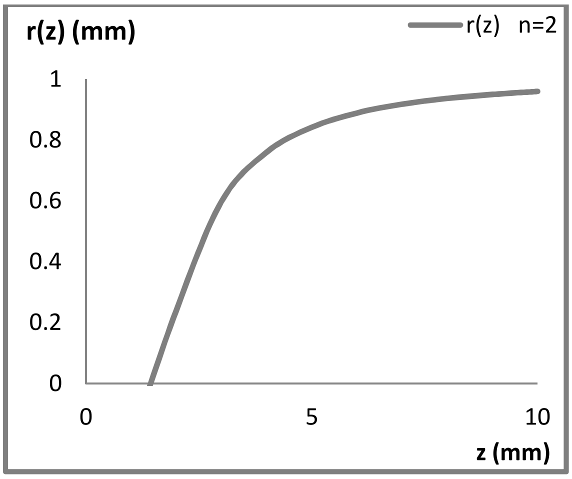
From this figure one can be observed that the magnetic field for n=2 and 4 have maximum value at z =0 while in the case n=3, the field is divided into two parts; one in the positive region and the other in the negative. This means that the domain of these fields (for n=2 and 4) is strong at this point (z=0) and it gradually decreases when it moves away from this point. Furthermore, increasing the power n leads to decrease the width of these fields.

4-2-2 Trajectory of electron beam

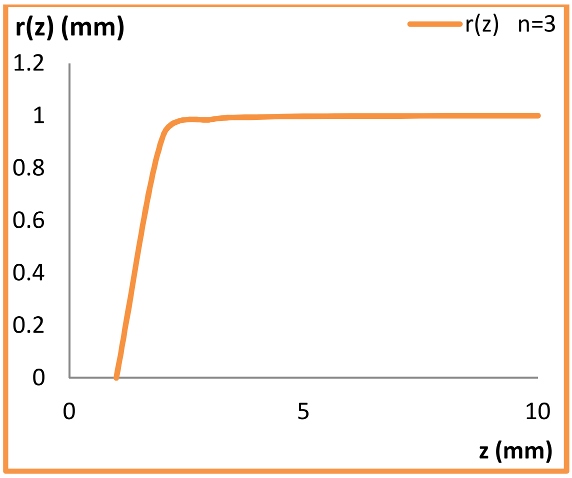
The trajectories of electron beam along the magnetic fields for lenses have been computed using Eq. (3-1)-Eq. (3-3) for n=2, 3 and 4, respectively at the value of the excitation parameter equal 13.424 ($\frac{\text{Amp.turn}}{(\text{Volt})^{0.5}}$) (i.e. k=5) as is shown in figure (4-2). The initial conditions for computing the trajectory of electron beam are given by:

$$\lim_{g(z_0)\to 0} g(z_0) = 1 \qquad \lim_{g(z_0)\to 0} g'(z_0) = 0$$

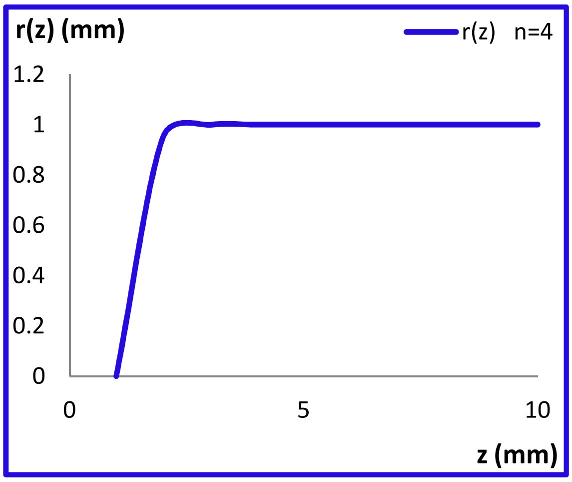
$$\lim_{h(z_0)\to 0} h(z_0) = 0 \qquad \lim_{h(z_0)\to 0} h'(z_0) = 1$$



(a) For n=2.



(b) For n=3.



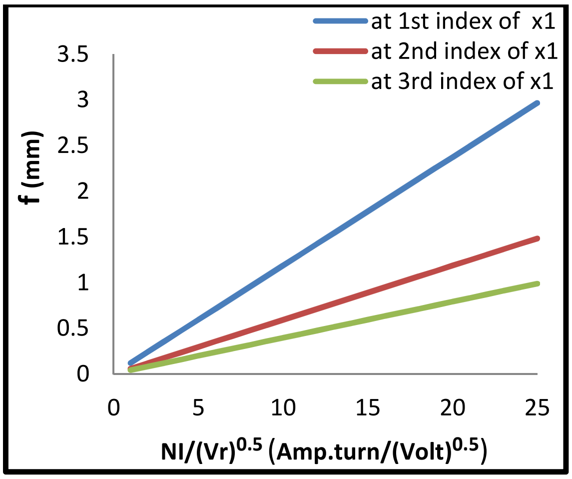
(c) For n=4.

Figure (4-2): Trajectory of electron beam at the excitation parameter 13.424 $(\frac{\text{Amp.turn}}{(\text{Volt})^{0.5}})$ for magnetic lenses with the field distributions of the form $B(z) \propto z^{-n}$ for n=2, 3 and 4.

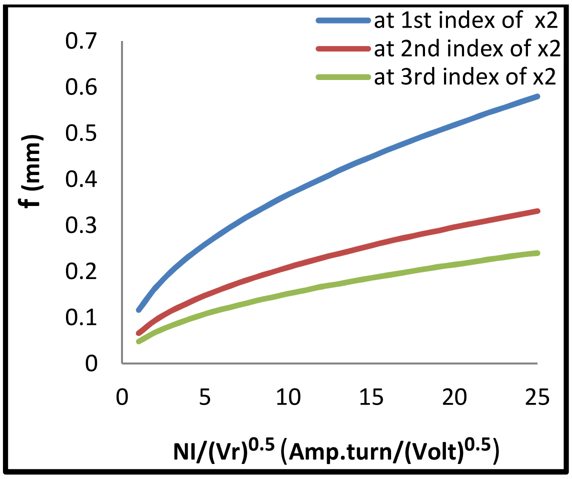
From figure (4-2), one can note that the trajectories of electron beam for different magnetic lenses have almost the same behavior in spite of different magnetic fields for each lens. This cause occurs because of the trajectories of the electron for these magnetic lenses described in terms of fractional Bessel functions. The behavior of these functions is somewhat similar to a large extent with each other. Therefore, the behavior of the trajectories is almost similar.

4-2-3 Focal length

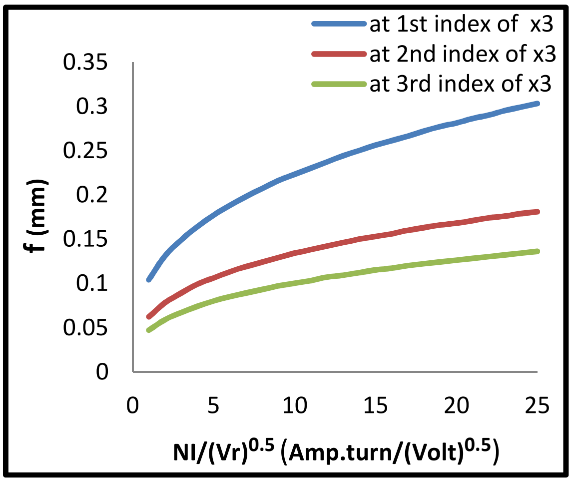
Eq. (3-27)-Eq. (3-29) are used to compute the focal length for magnetic lenses at the focus for n=2, 3 and 4, respectively. The results of calculations for these parameters are shown in figure (4-3) as a function of the excitation parameter.



(a) For n=2.



(b) For n=3.



(c) For n=4.

Figure (4-3): Focal length at the focus for first three indices of the zero as a function of excitation parameter $(\frac{NI}{\sqrt{V_r}})$ for magnetic lenses with the field distributions of the form $B(z) \propto z^{-n}$ for n=2, 3 and 4.

From the figure (4-3), one can notice that the focal length is directly proportional with the excitation parameter for each value of n. That means the increasing of the excitation parameter leads to increase the focal length.

Also, the focal length for each value of n has lowest value at third index of the zero corresponding to each one of x (x_1, x_2 and x_3).

By comparing figures (4-3a), (4-3b) and (4-3c), one can find that increasing of the power n leads to decrease the focal length. This case occurs because the focal lengths are proportional with $k, k^{1/2}$ and $k^{1/3}$ for n=2, 3 and 4, respectively. As the result, they are proportional with $\frac{NI}{\sqrt{V_r}}$, $(\frac{NI}{\sqrt{V_r}})^{1/2}$ and $(\frac{NI}{\sqrt{V_r}})^{1/3}$ for n=2, 3 and 4, respectively.

The calculation results of the relative focal length f/z_f for different values of n are shown in the table 1.

Table 1: The relative focal length, at the focus for first three indices of the zero, of monopole, dipole and quadrupole magnetic lenses (note that the results were calculated to the eighth decimal order).

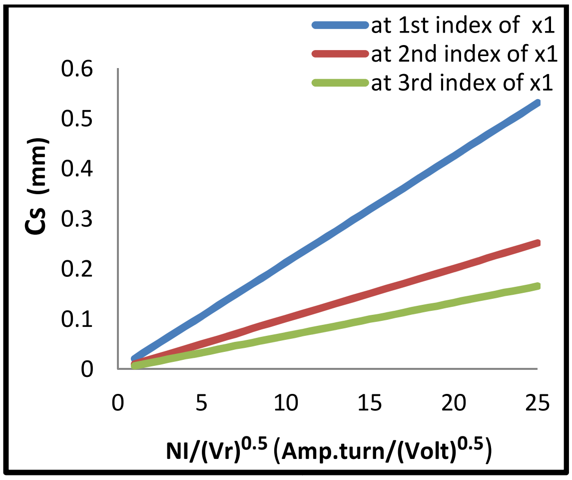
Index of the zero	Monopole (n=2)	Dipole (n=3)	Quadrupole (n=4)
First	1	0.44785219	0.28770505
Second	1	0.37244770	0.22321278
Third	1	0.33507053	0.19336092

From the table 1, it is clear that the lowest value of relative focal length is at third index of the zero for each case of x except in the case when n=2, where it has constant value for all indices of the zero. Also from the comparison among different magnetic lenses, it can be observed that the increasing of the power n leads to decrease f/z_f .

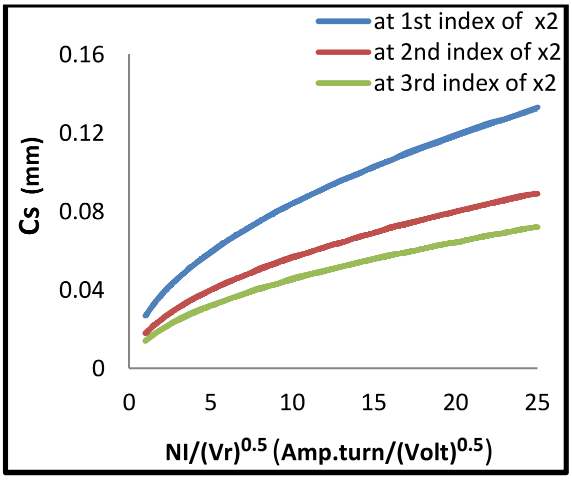
4-2-4 Aberration coefficients for magnetic lenses

4-2-4-1 spherical aberration coefficient

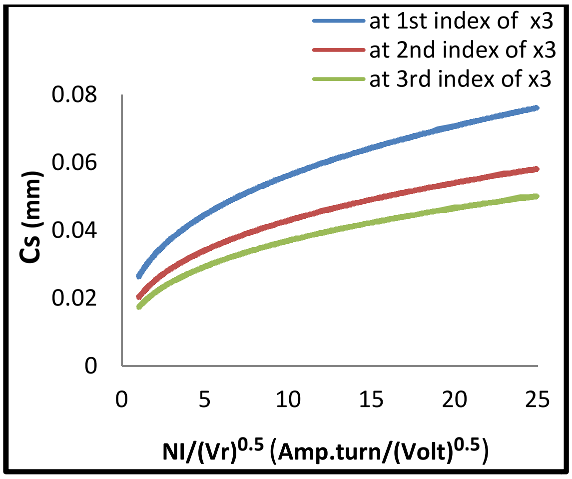
By using Eq. (3-39), Eq. (3-42) and Eq. (3-45), the spherical aberration coefficients at the focus can be found for n=2, n=3 and n=4, respectively. The results of calculations for these coefficients are shown in figure (4-4).



(a) For n=2.



(b) For n=3.



(c) For n=4.

Figure (4-4): The spherical aberration coefficients at the focus for first three indices of the zero as a function of excitation parameter $(\frac{NI}{\sqrt{V_r}})$ for magnetic lenses with the field distribution of the form $B(z) \propto z^{-n}$ for n=2, 3 and 4.

The figure (4-4) shows that the spherical aberration coefficient for different values of n, (n=2, 3 and 4), is directly proportional with excitation parameter. Additionally, the spherical aberration coefficient has lowest value at third index of the zero corresponding to each one of x (x_1, x_2 and x_3).

The comparison among figures (4-4a), (4-4b) and (4-4c) shows that the increasing of the power n leads to decrease the spherical aberration coefficient. This case occurs because the spherical aberration coefficients are proportional with $k, k^{1/2}$ and $k^{1/3}$ for n=2, 3 and 4, respectively. As a consequence, they are proportional with $\frac{NI}{\sqrt{V_r}}$, $(\frac{NI}{\sqrt{V_r}})^{1/2}$ and $(\frac{NI}{\sqrt{V_r}})^{1/3}$ for n=2, 3 and 4, respectively.

The relative spherical aberration coefficients $C_s\backslash z_f$ and $C_s\backslash f$ are computed. The results of calculations are listed in the table 2.

Table 2: The relative spherical aberration coefficients $C_s\backslash z_f$ and $C_s\backslash f$, at the focus for first three indices of the zero, of monopole, dipole and quadrupole magnetic lenses (note that the results were calculated to the eighth decimal order).

Magnetic lenses	Index of the zero	$C_s\backslash z_f$	$C_s\backslash f$
Monopole (n=2)	First	0.17933181	0.17933181
	Second	0.16983295	0.16983295
	Third	0.16807391	0.16807391
Dipole (n=3)	First	0.10269968	0.22931602
	Second	0.10066346	0.27027542
	Third	0.10029476	0.29932432
Quadrupole (n=4)	First	0.07242078	0.25171885
	Second	0.07167219	0.32109357
	Third	0.0715374	0.36996824

In the table 2, the value of $C_s\backslash z_f$ or $C_s\backslash f$ is of interest because they are generally used as figure of merit by which various types of lenses, both magnetic and electrostatic, can be compared [El-Kareh and El-Kareh 1970]. Furthermore, the figure of merit can be used in magnetic deflector.

The constant values of both $C_s \backslash z_f$ and $C_s \backslash f$ caused is that the foci position in case n=2, for example, is proportional with k and the spherical aberration coefficient in the same case also proportional with k. Therefore, the relative coefficients are constant values. This case is the same for different magnetic lenses (i.e. for n=3 and n=4).

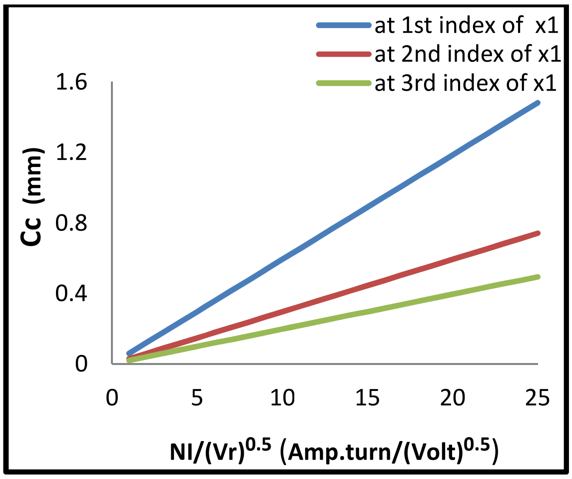
It is clear from the table 2 that the relative spherical aberration coefficient $C_s \backslash z_f$ has lowest value at third index for each one of x (x_1, x_2 and x_3). Through comparing among different magnetic lenses as in table 2, one can find that the increasing of the power n leads to decrease $C_s \backslash z_f$.

On the other hand, table 2 shows that the relative spherical aberration coefficient $C_s \backslash f$ has lowest value at first index for each value of x except the case n=2. Furthermore, the increasing of the power n leads to increase $C_s \backslash f$ except the case n=2.

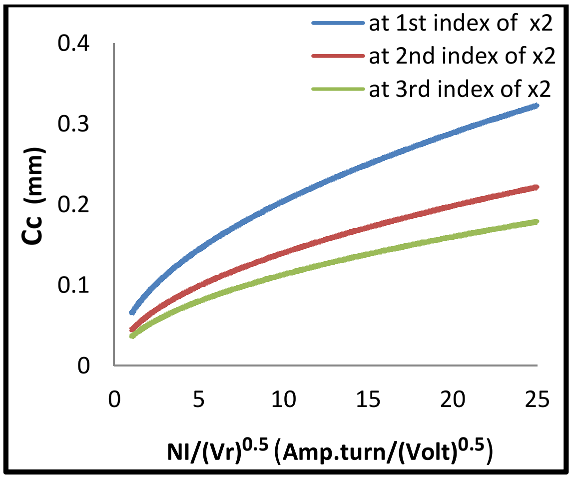
4-2-4-2 chromatic aberration coefficients

A-axial chromatic aberration coefficient

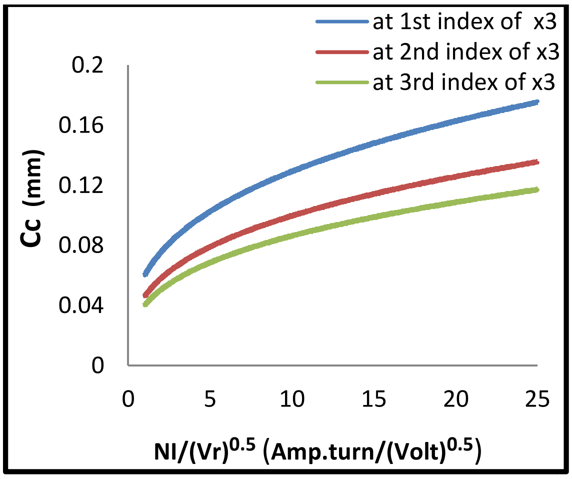
The computing axial chromatic aberration coefficients at the focus can be found by using Eq. (3-53), Eq. (3-55) and Eq. (3-57) for n=2, 3 and 4, respectively. The results of calculations for these coefficients are shown in figure (4-5).



(a) For n=2.



(b) For n=3.



(c) For n=4.

Figure (4-5): The axial chromatic aberration coefficients at the focus for first three indices of the zero as a function of excitation parameter ($\frac{NI}{\sqrt{V_r}}$) for magnetic lenses with the field distributions of the form $B(z) \propto z^{-n}$ for n=2, 3 and 4.

This figure shows that the axial chromatic aberration coefficient for each value of n is directly proportional with excitation parameter. As well, it has lowest value at third index of the zero corresponding to each one of x (x_1, x_2 and x_3).

By comparing figures (4-5a), (4-5b) and (4-5c), one can observe that the increasing of the power n leads to decrease the axial chromatic aberration coefficients. This case occurs because of the directly proportionality between the axial chromatic aberration coefficients and $k, k^{1/2}$ and $k^{1/3}$ for n=2, 3 and 4, respectively. As the result, they are proportional with $\frac{NI}{\sqrt{V_r}}$, $(\frac{NI}{\sqrt{V_r}})^{1/2}$ and $(\frac{NI}{\sqrt{V_r}})^{1/3}$ for n=2, 3 and 4, respectively.

The relative axial chromatic aberration coefficients $C_c \backslash z_f$ and $C_c \backslash f$ are computed. The results of calculations are shown in the table 3.

Table 3: The relative axial chromatic aberration coefficients $C_c\backslash z_f$ and $C_c\backslash f$, at the focus for first three indices of the zero, of monopole, dipole and quadrupole magnetic lenses (note that the results were calculated to the eighth decimal order).

Magnetic lenses	Index of the zero	$C_c\backslash z_f$	$C_c\backslash f$
Monopole (n=2)	First	0.5	0.5
	Second	0.5	0.5
	Third	0.5	0.5
Dipole (n=3)	First	0.24999999	0.55821988
	Second	0.24999999	0.98207718
	Third	0.24999999	1.35169873
Quadrupole (n=4)	First	0.16666665	0.57929693
	Second	0.16666665	1.10667753
	Third	0.16666665	1.58811633

The table 3 shows that the relative axial chromatic aberration coefficient C_c/z_f for each case of n has constant value for each value of index of the zero.

If magnetic lenses are compared with each other as in table 3, one can observe that the increasing of the power n leads to decrease C_c/z_f .

However, the relative axial chromatic aberration coefficient C_c/f for each case of n has lowest value at first index for each one of x except the case n=2.

By comparing between magnetic lenses as in table 3, one can be noted that the increasing of the power n leads to increase C_c/f except the case n=2.

B- magnification chromatic aberration coefficient

To compute the magnification chromatic aberration coefficients at the focus, one can be used Eq. (3-65), Eq. (3-67) and Eq. (3-69) for n=2, 3 and 4, respectively. The results of calculations for these coefficients are shown in table 4.

Table 4: The magnification chromatic aberration coefficients, at the focus for first three indices of the zero, of monopole, dipole and quadrupole magnetic lenses (note that the results were calculated to the eighth decimal order).

Index of the zero	Monopole (n=2)	Dipole (n=3)	Quadrupole (n=4)
First	0.5	0.55821988	0.57929693
Second	0.5	0.67123511	0.7466715
Third	0.5	0.74611155	0.8619459

From the table 4, one can find that the magnification chromatic aberration coefficients at the focus for each value of n have constant values, but the change in the magnification chromatic aberration coefficients due to vary the index of the zero except the case n=2, where it has constant value for all indices of the zero. The constant values of these coefficients due to independent on $(\frac{NI}{\sqrt{V_r}})$. Also, the magnification chromatic aberration coefficient has lowest value at first index of x (x_2 and x_3) for n=3 and 4.

Referring to the table 4, one can notice that the increasing of the power n leads to increase C_m.

C- rotation chromatic aberration coefficient

From Eq. (3-74)-Eq. (3-76), one can be computed the rotation chromatic aberration coefficients at the focus for n=2, 3 and 4, respectively. The results of calculations for these coefficients are shown in table 5.

Table 5: The rotation chromatic aberration coefficients, at the focus for first three indices of the zero, of monopole, dipole and quadrupole magnetic lenses (note that the results were calculated to the eighth decimal order).

Index of the zero	Monopole (n=2)	Dipole (n=3)	Quadrupole (n=4)
First	1.57079633	1.39044385	1.3287528
Second	3.14159265	2.9530713	2.88926995
Third	4.71238898	4.52119185	4.4567831

The table 5 shows that the rotation chromatic aberration coefficients at the focus for each case of n have constant values corresponding to each value of index of the zero. These constant values of C_θ due to the independent on $\frac{NI}{\sqrt{V_r}}$. As well, these coefficients have lowest values at first index of the zero corresponding to each one of x (x_1, x_2 and x_3).

The positive sign of these coefficients means that the tangential deviation of the image point due to an increasing of accelerating voltage is the same direction to that of image rotation.

From comparison among different magnetic lenses as in the table 5, one can be observed that the increasing of the power n leads to slightly decrease C_θ.

The results of calculation in the present work by analytical method are shown in table 6, 7, and 8 together with those calculated through the Hawkes's analytical method, Crewe's results and the Liu's DA results for comparison (the results through different methods are taken from [Liu 2003]).

Table 6: The optical properties of monopole lenses at the focus have been calculated by using different methods: a) DA method; b) Analytical method; and c) Crewe's method; d) Result from present study. * Sign "–" indicates that the aberration integral was performed from infinity to the focus.

Zero	Methods	f/z_f	C_s/z_f	C_c/z_f	C_s/f	C_c/f	C_m	C_θ
First	a	1.0000000	0.17933182	0.50001161				
	b	1.0000000	0.17933181	0.50000000				
	c	1.00	-0.18*	0.50				
	d	1	0.17933181	0.5	0.17933181	0.5	0.5	1.57079633
Second	a	0.9999999	0.16983296	0.50001167				
	b	1.0000000	0.16983295	0.50000000				
	c	1.00	-0.17*	0.50				
	d	1	0.16983295	0.5	0.16983295	0.5	0.5	3.14159265
Third	a	1.0000000	0.16807391	0.50001112				
	b	1.0000000	0.16807391	0.50000000				
	c	1.00	-0.17*	0.50				
	d	1	0.16807391	0.5	0.16807391	0.5	0.5	4.71238898

Table 7: The optical properties of dipole lenses at the focus have been calculated by using different methods: a) DA method; b) Analytical method; and c) Crewe's method; d) Result from present study. * Sign "–" indicates that the aberration integral was performed from infinity to the focus.

Zero	Methods	f/z_f	C_s/z_f	C_c/z_f	C_s/f	C_c/f	C_m	C_θ
First	a	0.44785218	0.10269970	0.25000056				
	b	0.44785219	0.10269968	0.25000000				
	c	0.45	-0.10*	0.25				
	d	0.44785218	0.10269968	0.24999999	0.22931602	0.55821988	0.55821988	1.39044385
Second	a	0.37244770	0.10066346	0.25000046				
	b	0.37244770	0.10066346	0.25000000				
	c	0.37	-0.10*	0.25				
	d	0.3724477	0.10066346	0.24999999	0.27027542	0.98207718	0.67123511	2.9530713
Third	a	0.33507052	0.10029477	0.25000034				
	b	0.33507053	0.10029476	0.25000000				
	c	0.34	-0.10*	0.25				
	d	0.3350705	0.10029476	0.24999999	0.29932432	1.35169873	0.74611155	4.52119185

Table 8: The optical properties of quadrupole lenses at the focus have been calculated by using different methods: a) DA method; b) Analytical method; and c) Crewe's method; d) Result from present study * Sign "–" indicates that the aberration integral was performed from infinity to the focus.

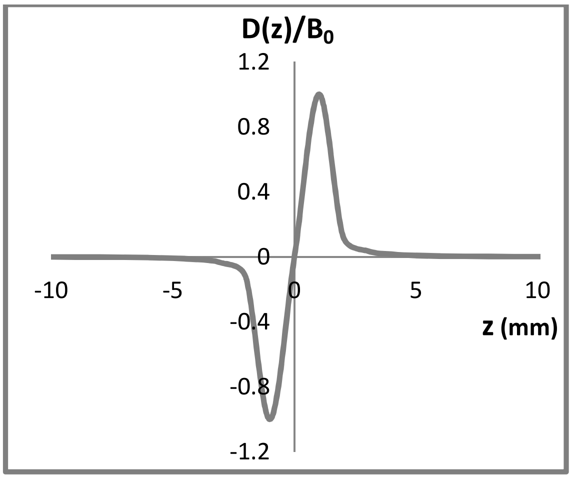
Zero	Methods	f/z_f	C_s/z_f	C_c/z_f	C_s/f	C_c/f	C_m	C_θ
First	a	0.28770505	0.072420788	0.16666670				
	b	0.28770505	0.072420784	0.16666667				
	c	0.29	-0.072*	0.166				
	d	0.28770505	0.07242078	0.16666665	0.25171885	0.5792969	0.5792969	1.3287528
Second	a	0.22321278	0.071672189	0.16666666				
	b	0.22321278	0.071672190	0.16666667				
	c	0.22	-0.072*	0.166				
	d	0.223212782	0.07167219	0.16666665	0.32109357	1.1066775	0.7466715	2.8892699
Third	a	0.19336092	0.071537400	0.16666665				
	b	0.19336092	0.071537398	0.16666667				
	c	0.19	-0.072*	0.166				
	d	0.193360918	0.0715374	0.16666665	0.36996824	1.5881163	0.8619459	4.4567831

4-3 Magnetic Deflectors

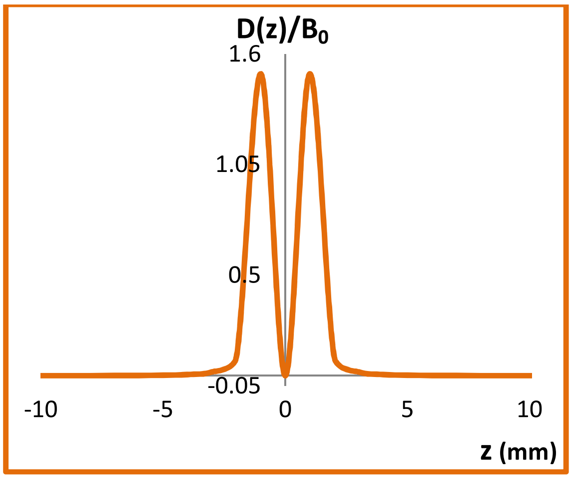
4-3-1 Magnetic field distribution by using the MOL concept

The MOL concept is used to find the deflection field of magnetic deflector. By using the MOL, one can find the design of magnetic deflector by knowing the design of magnetic lens or the axial magnetic field distribution of lens, depending on Eq. (2-6). In the present work, the field of magnetic deflector is found by knowing the axial magnetic field distributions of lens which are given by Eq. (2-3).

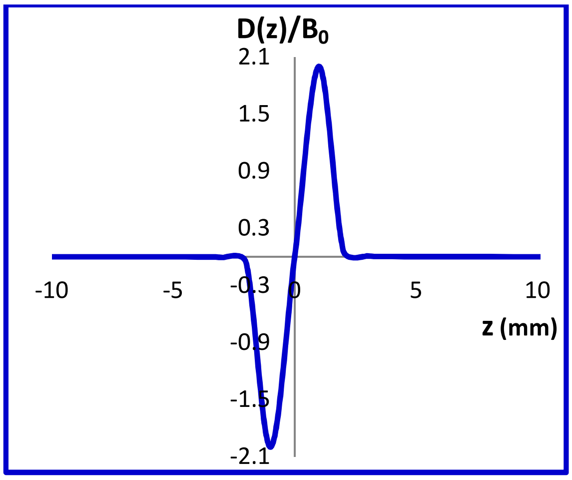
The axial flux density distribution of deflector D(z) is computed by applying Eq. (2-6) where $B'(z)$ is computed with the aid of Eq. (2-3). Then, the fields of magnetic deflector are shown in figure (4-6) for each value of n at constant (a) and (d).



(a) For n=2.



(b) For n=3.



(c) For n=4.

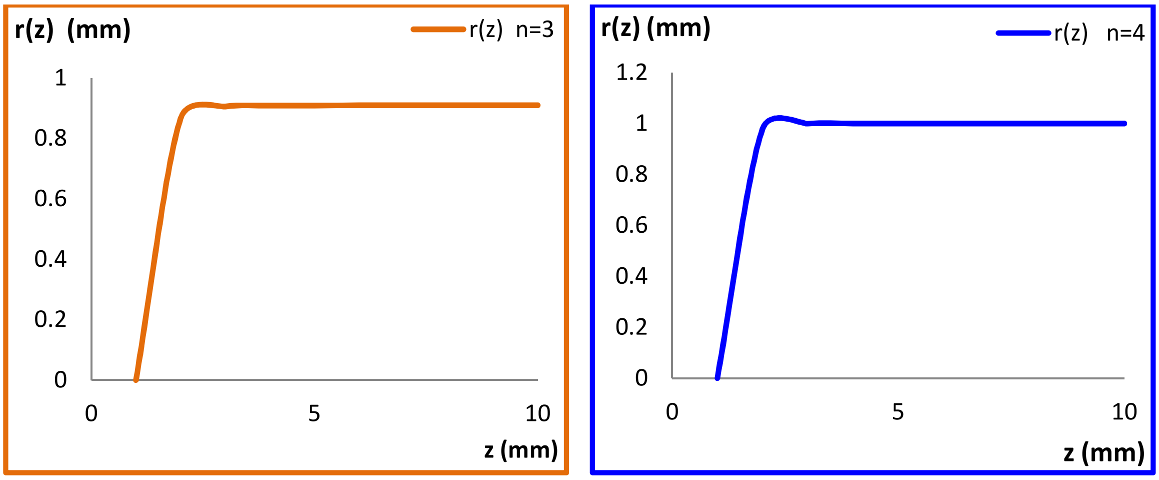
Figure (4-6): The axial flux density distributions of magnetic deflector with the fields of the form $D(z) \propto z^{-n-1}$ for n=2, 3 and 4.

4-3-2 Trajectory of electron beam

By applying Eq. (3-9)-Eq. (3-11) for n=2, 3 and 4, respectively, the trajectories of electron beam a long magnetic fields of deflectors can be computed at the value of the excitation parameter equal 13.424 ($\frac{\text{Amp.turn}}{(\text{Volt})^{0.5}}$) (i.e. k=5). The trajectory of electron beam when n = 2 for magnetic deflector is the same as the trajectory of electron beam for magnetic lens when n=3. This case occurs because of the field distribution of deflector in this case is the same as the field distribution of lens, so the trajectory in this case was shown previously by figure (4-2b). While the trajectories of electron beam for the case n=3 and 4 are shown in figure (4-7). The initial conditions for computing the trajectory of electron beam are given by:

$$\lim_{g(z_0)\to 0} g(z_0) = 1 \qquad \lim_{g(z_0)\to 0} g'(z_0) = 0$$

$$\lim_{h(z_0)\to 0} h(z_0) = 0 \qquad \lim_{h(z_0)\to 0} h'(z_0) = 1$$



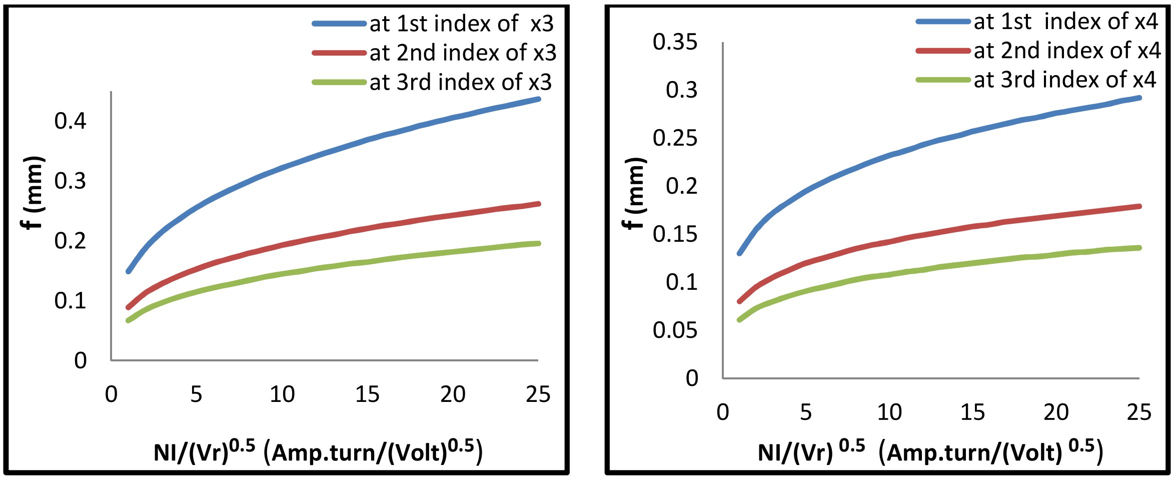
(a) For n=3. (b) For n=4.

Figure (4-7): Trajectories of electron beam at the excitation parameter 13.424 ($\frac{\text{Amp.turn}}{(\text{Volt})^{0.5}}$) for magnetic deflector with the field distributions of the form D(z) ∝ z^{-n-1} for n=3 and 4.

From this figure and figure (4-2b), one can note that the behavior of trajectory of electron beam in magnetic deflectors which is computed by using MOL concept like the behavior of trajectory of electron in magnetic lenses. This case occurs because of the magnetic fields in magnetic deflector by following MOL concept is a derivative of the magnetic field of the lens. At constant (d), When compute the magnetic field of deflector when n=2, it appears the magnetic field of the lens when n = 3 and therefore it will get on the same the trajectory of electron. While in the case of magnetic field of deflector when n=3, it almost the same as the magnetic field of the lens when n=4, so the trajectory of the electron almost the same, but there are some difference, ect.

4-3-3 Focal length

By employing Eq. (3-28), Eq. (3-30) and Eq. (3-31) for n=2, 3 and 4, respectively, the focal length for magnetic deflector at the focus has been computed. The focal length when n=2 for magnetic deflector is the same as the focal length for magnetic lens when n=3, so it was shown previously by figure (4-3b). While for the case n=3 and n=4, the focal lengths are shown in figure (4-8).



(a) For n=3. (b) For n=4.

Figure (4-8): Focal length at the focus for first three indices of the zero as a function of excitation parameter $\left(\frac{NI}{\sqrt{V_r}}\right)$ for magnetic deflectors with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=3 and 4.

Figure (4-8) and figure (4-3b) indicate that the focal length for each value of n is directly proportional with excitation parameter. Moreover, the focal length for magnetic deflector for each case of n has lowest value at third index of the zero corresponding to each value of x (x_2, x_3 and x_4). This behavior is like the behavior which occurs in magnetic lenses.

The comparison among figures (4-3b), (4-8a) and (4-8b) shows that the increasing of the power n leads to decrease the focal length. This case occurs because the focal lengths are proportional with $k^{1/2}, k^{1/3}$ and $k^{1/4}$ for n=2, 3 and 4, respectively. Accordingly, they are proportional with $(\frac{NI}{\sqrt{V_r}})^{1/2}, (\frac{NI}{\sqrt{V_r}})^{1/3}$ and $(\frac{NI}{\sqrt{V_r}})^{1/4}$ for n=2, 3 and 4, respectively.

The relative focal length f/z_f is computed for magnetic deflectors when n=3 and 4. While the relative focal length for magnetic deflector when n=2 is the same as the relative focal length for magnetic lens when n=3. The results of calculations are shown in table 9.

Table 9: The relative focal length, at the focus for first three indices of the zero, of magnetic deflector with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=2, 3 and 4 (note that the results were calculated to the eighth decimal order).

Index of the zero	$D(z) \propto z^{-3}$ (n=2)	$D(z) \propto z^{-4}$ (n=3)	$D(z) \propto z^{-5}$ (n=4)
First	0.44785219	0.31666038	0.21180208
Second	0.37244770	0.24567746	0.15842764
Third	0.33507053	0.21282123	0.13459751

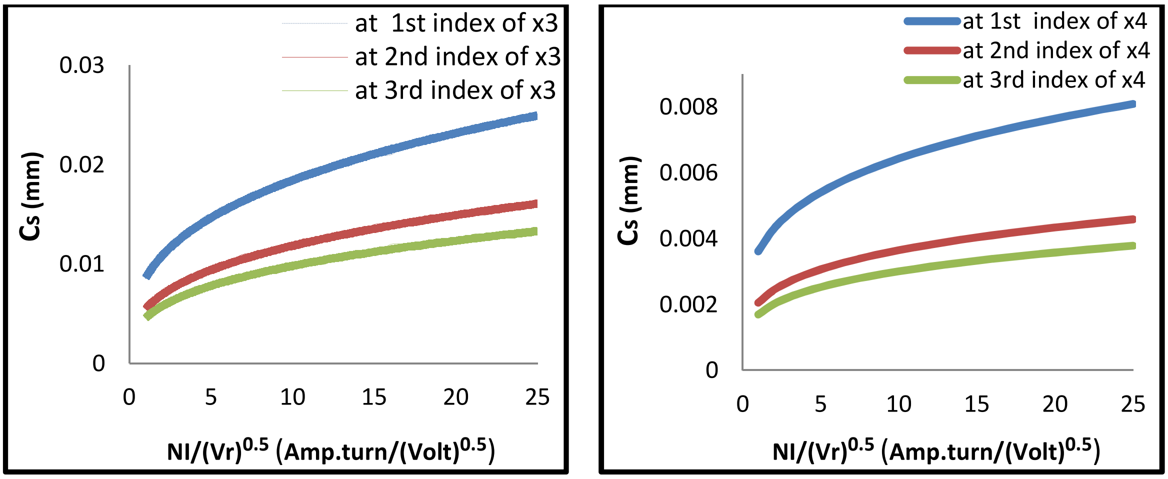
It is seen from table 9 that the relative focal length for each value of n has the lowest value at third index for each one of x (x_2, x_3 and x_4).

If the magnetic deflectors are compared with each other as in table 9, one can observe that the increasing of the power n leads to decrease the relative f/z_f . This behavior is like the behavior that it is occurred in magnetic lenses.

4-3-4 Aberration coefficients for magnetic deflectors

4-3-4-1 spherical aberration coefficient

The spherical aberration coefficients at the focus have been computed using Eq. (3-42), Eq. (3-46) and Eq. (3-47) for n=2, 3 and 4, respectively. The spherical aberration coefficient for deflector when n=2 is the same as the spherical aberration coefficient for magnetic lens when n=3, so it was shown previously by figure (4-4b). While the results of calculations for these coefficients when n=3 and 4 are shown in figure (4-9).



(a) For n=3. (b) For n=4.

Figure (4-9): The spherical aberration coefficients at the focus for first three indices of the zero as a function of excitation parameter ($\frac{NI}{\sqrt{V_r}}$) for magnetic deflectors with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=3 and 4.

The spherical aberration coefficient for each value of n is directly proportional with excitation parameter. This is confirmed by figures (4-4b) and (4-9). Also, the spherical aberration coefficient for each value of n has lowest value at third index for each one of x (x_2, x_3 and x_4). This behavior is like the behavior which is occurred in magnetic lenses.

By comparison among figures (4-4b), (4-9a) and (4-9b), one can observe that the increasing of the power n leads to decrease the spherical aberration coefficient. This case occurs because the spherical aberration coefficients are proportional with

$k^{1/2}$, $k^{1/3}$ and $k^{1/4}$ for n=2, 3 and 4, respectively. Consequently, they are proportional with $(\frac{NI}{\sqrt{V_r}})^{1/2}$,$(\frac{NI}{\sqrt{V_r}})^{1/3}$ and $(\frac{NI}{\sqrt{V_r}})^{1/4}$ for n=2, 3 and 4, respectively.

In the following table, the relative spherical aberration coefficient C_s/z_f and C_s/f are computed for n=3 and 4. While the relative spherical aberration coefficient for deflector when n=2 is the same as the relative spherical aberration coefficient for magnetic lens when n=3. The results of calculations are shown in the table 10.

Table 10: The relative spherical aberration coefficients $C_s\backslash z_f$ and $C_s\backslash f$, at the focus for first three indices of the zero, of magnetic deflector with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=2, 3 and 4 (note that the results were calculated to the eighth decimal order).

Magnetic deflectors	Index of the zero	$C_s\backslash z_f$	$C_s\backslash f$
$D(z) \propto z^{-3}$ (n=2)	First	0.10269968	0.22931602
	Second	0.10066346	0.27027542
	Third	0.10029476	0.29932432
$D(z) \propto z^{-4}$ (n=3)	First	0.0180782	0.05709018
	Second	0.01508382	0.06139685
	Third	0.01454465	0.06834212
$D(z) \propto z^{-5}$ (n=4)	First	0.00587673	0.02774631
	Second	0.00406311	0.02796766
	Third	0.00373729	0.03027953

The table 10 shows that the relative spherical aberration coefficient $C_s\backslash z_f$ for each value of n has lowest value at third index for each one of x (x_2, x_3 and x_4).

The comparison of magnetic deflectors with each other as in table 10 shows that the increasing of the power n leads to decrease $C_s\backslash z_f$.This behavior is like the behavior that it occurs in magnetic lenses.

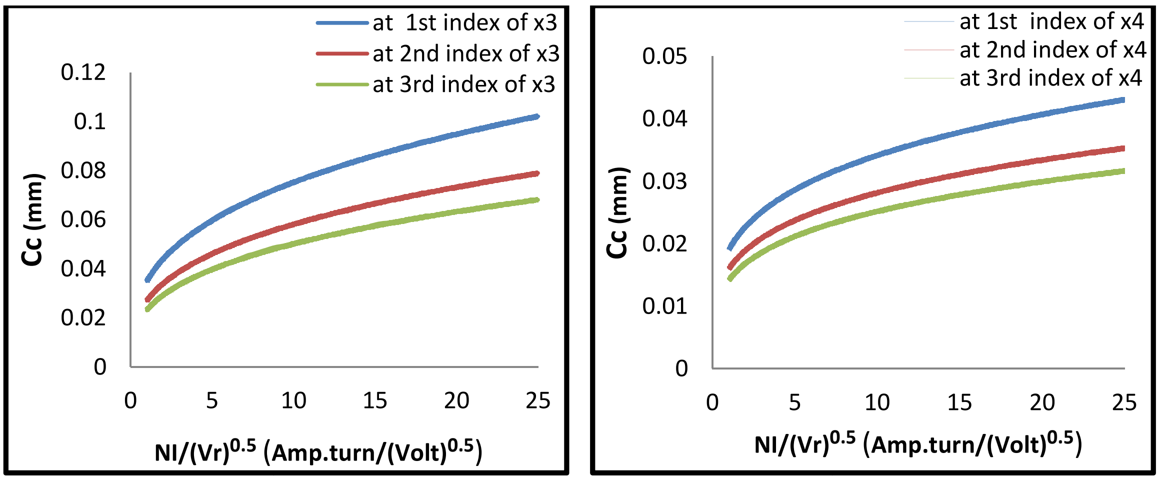
On the other hand, the relative spherical aberration coefficient $C_s\backslash f$ for each value of n has lowest value at first index for each one of x (x_2, x_3 and x_4).

From comparison of magnetic deflectors with each other as in table 10, one can note that the increasing of the power n leads to decrease $C_s\backslash f$. This behavior is different to the behavior which occurs in magnetic lenses.

4-3-4-2 chromatic aberration coefficients

A-axial chromatic aberration coefficient

To compute the axial chromatic aberration coefficient at the focus, Eq. (3-55), Eq. (3-58) and Eq. (3-59) are applied for n=2, 3 and 4, respectively. The axial chromatic aberration coefficient for deflector when n=2 is the same as the axial chromatic aberration coefficient for magnetic lens when n=3, so it was shown earlier by figure (4-5b). While the axial chromatic aberration coefficients for the case n=3 and 4 are shown in figure (4-10).



(a) For n=3. (b) For n=4.

Figure (4-10): The axial chromatic aberration coefficients at the focus for first three indices of the zero as a function of excitation parameter ($\frac{NI}{\sqrt{V_r}}$) for magnetic deflectors with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=3 and 4.

From this figure and figure (4-5b), one can note that the axial chromatic aberration coefficient for each value of n is directly proportional with excitation parameter. As well, the axial chromatic aberration coefficient for each value of n has lowest value at third index for each one of x (x_2, x_3 and x_4). This behavior is like the behavior that it is occurred in magnetic lenses.

If figures (4-5b), (4-10a) and (4-10b) are compared, they show that the increasing of the power n leads to decrease the axial aberration coefficient. This case occurs because the axial chromatic aberration coefficients are proportional with $k^{1/2}$, $k^{1/3}$ and $k^{1/4}$ for n=2, 3 and 4, respectively. Consequently, they are proportional with $(\frac{NI}{\sqrt{V_r}})^{1/2}$,$(\frac{NI}{\sqrt{V_r}})^{1/3}$ and $(\frac{NI}{\sqrt{V_r}})^{1/4}$ for n=2, 3 and 4, respectively.

The relative axial chromatic aberration coefficient C_c/z_f or C_c/f are computed for n=3 and 4. While the relative axial chromatic aberration coefficient for deflector when n=2 is the same as the relative axial chromatic aberration coefficient for magnetic lens when n=3. The results of the calculations for these relative coefficients are shown in table 11.

Table 11: The relative axial chromatic aberration coefficients $C_c\backslash z_f$ and $C_c\backslash f$, at the focus for first three indices of the zero, of magnetic deflector with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=2, 3 and 4 (note that the results were calculated to the eighth decimal order).

Magnetic deflectors	Index of the zero	$C_c\backslash z_f$	$C_c\backslash f$
$D(z) \propto z^{-3}$ (n=2)	First	0.24999999	0.55821988
	Second	0.24999999	0.98207718
	Third	0.24999999	1.35169873
$D(z) \propto z^{-4}$ (n=3)	First	0.07407407	0.23392275
	Second	0.07407407	0.30150937
	Third	0.07407407	0.3480577
$D(z) \propto z^{-5}$ (n=4)	First	0.03125	0.14754338
	Second	0.03125	0.19725087
	Third	0.03125	0.23217366

As shown in the table 11, the relative axial chromatic aberration coefficient C_c/z_f for each case of n has constant value for each indices of the zero.

From comparison among the different magnetic deflector as in table 11, one can note that the increasing of the power n leads to decrease C_c/z_f. This behavior is similar to the behavior which occurs in magnetic lenses.

However, the relative axial chromatic aberration coefficient C_c/f for each case of n has lowest value at first index for each one of x (x_2, x_3 and x_4).

By comparing between the different magnetic deflector as in table 11, one can observe that the increasing of the power n leads to decrease C_c/f. This behavior is different to the behavior that it occurs in magnetic lenses.

B- magnification chromatic aberration coefficient

For computing the magnification chromatic aberration coefficients at the focus, one can be used Eq. (3-67), Eq. (3-70) and Eq. (3-71) for n=2, 3 and 4, respectively. The magnification chromatic aberration coefficient for deflector when n=2 is the same as the magnification chromatic aberration coefficient for magnetic lens when n=3. The results of calculations for these coefficients are shown in table 12.

Table 12: The magnification chromatic aberration coefficients, at the focus for first three indices of the zero, of magnetic deflector with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=2, 3 and 4 (note that the results were calculated to the eighth decimal order).

Index of the zero	$D(z) \propto z^{-3}$ (n=2)	$D(z) \propto z^{-4}$ (n=3)	$D(z) \propto z^{-5}$ (n=4)
First	0.55821988	0.23392275	0.14754338
Second	0.67123511	0.30150937	0.19725087
Third	0.74611155	0.3480577	0.23217366

The table 12 is made clear that the magnification chromatic aberration coefficients at the focus for each value of n have constant value, but they are changed due to change the index of the zero. The reason of the magnification chromatic aberration coefficients have constant value is they are independent on $(\frac{NI}{\sqrt{V_r}})$. Also, the magnification chromatic aberration coefficient has lowest value at first index for each one of x (x_2, x_3 and x_4). This behavior is similar to the behavior which is occurred in magnetic lenses.

From comparison among the different magnetic deflectors as in table 12, one can be noticed that the increasing of the power n leads to decrease C_m. This behavior is different to the behavior which occurs in magnetic lenses.

C- rotation chromatic aberration coefficient

From Eq. (3-75), Eq. (3-77) and Eq. (3-78) for n=2, 3 and 4, respectively, one can be computed the rotation chromatic aberration coefficients at the focus. The rotation chromatic aberration coefficient for deflector when n=2 is the same as the rotation chromatic aberration coefficient for magnetic lens when n=3. The results of calculations for these coefficients are shown in table 13.

Table 13: The rotation chromatic aberration coefficients, at the focus for first three indices of the zero, of magnetic deflector with the field distributions of the form $D(z) \propto z^{-n-1}$ for n=2, 3 and 4 (note that the results were calculated to the eighth decimal order).

Index of the zero	$D(z) \propto z^{-3}$ (n=2)	$D(z) \propto z^{-4}$ (n=3)	$D(z) \propto z^{-5}$ (n=4)
First	1.39044385	0.8858352	0.64878
Second	2.9530713	1.92617997	1.428585
Third	4.52119185	2.97118873	2.2122225

It is appeared from this table that the rotation chromatic aberration coefficient at the focus for each value of n has constant value corresponding to each one of index of the zero, this mean that it is independent on $(\frac{NI}{\sqrt{V_r}})$. Moreover, this coefficient has lowest value at first index for each one of x (x_2, x_3 and x_4). This behavior is like the behavior that it occurs in magnetic lenses. By comparing among different magnetic deflector as in table 13, one can note that the increasing of the power n leads to decrease C_θ.

The results of calculation for magnetic deflectors at the focus for each case of n, which are computed by analytical method in the present work, are shown in table 14, 15 and 16.

Table 14: The optical properties of magnetic deflector which has the field in the form $D(z) \propto z^{-n-1}$ with n=2 at the focus calculated by using analytical method.

Zero	f/z_f	C_s/z_f	C_c/z_f	C_s/f	C_c/f	C_m	C_θ
First	0.44785219	0.10269968	0.24999999	0.22931602	0.55821988	0.55821988	1.39044385
Second	0.37244770	0.10066346	0.24999999	0.27027542	0.98207718	0.67123511	2.9530713
Third	0.33507053	0.10029476	0.24999999	0.29932432	1.35169873	0.74611155	4.52119185

Table 15: The optical properties of magnetic deflector which has the field in the form $D(z) \propto z^{-n-1}$ with n=3 at the focus calculated by using analytical method.

Zero	f/z_f	C_s/z_f	C_c/z_f	C_s/f	C_c/f	C_m	C_θ
First	0.31666038	0.0180782	0.07407407	0.05709018	0.23392275	0.23392275	0.8858352
Second	0.24567746	0.01508382	0.07407407	0.06139685	0.30150937	0.30150937	1.92617997
Third	0.21282123	0.01454465	0.07407407	0.06834212	0.3480577	0.3480577	2.97118873

Table 16: The optical properties of magnetic deflector which has the field in the form $D(z) \propto z^{-n-1}$ with n=4 at the focus calculated by using analytical method.

Zero	f/z_f	C_s/z_f	C_c/z_f	C_s/f	C_c/f	C_m	C_θ
First	0.21180208	0.00587673	0.03125	0.02774631	0.14754338	0.14754338	0.64878
Second	0.15842764	0.00406311	0.03125	0.02796766	0.19725087	0.19725087	1.428585
Third	0.13459751	0.00373729	0.03125	0.03027953	0.23217366	0.23217366	2.2122225

Chapter One

Introduction

It is axiomatic that any device which permits the discernment of details finer than those that are visible with the naked eye, is of great scientific value. Thus the discovery of the electron optics has initiated a new era of investigation into many aspects of physical and biological science.

In 1828 Hamilton was noted that there is the close analogy between classical mechanics and geometrical light. However at the third decade of the last century, this analogy strongly leads to give birth to a new branch of physics called later by charged particle optics or most common by Electron Optics [El-Kareh and El-Kareh 1970].

Electron optics, in fact, is based on two fundamental discoveries made in 1925 by de Broglie and in 1927 by Busch. De Broglie postulated on ground of theoretical considerations that one must attribute a wave with each moving particle. At about the same time, Busch discovered that the magnetic field of solenoid acts on electrons in exactly the same way as a glass lens on the light rays. Actually these two essential discoveries led Ruska to conclude the possibility to build up a microscope which uses electrons instead of photons. Hence, he successfully realized the first transmission electron microscope (TEM) in 1931 [Harald 2008].

Therefore, electron optics becomes a theory and practice of production, controlling and utilization of charged particle beam. In other word, one may say that the branch of physics that deals with the problems of charged particle beams motion throughout electromagnetic field is known as electron optics.

With the analogy between light and electron optics there are fundamental limitations that should be taken into account. Some of these limitations are listed as follows [El-Kareh and El-Kareh 1970]:

1- In light optics, refractive index of light lenses changes abruptly between materials of different indices of refraction. In electron optics, the changes are continuous.
2- Both the energy and momentum of the electron are continuously variable and can be changed arbitrarily. This is not the case in light optics.
3- A good vacuum must be satisfied for traveling of charged particle beams due to the rapid absorption and scattering of particles by gases, while light rays are free.
4- Almost all lenses in electron optics are convergent, while in light optics convergent as well as divergent lenses are used.

In many electron beam instruments, such as scanning electron microscopes and scanning electron beam lithography systems are usually use a magnetic lens to focus a charge particle beam and magnetic deflection coils mounted within the lens to purpose scanning the beam over a surface.

The most common and classical type of deflection is used in cathode ray tubes, lithography machines, scanning electron microscopes, electron accelerators, electron-beam manufacturing technologies and some other analytical instruments [Szilagyi 1988].

In the present work, we will address only to the magnetic lens and deflector in detail but for the other types of lenses and electrostatic deflector can be seen for example; [El-Kareh and El-Kareh 1970; Szilagyi 1988; Hawkes and Kasper 1989].

1-2 Electron Lenses

Electron lens, in general, can be defined as an instrument which collects a moving beam of charged particles or focuses them to the same point. The electron lens acts on the charged particle beams (electrons or ions) to be focused and imaged similar to that of glass lens on the light. A set of electrodes held at suitable distances and voltages will be produced electrostatic fields, while the magnetic fields can be produced by current-carrying coils [Szilagyi 1988].

The main three types of the electron lenses are electrostatic lenses, permanent magnetic lenses and magnetic lenses.

1-2-1 Magnetic lenses

Any axially symmetric magnetic field produced by circular coils with or without ferromagnetic materials acts as a magnetic lens [Szilagyi 1988]. When the electrons travel through the magnetic field in electron optical systems, this field enables electrons to be focused and imaged. This means that a magnetic field has an imaging property.

The optical properties of the magnetic lenses are dependent on the charge-to-mass quotient of the particle [Szilagyi 1988].

There are two effects of a magnetic electron lens on the moving electron beams, the first one, is a deflection towards the optical axis identically to the focusing effect of a converging lens in light optics. The second is an additional rotation around the optical axis [Labar 2002].

1-2-2 Common types of magnetic lenses

The magnetic lenses can be classified from many different points of view, for example; one can talk about bounded lenses or lenses immersed in fields, whether the boundaries of the lens can or cannot found; strong or weak lenses depending on whether their focal points are situated inside or outside the field; thick or thin lenses; symmetrical or asymmetrical lenses depending upon whether there exists middle plane perpendicular to the optical axis about which the geometrical arrangement of the lens is symmetrical or not [Hawkes 1982; Szilagyi 1988].

Magnetic electron lenses can be classified according to the number of their polepieces into three types: single polepiece lens, double polepiece lens and triple polepiece lens. Also iron-free lenses are one of magnetic lenses types. So these types will be explained as the following:

1-2-2-1 double polepiece magnetic lens

This type of magnetic lenses consists of insulated wire or tape windings made of conducting material (usually copper) surrounded by a ferromagnetic material core of high magnetic permeability which was designed by Ruska in 1933.

The core has coaxial circular bore of diameter *“d”* along the optical axis to allow the electrons beam to pass through an air gap of width “S” formed in the iron circuit between the two polepieces (Figure 1-1). The properties of these lenses are expressed in terms of the ratio (S/d) [Liebmann 1952].

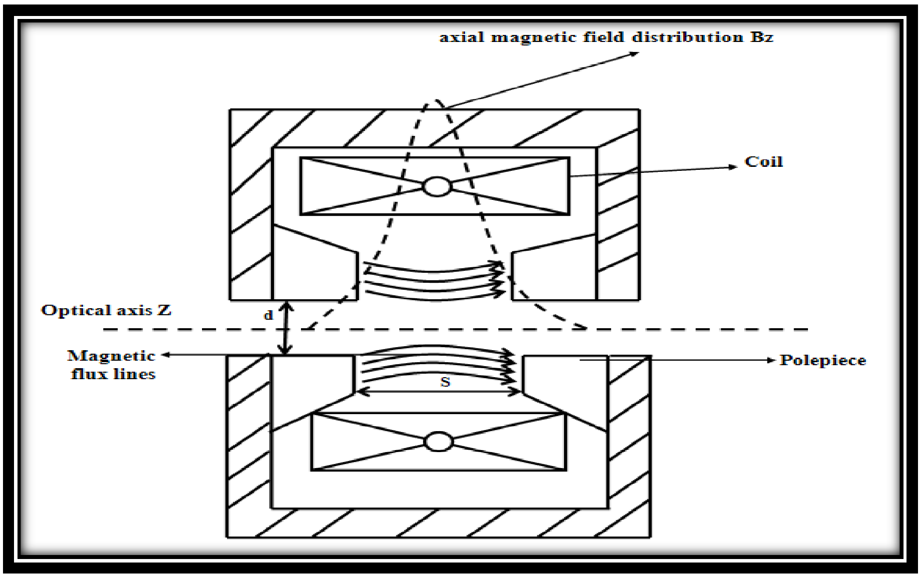


Figure (1-1): A schematic diagram of double polepiece magnetic lens [Ahmed 2007].

Symmetrical double polepiece magnetic lens could be achieved when the axial bores between the two polepieces are identical, while asymmetrical lens be held when the axial bores are not identical.

1-2-2-2 single polepiece magnetic lens

In 1972 Mulvey introduced a new design of magnetic lens named ‘Snorkel’ lens. When the double polepiece magnetic lens is divided into two halves from the middle a single polepiece magnetic lens can be obtained by removing one half. The single polepiece magnetic lens has been taken a great interest in the electron optical instruments [Mulvey and Newman 1973; 1974]. The absence of the bore in the single polepiece lens makes fabrication of the lens easier. The single polepiece lens has the advantage that the entire lens can be physically situated outside the vacuum chamber (Figure 1-2). The axial magnetic flux density distribution of the single polepiece lens can be pushed away from the lens profile itself making the optical properties of the lens less dependent on the imperfections in the iron circuit and the energizing coil.

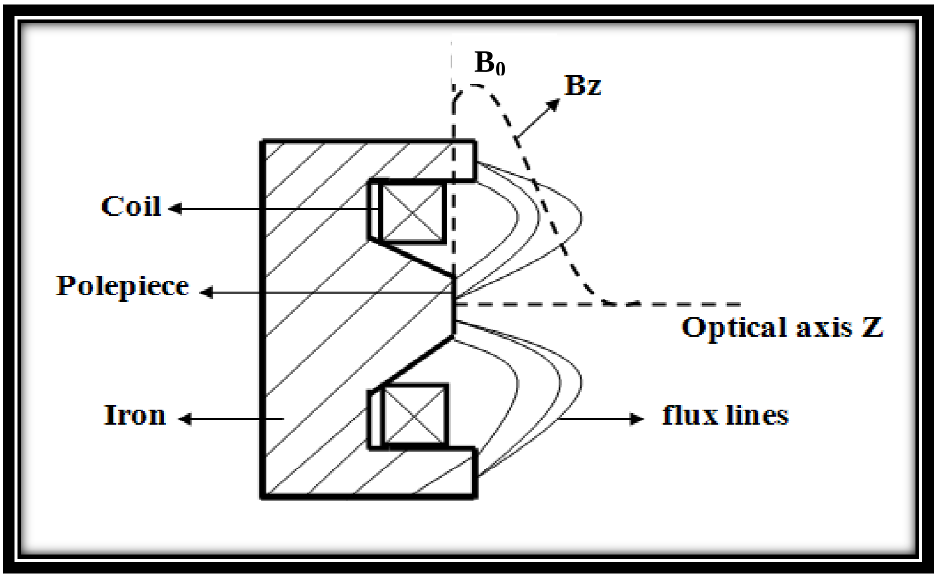


Figure (1-2): Cross-section of single polepiece magnetic lens [Ahmed 2007].

1-2-2-3 triple polepiece magnetic lens

Doublet lens [Juma 1975] consists of two magnetic electron lenses of two air gaps and it is also called triple polepiece magnetic lens [Tsuno and Harada 1981]. The two magnetic lenses of the triple lens may be symmetrical or asymmetrical depending on the design of each lens (Figure 1-3).

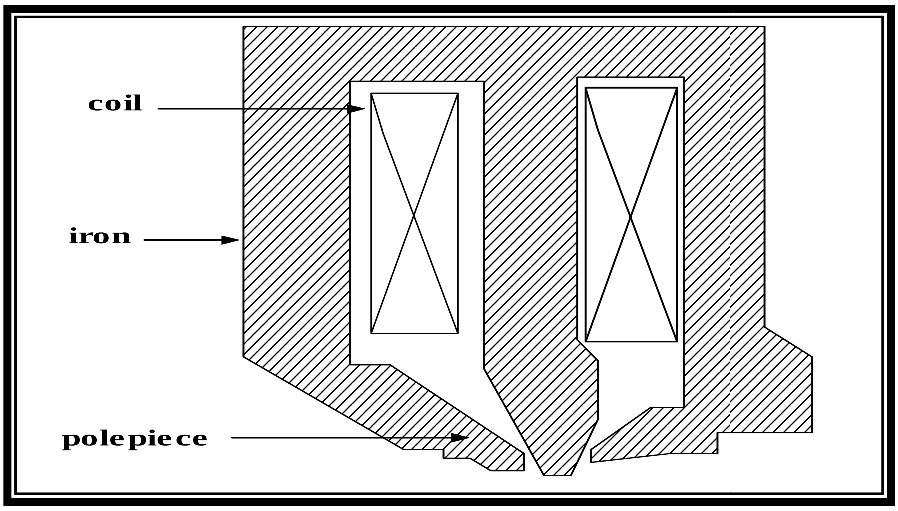


Figure (1-3): Cross-section of triple polepiece magnetic lens [Tsuno and Harada 1981].

1-2-2-4 iron-free lens

This lens is the simplest probe-forming device; it consists of coils made from metallic conductor wire or tape windings wound on non-magnetic core (Figure1-4).

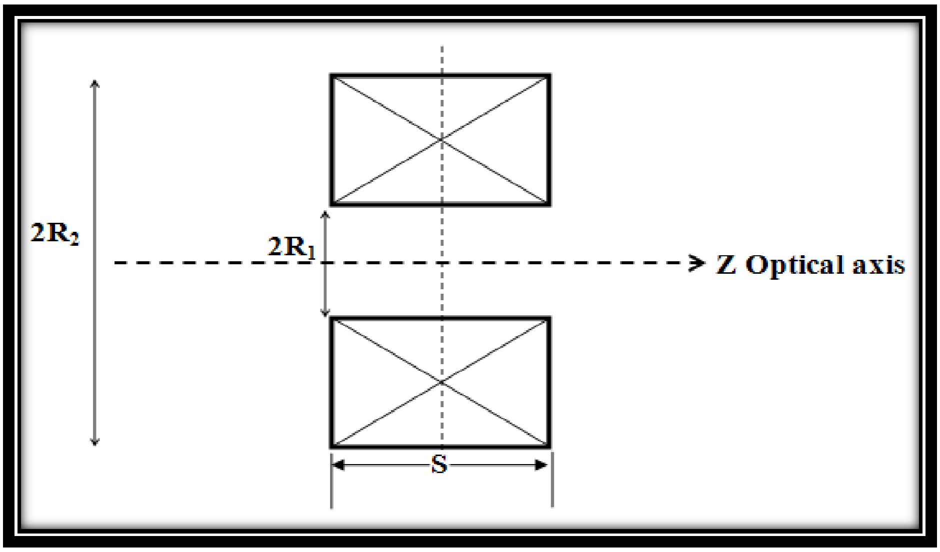


Figure (1-4): Iron-free rectangular cross-sections [Alamir 2000].

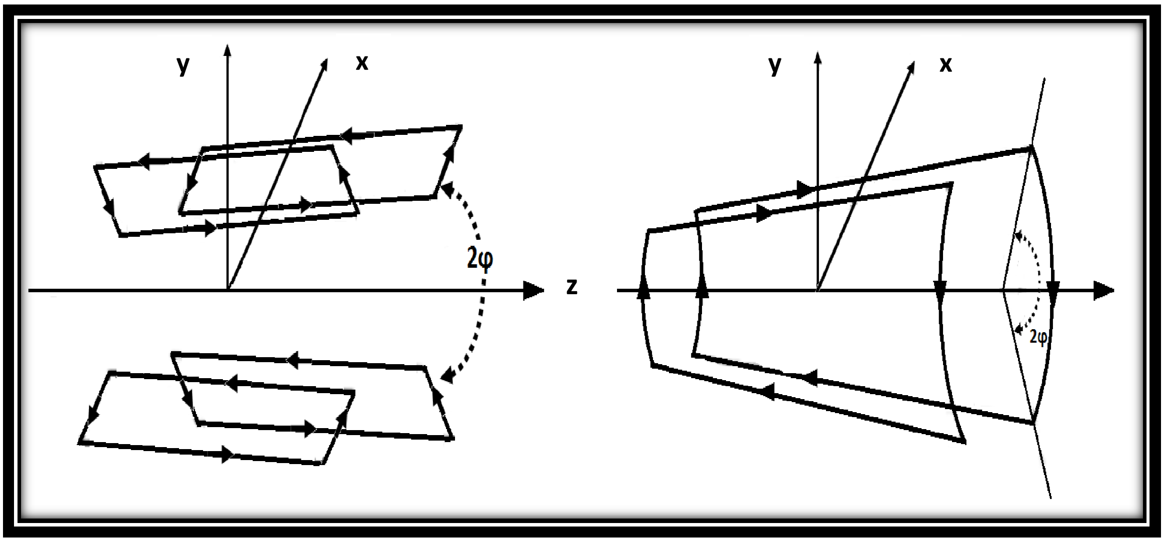
One advantage of the iron-free coils is their size reduction with respect to the iron circuit lenses. It is therefore desirable to describe some recent investigations of iron-free objective lenses that offer the possibility of developing electron optical instruments, both with and without the use of superconducting windings [Alamir 2000].

1-3 Deflection Systems

A deflection system is an arrangement of electrodes or coils by means of which it is possible to exert an influence on the path of an electron ray. The roles of magnetic deflection systems are different and these depend on the function of magnetic device. For example; in fixed-beam instruments, essentially in conventional transmission electron microscopes, deflection plays a minor role and is provided only to permit nonmechanical alignment of the column [Hawkes and Kasper 1989].

However, in scanning devices, the deflection systems are needed here for two purposes: one is deflection of the focused probe over the specimen in order to form an image or to position it at a particular point in order to make a measurement, and the other is for beam-blanking purposes. The former function is invariably carried out by magnetic deflection, whereas the latter is usually achieved through electrostatic deflection [Khursheed 2011]. Here, the design of deflection systems is at least as important as that of the lenses [Hawkes and Kasper 1989].

As mentioned before, the present study is concerned with the magnetic deflection only. A magnetic deflector usually has current-carrying conductors arranged on a surface of revolution about z-axis in the form of a cylinder tapered horn. The winding may be in the form of either a saddle or toroidal type, and the deflector itself can be placed next symmetric magnetic materials. A schematic diagram for toroidal and saddle deflection yoke are shown in figure (1-5). The arrows indicate the direction of current flow [Khursheed 2011].



Toroidal Coils Saddle Coils

Figure (1-5): Schematic drawing of primary beam magnetic deflectors [Khursheed 2011].

A significant difference between electrostatic and magnetic deflector is their relative deflection sensitivity. Electrostatic deflection has a low deflection sensitivity and thus needs high driving voltages, while magnetic field has a high deflection sensitivity and thus requires low driving currents. Scan generators for magnetic coils are simpler than electrostatic ones. The major disadvantage of a coil comes from its eddy currents, which limit its scan frequency [Khursheed 2011].

The study of the optics of deflection systems passes through essentially the same stages as those already encountered for round lenses or quadruples; the novel aspects arise from the new symmetry conditions [Hawkes and Kasper 1989].

1-4 Historical Development

1-4-1 Magnetic lenses

The theory and practice of electron focusing was developed principally with the needs of electron microscopy. In particular transmission electron microscopy, the electrons must pass through the electron optical system unimpeded and this precluded the use of any physical object on the axis. More recently, there had evolved a need for the use of low voltage electrons in the SEM. These electrons are so slow that they do not pass through the specimen and this means that the incident beam and the electrons containing the image information all occur in one hemisphere. The region below the specimen is then made available and there is no longer any restriction on placing iron on the axis [Crewe 2001].

This opens up the possibilities for new lenses and there is a wide variety of lens designs that could be used.

Mulvey studied the use of a single pole electromagnetic lens and indicated that it had good electron optical properties. He also indicated that a dipole magnetic field might be a good model for the field produced by such a lens, although that field would be severely modified by the coil excitation [Hawkes 1982].

In looking for new lens designs it was guided by the observation that if it was able to make a hyperbolic magnetic field ($B(z) \propto 1/z$), the trajectory that focuses at the singularity is a parabola and then the integrand for Cs is zero [Crewe 1977]. It could not make such a field, but one could make fields that approximate monopoles which the field in the form $B(z) \propto 1/z^2$, dipoles which the field in the form $B(z) \propto 1/z^3$ etc.

In 1996 Crewe and Kielpinski found that the focal length, spherical and chromatic aberration coefficients of dipoles magnetic lenses could be expressed simply when they were normalized to the dipole moment [Crewe and Kielpinski 1996]. Subsequently, Crewe extended the study to two other members of the multipole magnetic lenses family through numerical and digital methods and obtained some interesting results on the electron optical properties of monopole, dipole and quadrupole lenses [Crewe 2001].

In 2002 Hawkes used the properties of Bessel function to predict the most of Crewe's finding for the same magnetic lenses. In this study, he included the analytical method to obtain on the optical properties which found by Crewe [Hawkes 2002].

One year later, the optical properties of monopole, dipole and quadrupole magnetic lenses had been demonstrated by Liu. The differential algebraic (DA) method and analytical expression were adopted in this study [Liu 2003].

Meanwhile, the rotation free-system of projector magnetic lenses in the form of an inverse power law was studied by Alamir and the value of chromatic change of magnification and rotation were estimated [Alamir 2003a].

Also, the spiral distortion in for such projector magnetic lenses had been studied by Alamir [Alamir 2003b].

In 2004 Crewe studied the focusing properties of magnetic fields of the form $B(z) \propto z^n$ for all integer n except the cases when n=-1and n=0. The calculations in this study by using numerical ray tracing were carried out for first and second indexes of the zero only [Crewe 2004].

In the same year, Alamir studied the chromatic aberration coefficients for objective and projector magnetic lenses with the field distribution in the forms of inverse power law by using analytical and digital methods. Also, the chromatic aberration coefficients for one of the theoretical models that represent the single-pole magnetic lens were calculated to express the magnitude analytically [Alamir 2004].

One year later, the spherical and chromatic aberration coefficients for the same magnetic field model were calculated by Alamir to express the magnitude analytically. The results for this study were presented in a Tretner's form to determine the optimum performance of magnetic lenses [Alamir 2005].

Subsequently, the performance of magnetic lenses in the form $B(z) \propto z^n$, where n has positive integer, had been studied by Alamir in 2009. By using Tretner's form, the different objective lenses had been compared in this study [Alamir 2009a].

Also in 2009 by Alamir, the spiral and radial distortions of magnetic lenses with field distribution in the form $B(z) \propto z^n$ were analyzed by means of Scherzer's formula. In this study, the quality factor had been found for both the spiral and radial distortion to estimate the performance of the image in electron microscope for projector magnetic lenses [Alamir 2009b].

Later, the chromatic aberration coefficients for both objective and projector magnetic lenses which fields in the form $B(z) \propto z^n$ had been studied by Alamir [Alamir 2011].

1-4-2 Magnetic deflectors

New ideas had been introduced for magnetic deflector. Munro listed the geometric and chromatic aberration integrals for purely magnetic deflection and round lens systems and also derived the formulae for calculating the first-order optical properties [Munro 1974].

One year later, he introduced method for computing the optical properties of any combination of magnetic lenses and deflection yokes, including the most general case in which the lens and deflector fields may physically be superimposed [Munro 1975].

The general formulae had been expressed to include all possible focusing and deflection effects of both magnetic and electrostatic type. These formulae were derived by Soma [Soma 1977].

A systematic analysis of aberration for post-lens deflection, double deflection before the lens and “moving objective lens” (MOL) had been found by Ohiwa [Ohiwa 1978, 1979 and Ohiwa et al. 1971].

Also, the aberration coefficients of a double-deflection unit for which the second deflector coincides with a round magnetic lens field were listed in Kuroda [Kuroda 1980].

Lencova found the more general formulae for adding deflections aberrations [Lencova 1981]. These formulae were applied to several practical situations by Lencova [Lencova 1988].

An electron optical focusing and deflection system for electron beam lithography had been developed by Pfeiffer et al. which eliminates off-axis aberrations up to the third order including transverse chromatic errors by means of a variable axis lens (VAL) [Pfeiffer et al. 1981].

The series of papers had been introduced by Munro and Chu [Munro and Chu 1982 I, II, III and IV], the first two papers devoted to the numerical analysis of electron beam lithography systems so they concerned with field calculation. While the third gave a list of aberration integrals which could be used to study systems consisting of any combination of magnetic and electrostatic lenses and deflectors and the fourth concerned with computation optimization of complex systems.

Subsequently, the fifth order aberration coefficient formulae for deflective focusing systems had been derived by several researchers; [Kangyan and Tang 1999; Li et al. 1993 and Uno et al. 1995].

Meanwhile, variational deflection aberration theory had been further developed by Ximen et al. for deflection systems with curved axes at extra-large deflection angles (up to 120^{o}). The variational method allowed us to calculate second- and third-order deflection aberrations with respect to a curved axis by means of gradient operations on eikonal (the function of optical length) [Ximen et al. 1995].

One year later, a unified deflection aberration theory had been developed by Ximen et al. for nonhomogeneous magnetic deflection system with curvilinear or rectilinear axis. By using variable method, primary-order deflection aberrations with respect to curvilinear or rectilinear axis could be universally calculated by means of gradient operations on eikonal [Ximen et al. 1996a].

Also, by following the variational deflection aberration theory, a magnetic deflection system consisting of homogeneous deflection field and a homogeneous sextupole field had been further investigated by them. For such a magnetic deflection system, both the Gaussian trajectory and all second and third-order aberrations had been calculated analytically and expressed by algebraic-trigonometric formulae suitable for computer computations [Ximen et al. 1996b].

The variable axis lens (VAL) concept, which was based upon the power series expansion of the lens field, had been introduced by Zhao and Khursheed. Theoretically, this concept limited to situations where the focusing and in-lens deflection fields were of the same type, usually magnetic. In this study, a general VAL condition was derived from the paraxial trajectory equation that was applicable to any combined focusing and in-lens deflection system with mixed magnetic and electrostatic fields [Zhao and Khursheed 1999].

Wang et al. developed differential algebraic method (DA), which implement the DA method to arbitrary high order in visual C^{++}, and applied it to the analysis of electron lenses and deflection systems separately [Wang et al. 2000]. In fact the facility of differential algebraic method had been used widely by many researchers since the beginning of the present century see for example [Hosokawa 2002; Wang et al. 2002 and Kang et al. 2009].

The magnetic and electrostatic deflector by including the MOL concept had been studied by Oday in 2005. The optimum design of each one of them which give rise to the minimum spherical and chromatic aberration had been obtained. In this study, the saddle deflection coil used as the source of magnetic field and then the field distribution determined by using particular famous model such as Glaser bell shaped and Grivet models [Oday 2005].

The aberration theory of a new type of combined electron focusing-deflection system had been studied by Yan et al. [Yan et al. 2007].

One year later, the magnetic deflection and focusing system by following the MOL concept had been studied by Ahmed to find the optimum design of magnetic deflection and focusing system. In this study, the toroidal deflection coil used as the source of magnetic field and then the field distribution determined by using an exponential function [Ahmed 2008].

1-5 Aim of Work

The analytical method will be used in the present work to derive the formulae of optical properties for magnetic lenses with field distributions in the form $B(z) \propto z^{-n}$; n=2, 3, 4 and to derive the formulae of optical properties for magnetic deflector which magnetic fields will be computed by including MOL concept.

Also, the first- and third-order optical properties such as trajectory of electron beam, focal length, chromatic and spherical aberration coefficients for magnetic lens and deflector at the focus will be computed by using the deriving formulae. The results of calculations for these optical properties will be obtained by using MATHCAD 14 package, where the computer program is written to find the results. Also, the minimum values of aberration coefficients will be selected by taking different values of the power n and the index of the zero corresponding to each value of n.

1-6 Thesis Layout

The thesis is generally divided into five chapters with one appendix to supplement the calculation detail presented in the main body of the thesis. The contents of the various chapters are as follows:

The second chapter is devoted to the theoretical considerations in electron optics, each component of magnetic system (lens, deflector) is fully described in terms of different parameters such as trajectory of electron beam, position of foci, focal lengths and aberration coefficients. Furthermore, these parameters are detailed by the basic equation of electron optics appropriate to the situation under study.

In the chapter three, the formulae for expressing to each parameter are derived by implement the analytical method for every component of magnetic system.

In chapter four, the formulae which are derived in chapter three are used to find all parameters of magnetic system. Our results are obtained by using MATHCAD 14 package, where the computer program is written to find the results.

The fifth chapter has listed the remakes, interested observations and some of recommendations for future work.

The thesis ends with one appendix which includes the derivative of the trajectories equation of magnetic deflectors by following MOL concept.

Lastly, there is the bibliography.

Chapter Three

Mathematical Structures

3-1 Introduction

In many electron probe instruments, such as scanning electron microscopes, scanning electron beam microfabraction systems, an electron beam is focused on a surface by using a magnetic lens and simultaneously the beam is scanned across the surface by a magnetic deflector system [Munro 1974].

The lens and deflection fields in many systems are superimposed on each other. In such case, the effects of the lens and deflector fields are inseparable, and the system should be analyzed as a single entity. However, the lens and deflector are separate with studied theoretical basis. In this case, the properties of the lens and deflector can be calculated separately and the results can be cascaded to give the overall properties [Munro 1974].

In the present chapter, the formulae for calculating the optical properties of magnetic lens and deflector are derived separately using analytical method.

3-2 Trajectory Equations of The Electron Beam in The Axial Symmetric Magnetic System

The paraxial equation which describes electron trajectories in the axial symmetric magnetic system (lens and deflector) which is given by Eq. (2-1) can be solved in terms of Bessel functions for the fields are given by Eq. (2-3) and Eq. (2-6). For the magnetic lens, it was derived by Hansel [Hawkes and Kasper 1989].

3-2-1 Trajectory equations of the electron beam in magnetic lenses

The corresponding trajectories of electron beams in magnetic lenses for fields in the form $B(z) \propto z^{-2}$, $B(z) \propto z^{-3}$ and $B(z) \propto z^{-4}$ are given by [Hawkes 2002]:

$$r(z)= A\ z^{\frac{1}{2}}\ J_{\frac{1}{2}}\left(\frac{k}{z}\right) \quad \text{for monopole} \tag{3-1}$$

$$r(z)=B\ z^{\frac{1}{2}}\ J_{\frac{1}{4}}\left(\frac{k}{2z^2}\right) \quad \text{for dipole} \tag{3-2}$$

$$r(z)= C\ z^{\frac{1}{2}}\ J_{\frac{1}{6}}\left(\frac{k}{3z^3}\right) \quad \text{for quadrupole} \tag{3-3}$$

where A, B and C are constants. These constants can be inserted in previous trajectory equations when z is a large value [Hawkes 2002].

For the case n=2, the constant A becomes:

$$A=\left(\frac{2}{k}\right)^{\frac{1}{2}}\Gamma\left(\frac{3}{2}\right) \tag{3-4}$$

where $\Gamma(x)$ is Gamma function.

For the case n=3, the constant B becomes:

$$B=\left(\frac{4}{k}\right)^{\frac{1}{4}}\Gamma\left(\frac{5}{4}\right) \tag{3-5}$$

For the case n=4, the constant C becomes:

$$C=\left(\frac{6}{k}\right)^{\frac{1}{6}}\Gamma\left(\frac{7}{6}\right) \tag{3-6}$$

where k is given by [Hawkes 2002]:

$$k = \frac{\eta B_0}{2\sqrt{V_r}} \tag{3-7}$$

or it is given by [Alamir 2004]:

$$k = 0.3725\frac{NI}{\sqrt{V_r}} \tag{3-8}$$

where NI is magnetic field excitation.

3-2-2 Trajectory equations of the electron beam in magnetic deflectors

In the case of magnetic deflector, the paraxial equation which describes electron trajectories in the axial symmetric magnetic system which is given by Eq. (2-1) can be solved analytically also in terms of Bessel functions for fields which are given by Eq. (2-6) and the magnetic field of lenses are given by Eq. (2-3).

For fields in the form $D(z) \propto z^{-3}$, $D(z) \propto z^{-4}$ and $D(z) \propto z^{-5}$, the corresponding trajectories of electron beams in magnetic deflector are derived and the results are given by [see appendix]:

$$r(z) = B\, z^{\frac{1}{2}}\, J_{\frac{1}{4}}\left(\frac{k}{2z^2}\right) \qquad \text{for } n=2 \qquad (3\text{-}9)$$

$$r(z) = D\, z^{\frac{1}{2}}\, J_{\frac{1}{6}}\left(\frac{3k}{4z^3}\right) \qquad \text{for } n=3 \qquad (3\text{-}10)$$

$$r(z) = E\, z^{\frac{1}{2}}\, J_{\frac{1}{8}}\left(\frac{k}{z^4}\right) \qquad \text{for } n=4 \qquad (3\text{-}11)$$

where B, D and E are constants. These constants can be inserted in previous trajectory equations when z is a large value. The constant B is defined by Eq. (3-5) while the constants D and E are given by:

$$D = \left(\frac{3}{2k}\right)^{\frac{1}{6}} \Gamma\left(\frac{7}{6}\right) \qquad (3\text{-}12)$$

$$E = \left(\frac{2}{k}\right)^{\frac{1}{8}} \Gamma\left(\frac{9}{8}\right) \qquad (3\text{-}13)$$

3-3 Positions of Foci

The electron beams cross the axis of magnetic system at special points. These special points are called the position of foci or the focal points which is denoted by z_f.

When Bessel function passes through zero, the electron is focused on the axis of the lens [Hawkes 2002]. The various Bessel function passes through zero for specific values of the argument, depending upon the order (corresponding to monopole, dipole, ect.) and the index of the zero (first, second, ect.) [Liu 2003]. The analogy define for positon of foci can be used in the case of magnetic deflector.

Therefore, one should know the index of the zero and the position of the focus corresponding to each zero.

The first three indices of the zero can be taken from Mathematica [Liu 2003]:

$$\left.\begin{array}{ll} x_1 = \pi, 2\pi \text{ and } 3\pi & \text{for monopole} \\ x_2 = 2.7808877, 5.9061426 \text{ and } 9.0423837 & \text{for dipole} \\ x_3 = 2.657505, 5.7785399 \text{ and } 8.9135662 & \text{for quadrupole} \\ x_4 = 2.59512, 5.71434 \text{ and } 8.84889 & \end{array}\right\} \quad (3\text{-}14)$$

The values of x_4 are found in the present study.

The position of focus corresponding to each zero has been evaluated analytically, based on its definition, as the following:

3-3-1 Foci position for the magnetic lenses

I-At n=2

The position of focus corresponding to x_1 is derived and the result is given by:

$$z_f = \frac{k}{x_1} \qquad \text{for monopole} \qquad (3\text{-}15)$$

II-At n=3

The position of focus corresponding to x_2 is derived and the result is given by:

$$z_f = \left(\frac{k}{2x_2}\right)^{\frac{1}{2}} \qquad \text{for dipole} \qquad (3\text{-}16)$$

III-At n=4

The position of focus corresponding to x_3 is derived and the result is given by:

$$z_f = \left(\frac{k}{3x_3}\right)^{\frac{1}{3}} \qquad \text{for quadrupole} \qquad (3\text{-}17)$$

3-3-2 Foci position for the magnetic deflectors

I-At n=2

The position of focus in this case is the same as the position of focus for magnetic lens when n=3 and it is given by Eq. (3-16).

II-At n=3

The position of focus corresponding to x_3 is derived and the result is given by:

$$z_f = \left(\frac{3k}{4x_3}\right)^{\frac{1}{3}} \qquad (3\text{-}18)$$

III-At n=4

The position of focus corresponding to x_4 is derived and the result is given by:

$$z_f = \left(\frac{k}{x_4}\right)^{\frac{1}{4}} \qquad (3\text{-}19)$$

3-4 Focal Length

The formulae of the focal length for the axial symmetric magnetic system in the present work can be found by applying Eq. (2-8) as the following:

3-4-1 Focal length of magnetic lenses

The formulae of the focal length of magnetic lenses for the field distribution are given by Eq. (2-3) can be found as follows:

I-At n=2

The formula of the focal length at the focus for the magnetic field of the form $B(z)=\frac{B_o}{z^2}$ and the trajectory equation which is given by Eq. (3-1) can be found as follow:

Eq. (3-4) can be substituted in Eq. (3-1) to give:

$$r(z)=\left(\frac{2}{k}\right)^{\frac{1}{2}}\frac{\sqrt{\pi}}{2}\ z^{\frac{1}{2}}\,J_{\frac{1}{2}}\left(\frac{k}{z}\right) \tag{3-20}$$

the fractional Bessel function $J_{\frac{1}{2}}\left(\frac{k}{z}\right)$ can be written in term of circular function [Hawkes 2002]:

$$J_{\frac{1}{2}}\left(\frac{k}{z}\right)=\left(\frac{2z}{\pi k}\right)^{\frac{1}{2}}\sin\left(\frac{k}{z}\right) \tag{3-21}$$

by substituting Eq. (3-21) in Eq. (3-20), one may find:

$$r(z)=\frac{z}{k}\sin\left(\frac{k}{z}\right) \tag{3-22}$$

now, let $x_1=\frac{k}{z}$; then the Eq. (3-22) becomes:

$$r(z)=\frac{z}{k}\sin(x_1) \tag{3-23}$$

the partial differential equation is given by:

$$\frac{dr(z)}{dz}=\frac{\partial r(z)}{\partial x}\times\frac{\partial x}{\partial z} \tag{3-24}$$

Eq. (3-23) is substituted in Eq. (3-24) to find:

$$\frac{dr(z)}{dz}=\frac{-1}{z}\cos(x_1) \tag{3-25}$$

by substituting Eq. (3-25) in Eq. (2-8) to find:

$$|f|=\frac{z_f}{\cos(x_1)} \tag{3-26}$$

then Eq. (3-26) can be written as [Hawkes 2002]:

$$f = z_f = \frac{k}{x_1} \tag{3-27}$$

II- At n=3

To derive the formula of the focal length at the focus for the magnetic field of the form $B(z) = \frac{B_o}{z^3}$ and the trajectory equation which is previously given by Eq. (3-2), one can be followed the similar method when n=2 and the result is given by:

$$f = \frac{k^{\frac{1}{2}}}{2^{\frac{3}{4}}\Gamma(\frac{5}{4})\ (x_2)^{\frac{5}{4}}[J_{\frac{5}{4}}(x_2) - J_{\frac{-3}{4}}(x_2)]} \tag{3-28}$$

III- At n=4

The focal length at the focus for the magnetic field of the form $B(z) = \frac{B_o}{z^4}$ and the trajectory equation which is previously given by Eq. (3-3) can be described by the formula which is derived by similar method when n=2 and it is given by:

$$f = \frac{2^{\frac{5}{6}}\ k^{\frac{1}{3}}}{3^{\frac{4}{3}}\Gamma(\frac{7}{6})\ (x_3)^{\frac{7}{6}}\ [J_{\frac{7}{6}}(x_3) - J_{\frac{-5}{6}}(x_3)]} \tag{3-29}$$

3-4-2 Focal length of magnetic deflectors

The formulae of the focal lengths for magnetic deflector can be found by similar method in the magnetic lenses. This is done by applying Eq. (2-8) as the following:

I- At n=2

The formula of the focal length at the focus for the magnetic field of the form $D(z) = \frac{B_o}{z^3}$ and the trajectory equation which is previously given by Eq. (3-9) is the same as the formula of the focal length for magnetic lens when n=3. Then, it is given earlier by Eq. (3-28).

II- At n=3

For deriving the formula of the focal length at the focus for the magnetic field of the form $D(z)=\frac{3B_o}{2z^4}$ and the trajectory equation which is previously given by Eq. (3-10), one can be used the similar method which is applied in magnetic lenses. Then, the result of the deriving formula is given by:

$$f=\frac{2^{\frac{5}{6}}\ k^{\frac{1}{3}}}{3\Gamma(\frac{7}{6})\ (x_3)^{\frac{7}{6}}[J_{\frac{7}{6}}(x_3)-J_{\frac{-5}{6}}(x_3)]} \tag{3-30}$$

III- At n=4

The formula of the focal length at the focus for the magnetic field of the form $D(z)=\frac{2B_o}{z^5}$ and the trajectory equation which is previously given by Eq. (3-11) is derived and the formula is given by:

$$f=\frac{k^{\frac{1}{4}}}{2^{\frac{9}{8}}\Gamma(\frac{9}{8})(x_4)^{\frac{9}{8}}[J_{\frac{9}{8}}(x_4)-J_{\frac{-7}{8}}(x_4)]} \tag{3-31}$$

3-5 Spherical Aberration

The formulae of the spherical aberration coefficient for magnetic lenses, deflectors and the combination of magnetic lens and deflector can be obtained by applying Eq. (2-12). After that, it can be derived the general formulae to the spherical aberration coefficient for the magnetic lens and deflector. As a consequence, one can find the aberration coefficient of each value of n in magnetic lenses, deflectors and the combination of magnetic lens and deflector as the following:

3-5-1 General formulae of spherical aberration

In the case of magnetic lenses, by substituting Eq. (2-3) in Eq. (2-12), one can find [Hawkes and Kasper 1989]:

$$C_s = \frac{1}{48}\int_{z_f}^{\infty}\left\{5\frac{\eta^2}{V_r}\left(B'(z)\right)^2 - \frac{\eta^2}{V_r}B(z)B''(z) + 4\frac{\eta^4}{V_r^2}B^4(z)\right\} h^4(z)dz$$

$$= \frac{\eta^2}{48V_r}\int_{z_f}^{\infty}\left\{5\left((B_0)^2\frac{n^2}{z^{2n+2}}\right) - (B_0)^2\frac{1}{z^n}\frac{n(-n+1)}{z^{n-2}} + 4\frac{\eta^2}{V_r}\frac{(B_0)^4}{z^{4n}}\right\} h^4(z)dz \quad (3\text{-}32)$$

where $B'(z)$ and $B''(z)$ are the first and second derivative with respect to z of $B(z)$ respectively. Then, Eq. (3-32) can be written as:

$$C_s = \frac{\eta^2(B_0)^2}{48V_r}\int_{z_f}^{\infty}\left\{\frac{5n^2}{z^{2n+2}} - \frac{(-n^2+n)}{z^{2n-2}} + 4\frac{\eta^2}{V_r}\frac{(B_0)^2}{z^{4n}}\right\} h^4(z)dz \quad (3\text{-}33)$$

Eq. (3-7) is substituted in Eq. (3-33) to find [Hawkes 2002]:

$$C_s = \frac{k^2}{12}\int_{z_f}^{\infty}\left\{\frac{4n^2-n}{z^{2n+2}} + 16\frac{k^2}{z^{4n}}\right\} h^4(z)dz \quad (3\text{-}34)$$

Equation (3-34) is the general formula for calculating the spherical aberration coefficients of magnetic lenses which field distributions are given by Eq. (2-3).

On the other hand, the formulae of the spherical aberration coefficient for magnetic deflector are obtained by using a similar method for magnetic lenses. So, they are derived and the formula is given by:

$$C_s = \frac{n^2k^2}{48}\int_{z_f}^{\infty}\left\{\frac{4n^2+7n-1}{z^{2n+4}} + 4\frac{n^2k^2}{z^{4n+4}}\right\} h^4(z)dz \quad (3\text{-}35)$$

Equation (3-35) is the general formula for calculating the spherical aberration coefficients for magnetic deflectors which field distributions are given by Eq. (2-6) and the magnetic fields of lenses are given by Eq. (2-3).

3-5-2 Spherical aberration for magnetic lenses

I- At n=2

The value of n (n=2) is substituted in Eq. (3-34) to get:

$$C_s = \frac{k^2}{12}\int_{z_f}^{\infty}\left(\frac{14}{z^6}+16\frac{k^2}{z^8}\right)h^4(z)dz \tag{3-36}$$

by substituting Eq. (3-1) and Eq. (3-33) in Eq. (2-13):

$$h(z) = \frac{z}{x_1}\sin\left(\frac{k}{z}\right) \tag{3-37}$$

inserting Eq. (3-37) into Eq. (3-36) gives:

$$C_s = \frac{k^2}{12}\int_{z_f}^{\infty}\left(\frac{14}{z^6}+16\frac{k^2}{z^8}\right)\frac{z^4}{(x_1)^4}\sin^4\left(\frac{k}{z}\right)dz$$

$$= \frac{k^2}{12(x_1)^4}\int_{z_f}^{\infty}\left(\frac{14}{z^2}+16\frac{k^2}{z^4}\right)\sin^4\left(\frac{k}{z}\right)dz \tag{3-38}$$

now, let $u = \frac{k}{z}$

then $z = \frac{k}{u}$ and $dz = -\frac{k}{u^2}du$

when $z = z_f \rightarrow u = x$

$z = \infty \rightarrow u = 0$

these suppositions are applied in Eq. (3-38) to get:

$$C_s = \frac{k^2}{12(x_1)^4}\int_{x_1}^{0}\left(\frac{14u^2}{k^2}+16\frac{k^2u^4}{k^4}\right)\frac{-k}{u^2}\sin^4(u)du$$

after some rearranging, then above equation can be written as:

$$C_s = \frac{k}{6(x_1)^4}\int_{0}^{x_1}(7+8u^2)\sin^4(u)du \tag{3-39}$$

II- At n=3

The value of n (n=3) is substituted in Eq. (3-34) to get [Hawkes 2002]:

$$C_s = \frac{k^2}{12}\int_{z_f}^{\infty}\left(\frac{33}{z^8}+16\frac{k^2}{z^{12}}\right)h^4(z)dz \tag{3-40}$$

Eq. (3-2) is substituted in Eq. (2-13) to obtaine:

$$h(z) = f\ B\ z^{\frac{1}{2}}\ J_{\frac{1}{4}}\left(\frac{k}{2z^2}\right) \tag{3-41}$$

f in this case is defined by Eq. (3-28).

The formula of the spherical aberration coefficient at the focus is derived by using the similar method when n=2. Then, the result of deriving for this coefficient is given by:

$$C_s = \frac{k^{\frac{1}{2}}}{6\sqrt{2}(x_2)^5\,[J_{\frac{5}{4}}(x_2) - J_{\frac{-3}{4}}(x_2)]^4}\int_0^{x_2}\left(33+64u^2\right)u^{\frac{2}{3}}\left(J_{\frac{1}{4}}(u)\right)^4 du \tag{3-42}$$

III- At n=4

The value of n (n=4) is substituted in Eq. (3-34) to find:

$$C_s = \frac{k^2}{12}\int_{z_f}^{\infty}\left(\frac{60}{z^{10}}+16\frac{k^2}{z^{16}}\right)\ h^4(z)dz \tag{3-43}$$

Eq. (3-3) is substituted in Eq. (2-13) to get:

$$h(z) = f\ C\ z^{\frac{1}{2}}\ J_{\frac{1}{6}}\left(\frac{k}{3z^3}\right) \tag{3-44}$$

f in this case is defined by Eq. (3-29).

If it follows the similar method when n=2, the formula of coefficient at the focus is derived and the formula is given by:

$$C_s = \frac{16\,k^{\frac{1}{3}}}{3^{\frac{10}{3}}\,(x_3)^{\frac{14}{3}}\,[J_{\frac{7}{6}}(x_3) - J_{\frac{-5}{6}}(x_3)]^4} \int_0^{x_3} \left(5+12u^2\right) u^{\frac{4}{3}} \left(J_{\frac{1}{6}}(u)\right)^4 du \qquad (3\text{-}45)$$

3-5-3 Spherical aberration for magnetic deflectors

To find the formulae of spherical aberration coefficients for magnetic deflectors, it would be used the similar method which is applied in magnetic lenses. Therefore, they are derived as the following:

I-At n=2

In this case, the formula of the spherical aberration coefficient at the focus is the same as the formula of the coefficient for magnetic lens when n=3 and it is given earlier by Eq. (3-42).

II- At n=3

The formula of the spherical aberration coefficient at the focus is derived and the result is given by:

$$C_s = \frac{2^{\frac{19}{3}}\,k^{\frac{1}{3}}}{3^{\frac{17}{3}}\,(x_3)^{\frac{14}{3}}\,[J_{\frac{7}{6}}(x_3) - J_{\frac{-5}{6}}(x_3)]^4} \int_0^{x_3} \left(7+8u^2\right) u^{\frac{4}{3}} \left(J_{\frac{1}{6}}(u)\right)^4 du \qquad (3\text{-}46)$$

III- At n=4

The spherical aberration coefficient at the focus is described by the formula which is derived and it is given by:

$$C_s = \frac{k^{\frac{1}{4}}}{192\,(x_4)^{\frac{9}{2}}\,[J_{\frac{9}{8}}(x_4) - J_{\frac{-7}{8}}(x_4)]^4} \int_0^{x_4} \left(91+64u^2\right) u^{\frac{5}{4}} \left(J_{\frac{1}{8}}(u)\right)^4 du \qquad (3\text{-}47)$$

3-6 Chromatic Aberration

The formulae for all types of the chromatic aberration coefficients will be derived as the following:

3-6-1 Axial chromatic aberration

To find the axial chromatic aberration coefficient for magnetic lenses, deflectors and the combination of magnetic lens and deflector, it can be started from Eq. (2-15). Consequently, the general formulae of the axial chromatic aberration coefficient will be derived for magnetic system (lens and deflector). As well, the formulae for the combination of magnetic lens and deflector will be obtained.

3-6-1-1 general formulae of axial chromatic aberration

In the case of magnetic lenses, by substituting Eq. (2-3) in Eq. (2-15), one can find:

$$C_c = \frac{e}{8mV_r} \int_{z_f}^{\infty} \frac{(B_0)^2}{z^{2n}} h^2(z)dz \tag{3-48}$$

by substituting Eq. (3-7) in Eq. (3-48) to give:

$$C_c = k^2 \int_{z_f}^{\infty} \frac{1}{z^{2n}} h^2(z)dz \tag{3-49}$$

where h(z) is defined previousely by Eq. (2-13).

Equation (3-49) is the general formula for calculating the axial chromatic aberration coefficients for magnetic lenses which field distributions are given by Eq. (2-3).

On the other hand, the formulae of the axial chromatic aberration coefficient for magnetic deflectors can be found by following the similar method which is applied in magnetic lenses. So, the general formula is derived and the result is given by:

$$C_c = \frac{k^2}{4}\int_{z_f}^{\infty}\frac{n^2}{z^{2n+2}}h^2(z)dz \tag{3-50}$$

Equation (3-50) is the general formula for calculating the axial chromatic aberration coefficients for magnetic deflectors which field distributions are given by Eq. (2-6) and the magnetic fields of lenses are given by Eq. (2-3).

3-6-1-2 axial chromatic aberration for magnetic lenses

I-At n=2

The value of n (n=2) is substituted in Eq. (3-49) to find:

$$C_c = k^2\int_{z_f}^{\infty}\frac{1}{z^4}h^2(z)dz \tag{3-51}$$

by substituting $h(z)$ from Eq. (3-37) in Eq. (3-51) to find:

$$C_c = k^2\int_{z_f}^{\infty}\frac{1}{z^4}\frac{z^2}{(x_1)^2}\sin^2(\frac{k}{z})dz$$

$$= \frac{k^2}{(x_1)^2}\int_{z_f}^{\infty}\frac{1}{z^2}\sin^2(\frac{k}{z})dz \tag{3-52}$$

now, let $u = \frac{k}{z}$

then $z = \frac{k}{u}$ and $dz = -\frac{k}{u^2}du$

when $z = z_f \rightarrow u = x$

$z = \infty \rightarrow u = 0$

these suppositions are applied in Eq. (3-52) to find:

$$C_c = \frac{k^2}{(x_1)^2}\int_{x_1}^{0}\frac{u^2}{k^2}\frac{(-k)}{u^2}\sin^2(u)du$$

$$= \frac{k}{(x_1)^2}\int_{0}^{x_1}\sin^2(u)du \tag{3-53}$$

Equation (3-53) is described the axial chromatic aberration coefficient for magnetic lens at the focus when n=2.

II-At n=3

The value of n (n=3) is substituted in Eq. (3-49) to find:

$$C_c = k^2 \int_{z_f}^{\infty} \frac{1}{z^6} h^2(z)dz \tag{3-54}$$

Eq. (3-28) is substituted in Eq. (3-41) to find $h(z)$. After that, $h(z)$ can be substituted in Eq. (3-54). Consequently, the formula of the axial chromatic aberration coefficient at the focus is derived by applying the similar method when n=2 and it is given by:

$$C_c = \frac{\sqrt{2}\ k^{\frac{1}{2}}}{(x_2)^{\frac{5}{2}}\ [J_{\frac{5}{4}}(x_2) - J_{\frac{-3}{4}}(x_2)]^2} \int_0^{x_2} u \left(J_{\frac{1}{4}}(u)\right)^2 du \tag{3-55}$$

III-At n=4

The value of n (n=4) is substituted in Eq. (3-49) to find:

$$C_c = k^2 \int_{z_f}^{\infty} \frac{1}{z^8} h^2(z)dz \tag{3-56}$$

by substituting Eq. (3-29) in Eq. (3-44) to find $h(z)$ and after some mathematical substitution and rearranging as the case when n=2, the formula of the axial chromatic aberration coefficient at the focus is derived and it is given by:

$$C_c = \frac{4\ k^{\frac{1}{3}}}{3^{\frac{4}{3}}(x_3)^{\frac{7}{3}}\ [J_{\frac{7}{6}}(x_3) - J_{\frac{-5}{6}}(x_3)]^2} \int_0^{x_3} u \left(J_{\frac{1}{6}}(u)\right)^2 du \tag{3-57}$$

3-6-1-3 axial chromatic aberration for magnetic deflectors

For deriving the formulae of the axial chromatic aberration coefficient for magnetic deflectors, it should be applied the similar method which is used in magnetic lenses. As a consequence, the formulae have been derived as follows:

I-At n=2

The formula of the axial chromatic aberration coefficient at the focus in this case is the same as the formula of the coefficient for magnetic lens when n=3 and it is given previousely by Eq. (3-55).

II-At n=3

The axial chromatic aberration coefficient at the focus is described by formula which is derived and the result is given by:

$$C_c = \frac{2^{\frac{10}{3}}\, k^{\frac{1}{3}}}{3^{\frac{8}{3}} (x_3)^{\frac{7}{3}}\, [J_{\frac{7}{6}}(x_3) - J_{\frac{-5}{6}}(x_3)]^2} \int_0^{x_3} u \left(J_{\frac{1}{6}}(u) \right)^2 du \qquad (3\text{-}58)$$

III-At n=4

The result of derivation for the formula which is described the axial chromatic aberration coefficient at the focus is given by:

$$C_c = \frac{k^{\frac{1}{4}}}{4\, (x_4)^{\frac{9}{4}}\, [J_{\frac{9}{8}}(x_4) - J_{\frac{-7}{8}}(x_4)]^2} \int_0^{x_4} u \left(J_{\frac{1}{8}}(u) \right)^2 du \qquad (3\text{-}59)$$

3-6-2 Field chromatic aberration

3-6-2 -1 isotropic chromatic aberration coefficient

For deriving the formulae of the magnification chromatic aberration coefficient for magnetic lenses, deflectors and the combination of magnetic lens and deflector, one should be applied Eq. (2-14). After that, the general formulae of the magnification chromatic aberration coefficient for the magnetic lenses and deflectors can be found. As the result, one can find the aberration coefficient for each value of n in magnetic lenses, deflectors and the combination of magnetic lens and deflector as follows:

A-general formula of magnification chromatic aberration

In the case of magnetic lenses, by substituting Eq. (2-3) in Eq. (2-16), one can find:

$$C_m = \frac{e}{8mV_r} \int_{z_f}^{\infty} \frac{(B_0)^2}{z^{2n}} h(z) g(z) dz \tag{3-60}$$

Eq. (3-7) is substituted in Eq. (3-60) to get:

$$C_m = k^2 \int_{z_f}^{\infty} \frac{1}{z^{2n}} h(z) g(z) dz \tag{3-61}$$

Equation (3-61) is the general formula for calculating the magnification chromatic aberration coefficients for magnetic lenses which field distributions are given by Eq. (2-3).

However, the general formula of the magnification chromatic aberration coefficient for magnetic deflectors is derived by following the similar method which is applied in magnetic lenses. Consequently, it is given by:

$$C_m = \frac{k^2}{4} \int_{z_f}^{\infty} \frac{n^2}{z^{2n+2}} h(z) g(z) dz \tag{3-62}$$

Equation (3-62) is the general formula for calculating the magnification chromatic aberration coefficients for magnetic deflectors which field distributions are given by Eq. (2-6) and the magnetic fields of lenses are given by Eq. (2-3).

B- magnification chromatic aberration for magnetic lenses

I-At n=2

The value of n (n=2) is substituted in Eq. (3-61) to find:

$$C_m = k^2 \int_{z_f}^{\infty} \frac{1}{z^4} h(z) g(z) dz \tag{3-63}$$

Eq. (3-22) and Eq. (3-37) are applied in Eq. (3-63) to get:

$$C_m = k^2 \int_{z_f}^{\infty} \frac{1}{z^4} \frac{z}{x} \sin(\frac{k}{z}) \frac{z}{k} \sin(\frac{k}{z}) dz$$

$$= \frac{k}{x_1} \int_{z_f}^{\infty} \frac{1}{z^2} \sin^2(\frac{k}{z}) dz \tag{3-64}$$

now, let $u = \frac{k}{z}$

then $z = \frac{k}{u}$ and $dz = -\frac{k}{u^2} du$

when $z = z_f \rightarrow u = x$

$z = \infty \rightarrow u = 0$

these suppositions are substituted in Eq. (3-64) to find:

$$C_m = \frac{k}{x_1} \int_{x_1}^{0} \frac{u^2}{k^2} \frac{(-k)}{u^2} \sin^2(u) du$$

$$= \frac{1}{x_1} \int_{0}^{x_1} \sin^2(u) du \tag{3-65}$$

Equation (3-65) is described the magnification chromatic aberration coefficient at the focus when n=2.

II-At n=3

The value of n (n=3) is substituted in Eq. (3-61) to find:

$$C_m = k^2 \int_{z_f}^{\infty} \frac{1}{z^6} h(z) g(z) dz \tag{3-66}$$

$h(z)$ can be found by substituting Eq. (3-28) in Eq. (3-41). After that, $h(z)$ and Eq. (3-2) can be substituted in Eq. (3-66). As a consequence, the derivative formula of the magnification chromatic aberration coefficient at the focus is given by:

$$C_m = \frac{2^{\frac{5}{4}}\,\Gamma(\frac{5}{4})}{(x_2)^{\frac{5}{4}}\,[J_{\frac{5}{4}}(x_2) - J_{\frac{-3}{4}}(x_2)]} \int_0^{x_2} u\left(J_{\frac{1}{4}}(u)\right)^2 du \tag{3-67}$$

III- At n=4

The value of n (n=4) is substituted in Eq. (3-61) to find:

$$C_m = k^2 \int_{z_f}^{\infty} \frac{1}{z^8} h(z) g(z) dz \tag{3-68}$$

by substituting Eq. (3-29) in Eq. (3-44) to find $h(z)$. After some mathematical substitution and rearranging as the case when n=2, the formula of the magnification chromatic aberration coefficient at the focus is derived and the result is given by:

$$C_m = \frac{2^{\frac{7}{6}}\,\Gamma(\frac{7}{6})}{(x_3)^{\frac{7}{6}}\,[J_{\frac{7}{6}}(x_3) - J_{\frac{-5}{6}}(x_3)]} \int_0^{x_3} u\left(J_{\frac{1}{6}}(u)\right)^2 du \tag{3-69}$$

C- magnification chromatic aberration for magnetic deflectors

By using the similar method in the magnetic lenses, the formulae of magnification chromatic aberration coefficient for magnetic deflectors are derived as follows:

I-At n=2

The formula of the magnification chromatic aberration coefficient at the focus is similar to the formula of the magnification chromatic aberration coefficient for magnetic lens when n=3 and it is given by Eq. (3-67).

II-At n=3

The formula of the magnification chromatic aberration coefficient at the focus is derived and it is given by:

$$C_m = \frac{2^{\frac{5}{2}}\,\Gamma(\frac{7}{6})}{3^{\frac{5}{3}}(x_3)^{\frac{7}{6}}\,[J_{\frac{7}{6}}(x_3) - J_{\frac{-5}{6}}(x_3)]}\int_0^{x_3} u\left(J_{\frac{1}{6}}(u)\right)^2 du \tag{3-70}$$

III-At n=4

The magnification chromatic aberration coefficient at the focus is described by the formula which is derived and the result is given by:

$$C_m = \frac{\Gamma(\frac{9}{8})}{2^{\frac{7}{8}}(x_4)^{\frac{9}{8}}\,[J_{\frac{9}{8}}(x_4) - J_{\frac{-7}{8}}(x_4)]}\int_0^{x_4} u\left(J_{\frac{1}{8}}(u)\right)^2 du \tag{3-71}$$

3-6-2 -2 anisotropic chromatic aberration coefficient

To derive the formulae of the rotation chromatic aberration coefficient, one can be started from Eq. (2-17). Consequently, one can be found the formulae of coefficient for the magnetic lenses, deflectors and the combination of magnetic lens and deflector as the following:

A- roation chromatic aberration for magnetic lenses

I-At n=2

The value of n (n=2) is substituted in Eq. (2-17) to find:

$$C_\theta = \frac{1}{2}\left(\frac{e}{8mV_r}\right)^{\frac{1}{2}}\int_{z_f}^{\infty}\frac{B_0}{z^2}dz$$

$$= \frac{B_0}{2}\left(\frac{e}{8mV_r}\right)^{\frac{1}{2}}\times\left(-\left|z^{-1}\right|_{z_f}^{\infty}\right) \tag{3-72}$$

by substituting Eq. (3-7) in Eq. (3-72) to get on:

$$C_\theta = \frac{k}{2}\left(\frac{1}{z_f} - \frac{1}{\infty}\right) = \frac{k}{2z_f} \quad (3\text{-}73)$$

Eq. (3-16) is inserted into Eq. (3-73) to find:

$$C_\theta = \frac{x_1}{2} \quad (3\text{-}74)$$

II-At n=3

The formula of the rotation chromatic aberration coefficient at the focus is derived and the result is given by:

$$C_\theta = \frac{x_2}{2} \quad (3\text{-}75)$$

III-At n=4

Formula which is described the rotation chromatic aberration coefficient at the focus is derived and it is given by:

$$C_\theta = \frac{x_3}{2} \quad (3\text{-}76)$$

B- rotation chromatic aberration for magnetic deflectors

I-At n=2

The formula of the rotation chromatic aberration coefficient at the focus is similar to the formula of the rotation chromatic aberration coefficient for magnetic lens when n=3 and it is given by Eq. (3-75).

II-At n=3

The rotation chromatic aberration coefficient at the focus is described by the formula which is derived and the result is given by:

$$C_\theta = \frac{x_3}{3} \quad (3\text{-}77)$$

III-At n=4

The formula of the rotation chromatic aberration coefficient at the focus is derived and it is given by:

$$C_\theta = \frac{x_4}{4} \quad (3\text{-}78)$$

Chapter Two

Theoretical Considerations

2-1 Introduction

In the present work, a magnetic lens and deflector have been adopted for the purpose of the theoretical study. Each component of magnetic system (lens and deflector) is fully described in terms of different parameters such as trajectory of electron beam, position of foci, focal lengths and aberration coefficients.

2-2 Paraxial-Ray Equation in Magnetic Field

The motion of an electron in an axially symmetrical field can be derived starting from many departure points. One may start from the Lagrangian [Hawkes 1982; Silagyi 1988] or from a more familiar method of elementary mechanics [El-Kareh and El-Kareh 1970; Klemperer and Barnett 1971]. The paraxial-ray equation of an electron in a magnetic field of axial symmetry is given by [El-Kareh and El-Kareh 1970]:

$$r''(z) + \left(\frac{e}{8mV_r}\right)B^2(z)r(z) = 0 \qquad (2\text{-}1)$$

where r(z) is the radial displacement of the beam from the optical axis z, e and m are the charge and mass of the electron, respectively, $r''(z)$ the second derivative of r(z) with respect to z and V_r is the relativistically corrected accelerating voltage which is given by [El-Kareh and El-Kareh 1970]:

$$V_r = V_a\left[1+\frac{eV_a}{2mc^2}\right] = V_a\left[1+0.978\times10^{-6}V_a\right] \qquad (2\text{-}2)$$

where V_a is the accelerating voltage and c is the speed of light in space.

An important deduction can be made from Eq. (2-1). The force driving the electrons towards the axis is directly proportional to the radial distance r. This is the principle of a focusing field. Futhermore, this force is proportional with the square of the magnetic flux density. It means that if the direction of the magnetic field is reversed by reversing the current, the direction of the force towards the axis should not change. In other words, there will be no change in the focus [El-Kareh and El-Kareh 1970].

2-3 Mathematical Models for Magnetic Lens

Magnetic field models of magnetic lenses are analytical expressions, that represent the axial field distribution B(z) of magnetic lenses. Each model gives a design basis for a lens, especially when its model is capable of being realized physically.

Many field models deal exclusively with double-pole lenses, such as the Glaser bell shaped field and the Grivet-Lenz field. While the exponential field and the spherical field deal with the single pole lenses.

In the present work, the mathematical model of the form inverse power law is chosen to represent the magnetic flux distribution which is given by [Alamir 2005]:

$$B(z) = B_0\left(\frac{a}{z}\right)^n \qquad ; n= 2, 3, 4 \qquad (2\text{-}3)$$

where B_0 is the maximum value of axial magnetic field, a is the half width at half maximum field and n is any positive or negative numbers, not necessarily an integer, expect unity. Here, n is taken as positive numbers, where n=2 for monopole lens, n=3 for dipole lens and n=4 for quadrupole lens [Crewe 2001].

2-4 Moving Objective Lens (MOL) Concept

The concept of moving objective lens (MOL) was introduced first by Ohiwa et al. [Ohiwa et al. 1971] for description of a system in which the deflectors are placed in the lens. In a moving objective lens (MOL) system, a lens is placed before the image plane and is electromagnetically moved is synchronization with the deflector. Such displacement of the lens can be expressed as the first-order Taylor expansion of the magnetic scalar potential of the lens [Ohiwa 1978]:

$$\Phi(x-d,y,z)=\Phi(x,y,z)-d(\frac{\partial\Phi}{\partial x}) \qquad (2\text{-}4)$$

This shows that superposition of the deflection field $-d(\frac{\partial\Phi}{\partial x})$ on the lens field Φ is equivalent to the displacement of the lens by a distance d. Let one assume that the focusing lens is composed of an axially symmetric magnetic field. Then, the off-axis potential can be expressed up to the second order as [Ohiwa 1978]:

$$\Phi(r,z)=\Phi(r)-(\frac{r^2}{4})\frac{d^2\Phi(z)}{dz^2} \qquad (2\text{-}5)$$

Let B(z) is the axial flux density distribution for the lens and D(z) is the deflection flux density required at the axis. Then, the following relation holds from Eqs. (2-4) and (2-5) [Ohiwa 1979]:

$$D(z)=\frac{-d}{2}B'(z) \qquad (2\text{-}6)$$

where d is the displacement by the first deflector (pre-deflection), and the $B'(z)$ is the first derivative of B(z) with respect to z.

The MOL concept is illustrated in figure (2-1). A point source of electrons, emitted from z_o, is imaged to z_i by a lens. The beam is deflected by the first deflector so that it enters the lens off-axis. The second deflector, placed inside the lens, shifts the electrical center of the lens off-axis. The Moving Objective Lens (MOL) reduces the effect of the off-axis lens aberrations [Khursheed 2011].

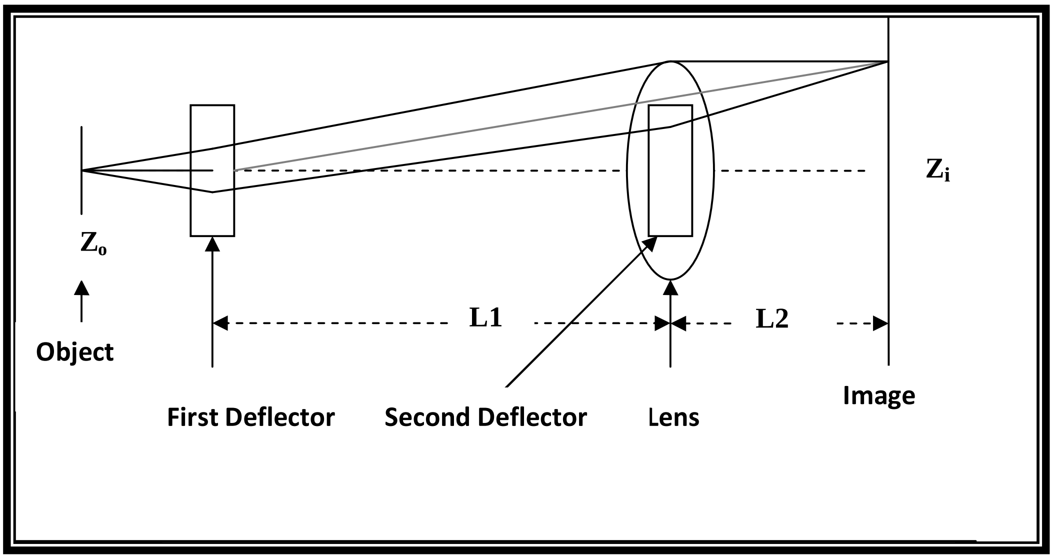


Figure (2-1): MOL arrangement [Ohiwa 1978].

2-5 Focal Lenght

Focal length is defined as the distance between the principal plane and the focus. Assume an incident electron moving initially parallel to the axis, the focal lenght can be obtained by direct integration of Eq. (2-1) [Crewe 2004]:

$$\frac{1}{f}=\frac{e}{8mV_r}\int_{-\infty}^{\infty}B^2(z)r(z)dz \tag{2-7}$$

or the focal lenght can be obtained by differentiating the expression of a ray which is parallel to the axis far from the axial symmetric system.Therefore, it is easy to show that the focal lenght of the ray is given by [Hawkes 2002]:

$$f_{(z)}=\frac{1}{r'(z)} \tag{2-8}$$

where $r'(z)$ is the first derivative of r(z) with respect to z.

2-6 Defects of Electron Optical System

In general, when speaking of aberration in electron optics, one refers to the case in which rays emanating from one point for the same object point converge to different image points. If an electron path is leaving an object point, at a finite distance from the axis with a particular direction and electron velocity, it intersects the Gaussian image plane at a point displaced from the Gaussian image point; this displacement is defined as the aberration [El-Kareh and El-Kareh 1970].

The operation of axially symmetric electron and ion systems (focusing systems or deflector systems) is based on the paraxial (first-order) theory. Practically, the trajectories always have both finite displacements r and finite slope r' with respect to the axis. Even if they are small, the omission of the higher-order terms in series expansions that leading to the paraxial ray equation causes some error. Therefore, the paraxial theory is always an approximation [Szilagyi 1988].

The assumption by means of equation (2-1) constricted is concerned with rays that are close to the system axis. However, if one calculating the second term (i.e. the third order-term) in series expansions of magnetic field that it is given by [Szilagyi 1988]:

$$B_r(r,z)=\sum_{n=1}^{\infty}\frac{(-1)^n}{(n)!(n-1)!}B_z^{(2n-1)}(z)\left[\frac{r}{2}\right]^{2n-1} \tag{2-9}$$

Then the equation of motion of an electron in an axially symmetric magnetic field for relativistic potential will take the form [Szilagyi 1988]:

$$r''+\frac{e}{8mV_r}\left[rB_Z^2-\frac{r^3B_ZB_Z''}{2}-r^2r'B_ZB_Z'+rr'^2B_Z^2+\frac{er^3B_Z^4}{8m}\right]=0 \tag{2-10}$$

Equation (2-10) is usually called the "third-order trajectory equation". This equation reduces to Eq. (2-1) if all the third-order terms in r and its derivatives (i.e. r^3, r^2r', rr'^2), are removed. The presence of these terms in Eq. (2-10) results in the appearance of aberrations. Aberrations are referred to as third-order, fifth-order and so on according to the order of the term in Eq. (2-9). Only the third-order aberrations are of an importance in most analyses and calculations in the field of electron and ion optics since they outweigh the aberrations of higher orders [Szilagyi 1988].

The electron magnetic system suffers from many different types of defects through the image formation process. The geometrical aberrations occur when a point object is imaged not by a point image but by a blurred spot produced by different paths of different slopes that focused at different image points. These paths intersect the Gaussian image plane at different points. The geometrical aberrations are arising from higher order terms in the field expansions. In general, there are eight different types of the geometrical aberrations namely; spherical, field curvature, radial distortion, spiral distortion, coma, anisotropic coma, astigmatism and anisotropic astigmatism aberrations.

In addition to the geometrical aberrations in the magnetic system, the chromatic aberration is the other important type of defects from which the magnetic system is suffered.

Space charge is another source of image aberration that it is occurred because of electrostatic repulsion forces between the same charge particles.

When the properties of the system are analyzed using the nonrelativistic approximation, the disparities between the relativistic and nonrelativistic can be conveniently regarded as relativistic aberration.

Finally, one has be mentioned to the mechanical aberrations which occur due to misalignment, material in homogeneity, mechanical imperfections, ect. [Szilagyi 1988; Hawkes and Kasper 1989].

Their implementation provides the creation of an ideal lens that forms stigmatic and similar images. Let us recall these assumptions: (1) rigorous axial symmetry; (2) paraxial trajectory approximation; (3) energy homogeneity, including the absence of time-dependent processes; and (4) negligible space-charge fields and small effects of electron diffraction. Violation of at least one of these conditions leads to aberrations that are responsible for blurred or distorted images and complicate beam transport problems [Tsimring 2007].

The most important aberrations for objective magnetic lens, which are limiting the resolution in an electron-optical system, are spherical and chromatic aberration. Thus, the present work has been determined these two aberrations for each magnetic lens and deflector.

2-6-1 Spherical aberration

The spherical aberration is one of the most important geometrical aberrations; this aberrations is sometime, called aperture defect. It is one of the principal factors that limit the resolution of the optical system. This defect occurs because the power of the optical magnetic system is greater for off-axis rays than the paraxial rays [Zhigarev 1975], as is shown in figure (2-2).

When these non-paraxial electrons arrive at the Gaussian image plane, they will be displaced radically from the optic axis by an amount r_s given by [Egerton 2007]:

$$r_s = C_s \ \alpha^3 \tag{2-11}$$

where C_s is the spherical aberration coefficient and α is acceptance half angle. Since α (in radian) is dimensionless, therefore, C_s has the dimensions of length.

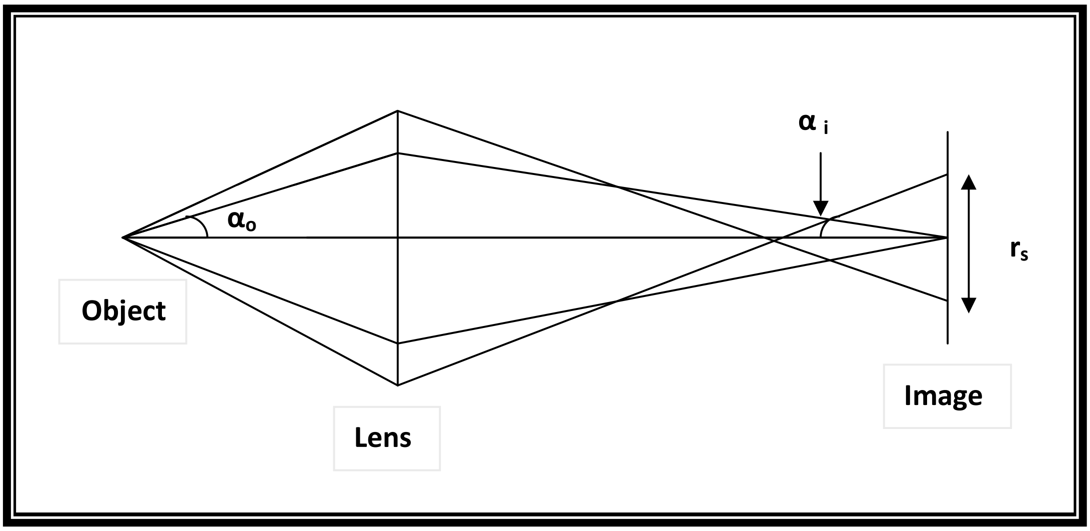


Figure (2-2): Ray diagram illustrating spherical aberration in the Gaussian image plane [Oday 2005].

The spherical aberration coefficient C_s of axial symmetric magnetic optical element (lens or deflector) is calculated using the following integral formula [Hawkes 2002]:

$$C_s = \frac{1}{48}\int_{-\infty}^{\infty}\left\{5\,(b')^2 - b\,b'' + 4\,b^4\right\} h^4(z)\,dz \tag{2-12}$$

$$b = \frac{\eta B(z)}{\sqrt{V_r}}$$

where $\eta = \sqrt{e/2m}$ is the charge -to- mass quotient of electron and h(z) is proportional to the solution r(z) of the paraxial equation which is given by Eq. (2-1), the constant of proportionality being equal to the focal length in order to ensure that the gradient of h(z) at the object position, here the focus, is unity. Thus, h(z) is given by [Hawkes 2002]:

$$h(z) = f\, r(z) \tag{2-13}$$

where f is the focal length.

2-6-2 Chromatic aberration

From the expression of focal length which is given previously by Eq. (2-7), one can note that the focal length is related to the accelerating potential. Then the variation of that potential will result in the variation of focal length. Thus, the image will be distorted and this type of distortion is referred to as chromatic aberration [El-Kareh and El-Kareh 1970].

The main reason for chromatic aberration is the fact that electrons with higher initial energy are less influenced by the imaging field than lower-energy electrons. Therefore, if all electrons leave the object point with the same slope, the high-energy electrons will form an image at a greater distance from the object than the low-energy electrons and the image will be blurred like in case of spherical aberration. Additionally, the rotation of electrons with different energies in a magnetic field will also be different [Szilagyi 1988]. There are several reasons for the energy spread of electrons and it occurs if [El-Kareh and El-Kareh 1970]:

a. The accelerating potential as provided by regulated power supply changes or the current in the energized coil is not highly stabilized.

b. The initial velocity of the electrons is not the same for all particles.

c. The passage of the electron beam through a target results in an inelastic collision of the particles with the specimen and thus the electron leave the specimen with different velocities.

Figure (2-3) represents a simplified diagram defining this defect. A ray of energy $E_0 + \Delta E_0$ will reach the image plane for energy E_0 at a distance Δr from the axis. At a plane about halfway between the points where the two rays intersect the axis in image space, the bundle of rays having energies between E_0 and $E_0 + \Delta E_0$ are contained within a disc of confusion of radius about one-half where [Hall 1966]:

$$\Delta r = \alpha \, C_c \left(\frac{\Delta E_0}{E_0} \right) \tag{2-14}$$

C_c is the chromatic aberration coefficient.

The chromatic aberration coefficients of magnetic axial symmetric system are divided into axial chromatic aberration coefficient C_c and chromatic field aberration (Chromatic aberration of magnification C_m and rotation C_θ) [Hawkes and Kasper 1989].

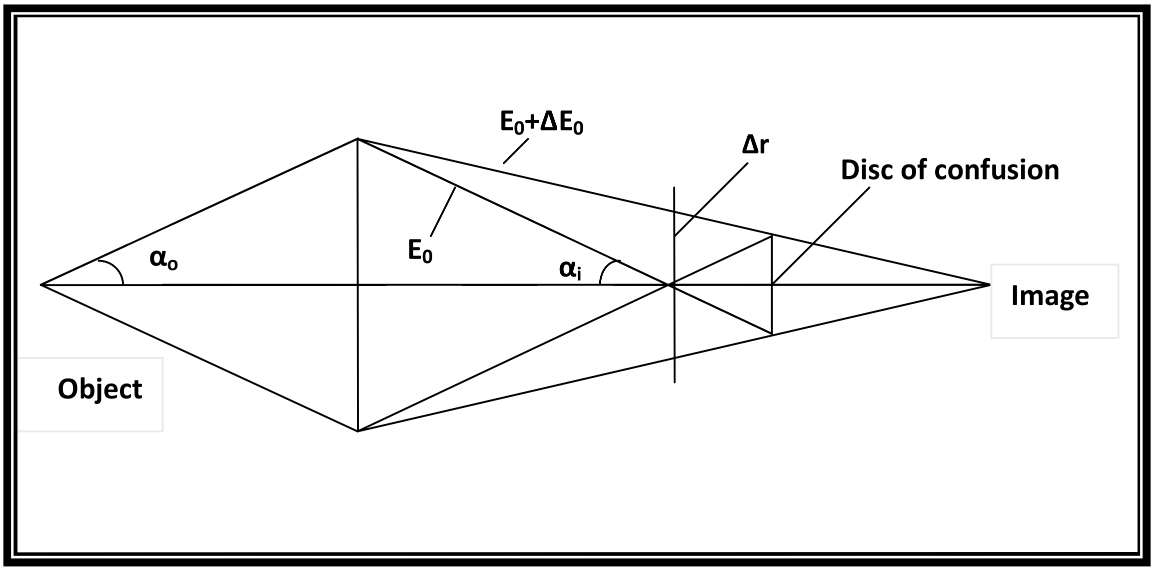


Figure (2-3): Ray diagram illustrating chromatic aberration [Hall 1966].

All types of the chromatic aberration coefficients will be mentioned as follows:

1-6-2-1 axial chromatic aberration

This aberration coefficient like the spherical aberration coefficient creates an aberration disc; its radius in the image plane is the same for all points in the object plane. It is of most concern when the angels are comparatively large [Hawkes and Kasper 1989].

The axial chromatic aberration coefficient C_c of axial symmetric magnetic optical element (lens or deflector) is calculated using the following integral formula [Hawkes and Kasper 1989]:

$$C_c = \frac{e}{8mV_r}\int_{-\infty}^{\infty} B^2(z)h^2(z)dz \tag{2-15}$$

1-6-2-2 field chromatic aberration

The field chromatic aberration is divided into the isotropic and anisotropic chromatic aberration. It is of most concern when the angles are small and the rays are off the axis [Hawkes and Kasper 1989]. This type of aberration will be mentioned as the following:

A- isotropic chromatic aberration

The isotropic chromatic aberration coefficient denoted by C_m and is commonly known as the chromatic aberration of magnification.

The magnification chromatic aberration coefficient C_m of axial symmetric magnetic optical element (lens or deflector) is calculated using the following integral formula [Hawkes and Kasper 1989]:

$$C_m = \frac{e}{8mV_r}\int_{-\infty}^{\infty} B^2(z)h(z)g(z)dz \tag{2-16}$$

where h(z) and g(z) are two independent solutions of the paraxial ray. h(z) is defined by Eq. (2-13) and g(z) is the same trajectory of electron in the magnetic axial symmetric system.

B- anisotropic chromatic aberration

The anisotropic chromatic aberration coefficient denoted by C_θ and is commonly known as the chromatic aberration of rotation, it causes a small change in the image rotation in magnetic systems [Hawkes and Kasper 1989].

The rotation chromatic aberration coefficient C_θ of axial symmetric magnetic optical element (lens or deflector) is calculated by using the following integral formula [Hawkes and Kasper 1989]:

$$C_\theta = \frac{1}{2}\left(\frac{e}{8mV_r}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} B(z)\,dz \qquad (2\text{-}17)$$

Contents

Republic of Iraq
Ministry of Higher Education
and Scientific Research
Al-Nahrain University
College of Science
Department of Physics

Computational Study of Magnetic Lens and Deflector with a Field Distribution of The Form $B(z) \propto z^{-n}$

A Thesis
Submitted to the College of Science / Al-Nahrain University as a Partial Fulfillment of the Requirements for the Degree of Master of Science in Physics

By

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جمهورية العراق
وزارة التعليم العالي والبحث العلمي
جامعة النهرين
كلية العلوم
قسم الفيزياء

الدراسة الحسابية لعدسة و حارف مغناطيسي ذو توزيع جهد يخضع للعلاقة $B(z) \propto z^{-n}$

رسالة
مقدمة الى كلية العلوم / جامعة النهرين
جزء من متطلبات نيل درجة الماجستير في علوم الفيزياء

من قبل

بهاء عبد الحسن جواد

إشراف

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Dedication

To

The lights of my life

Father & mother

The light of my eyes

Brother and sisters

The all faithful hearts which help me

Bahaa

Committee Certification

We, the examining committee certify that we have read the thesis entitled "**Computational Study of Magnetic Lens and Deflector with a Field Distribution of The Form B(z) $\propto$ z^{-n}** " and examined the student "**'Bahaa A. H. Jawad"** in its contents and that in our opinion, it is accepted for the Degree of Master of Science in **Physics**.

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List of Symbols

Symbol	*Definition*
a	the half width at half maximum field (m).
A, B, C, D and E	Constants inserted in the trajectory equation when z is a large value.
B(z)	The axial distribution of Magnetic field of lens (Tesla).
B_0	Maximum of the magnetic flux density (Tesla).
$B_r(r,z)$	Radial component of the magnetic flux density (Tesla).
c	The speed of light in space ($c = 3\times 10^8$ m/sec).
Cc	Axial chromatic aberration coefficient (m).
C_m	Magnification chromatic aberration coefficient.
C_S	Spherical aberration coefficient (m).
C_θ	Rotation chromatic aberration coefficient.
d	Diameter of axial bore (m).
d	Displacement by the first magnetic deflector (m).
D(z)	Deflection magnetic flux density (Tesla).
e	The charge of the electron ($e = 1.6\times 10^{-19}$ C).
E_0	Energy of electron beam (Joule).
f	Focal length (m).
h(z), g(z)	Two independent solutions of the paraxial-ray equation.
I	Electric current (Ampere).
$J_n(x)$	First kind Bessel polynomial.
k	Excitation parameter.

L	Lens length (m).
L_1	The distance between pre-deflection and the MOL system (m).
L_2	The distance between the MOL system and the screen (m).
m	The mass of the electron (m = 9.1×10^{-31} Kg).
n	Order of multipole or the power of the multipole.
NI	Magnetic lens excitation (Amp.turn).
$\frac{NI}{\sqrt{V_r}}$	Magnetic lens excitation parameter ($\frac{\text{Amp.turn}}{(\text{Volt})^{0.5}}$).
r(z)	Trajectory radial height along the lens axis (m).
R_1, R_2	Inner and outer radii of coil respectively (m).
r_s	Fluctuation in the electron beam focus (m).
S	Width of air gap of polepieces (m).
V_a	The accelerating voltage (Volt).
V_r	Relativistically corrected accelerating voltage (Volt).
x_1, x_2, x_3 and x_4	The index of the zero for each case of n.
z	The optical axis of system (m).
z_f	The position of focus or focal point (m).
z_i	Image plane position (m).
z_o	Object plane position (m).
α	Trajectory angle with system axis (degree).
α_i	α in image side (degree).
α_o	α in object side (degree).
Γ(x)	Gamma function.
ΔE_0	Fluctuation in the electron beam energy (Joule).
Δr	Fluctuation in the electron beam focus (m).

η	Electron charge to mass quotient $(e/2m)^{1/2}$ $(C/Kg)^{1/2}$.
φ	Angle of the coil (degree).
$\Phi(x,y,z)$	The magnetic scalar potential of the lens (Volt).
$\Phi(r,z)$	The off-axis magnetic scalar potential of the lens (Volt)
$'$, $''$	First and second derivative with respect to z-axis (d/dz) and (d^2/dz), respectively.

List of Abbreviations

Abbreviation	*Definition*
TEM	Transmission electron microscope.
SEM	Scanning electron microscope.
DA	Differential algebraic.
MOL	Moving objective lens.
VAL	Variable axis lens.
AEM	Analytical electron microscope.
STEM	Scanning transmission electron microscope.

بِسْمِ اللّهِ الرَّحْمَنِ الرَّحِيمِ

إِنَّمَا إِلَهُكُمُ اللَّهُ الَّذِي لاَ إِلَهَ إِلاَّ

هُوَ وَسِعَ كُلَّ شَيْءٍ عِلْمًا

صدقَ اللهُ العظيم

طه (٩٨)

الخلاصة

تم استخدام الطريقة التحليلية لإشتقاق صيغ الخصائص البصرية لكل من العدسة والحارف المغاطسي. من هذه الصيغ، تم حساب الخصائص البصرية لكل من العدسة والحارف المغاناطسي. تم اختيار معاملات الزيوغ الكروية واللونية (معاملات الزيغ اللوني المحوري، التكبيري و الدوراني) ذات القيم الاقل من نتائج الحسابات لكل من العدسة والحارف المغناطسي.

أعتمد أنموذج الأس العكسي للحصول على توزيع المجال المحوري للعدسة المغناطيسية. تم استخدام فكرة العدسة الشيئية المتحركة في حساب توزيع مجال الانحراف للحارف المغناطيسي.

تم دراسة الخواص البصرية ذات الرتب الأولى والثالثة لكل من العدسة والحارف المغناطسي. أيضاً، تم الحصول على الخصائص البصرية ذات القيم الاقل عن طريق تغير الرتبة n والمعامل الصفري المصاحب لكل قيمة لل n.

أظهرت الحسابات لكل من العدسة والحارف المغناطسي بإن البعد البؤري، معاملات الزيوغ الكروي واللوني المحوري تتناسب مع قيم ال z عند البؤرة. أيضا، وجد بإن افضل عدسة وحارف مغناطسي التي تعطي اقل زيوغ كروية، لونية محورية هي عندما تكون ال n = 4. كذلك، لوحظ ان زيادة قيم المعامل الصفري لكل حالة لل n، للعدسات والحوارف المغناطسية، تؤدي الى تقليل معاملات الزيوغ الكروية واللونية المحورية. بالاضافة الى ذلك لكل من العدسة والحارف المغناطسي، تمتلك معاملات الزيوغ اللونية التكبيرية والدورانية قيم ثابتة، لكل حالة لل n، تقابل كل قيمة للمعامل الصفري.