Abstract

The One-dimension theory is applied to study the performance of free electron laser amplifier when the electron beam have a uniform density distribution in the direction perpendicular to the helical undulator axis with initial phases uniformly distributed along the interval (0-2π). The matlab 6.1 program is used to built the computer programs that used in this work.

In the present work, the motion of electron are considered inside the helical undulator. The equations of motion for electron inside the undulator, show that three effecting parameters, they are:

1- Undulator magnetic field

2- Undulator period

3- Lorentz factor

The increase in undulator magnetic field leads to increase in the radius of electron trajectory, and the magnitude of transverse electron velocity and at same time leads to decreasing the magnitude of the longitudinal electron velocity.

The above effects of undulator magnetic field be reapted in the case of undulator period. Owing to the differences in the equations that used in this case, it is found differences in waves behavior of electron.

At last, the increase in Lorentz factor leads to decreasing in the radius of electron trajectory and magnitude of transverse electron velocity and also causes an increasing in magnitude of the longitudinal electron velocity.

The effects above discussed as a result of the law of the conservation of energy inside the undulator.

The motion of electron is also considered under the effect of the presence of electric field that may be as an electrical component of electromagnetic wave (laser).

As comparing with first case (when there is no electric field), there are increasing in the magnitude of longitudinal electron velocity and a variation in electron energy. Within this case, the effect of the phase of electron, undulator magnetic field and undulator period are studied.

Acknowledgement

A Praise and thanks be to God; who give me ((faith, patience and courage)) to get along completing this work.

I would like to express my deep appreciation to my supervisors Dr. Mohammed I. Sanduk for suggesting this project, valuable guidance, and encouragement during the development of this project.

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My deep appreciation to my family, friends, and every one who gave me the support.

Zina

Chapter Four

Conclusions And Future Work

4.1 Conclusion

While we have not considered all the possible trajectories that are likely to occur in the proposed experiment, we are confident that the major part of them will be correctly described by our analytic approach. The even simpler approximation appears to be adequate for most purposes.

According to the results obtained in chapter three, conclusion can be found as the following important points:

1. The velocities of electron in the helical undulator in the amplifier FEL depend on many factors like the laser electric field, the undulator magnetic field, Lorentz factor, undulator period, penderomotive phase, time and position.

2. The electron energy depent on the penderomotive phase and the electrons position inside the undulator.

2. Future Work

In the end of this work and according to the results obtianed and conclusions made through out, it is likely to suggest the following point of view to be considered. These suggestions are:

1. Using the three dimentional theory for the simulation of the free electron laser.
2. Study the trajectory of electron in helical undulator with non-uniform electron beam.

Chapter One

Concept of Free Electron Laser

1.1 Introduction

FELs originate in the work carried out in the 1950 and 1960 on the generation of coherent electromagnetic radiation from electron beams in the microwave region. As scientists tried to push power sources to shorter and shorter wavelengths, it became apparent that efficiency of the microwave tube, and the power they produced, dropped rapidly in the millimeter region. It was then realized that this problem could be overcome by using an undulator magnet to modify the beam trajectory, making it possible for the beam to interact with a wave, a way from any metallic boundary [1].

In the middle of the 1970 John Madey and colleagues constructed the first free electron laser, as shown in figure (1.1), operating in the infrared wavelength range.Although Madey first predicted the operation of a free electron laser by means of quantum mechanics.

The operational principle of FEL is similar to that of conventional vacuum-tube devices: it is based on the interaction of electron beams with radiation in vacuum. From this point of view FELs from a separate class of vacuum-tube devices capable of generating powerful coherent radiation at any wavelength from the millimeter to the X-ray part of the spectrum similar to the vacuum-tube devices which generate coherent radiation at any wavelength , from the kilometer to the millimeter range. Also, free electron lasers possess all the attractive features of vacuum-tube devices. FEL radiation is always totally polarized and has ideal, i.e. diffraction, dispersion [2].

FELs have applications in a wide variety of fields due to their special properties of wide tunability, high power, ultra short pulses and high brightness. The wide tenability of FELs enables them to fill the gaps in the electromagnetic spectrum left by conventional lasers and other conventional sources-these are in the far-infrared and X-ray. The capability of ultra-short pulses and high peak power enables applications even in the IR and visible. The high brightness enables a wide variety of application in spectroscopy and related field. So FELs have many applications in condensed-matter physics, chemistry, biology and medicine.

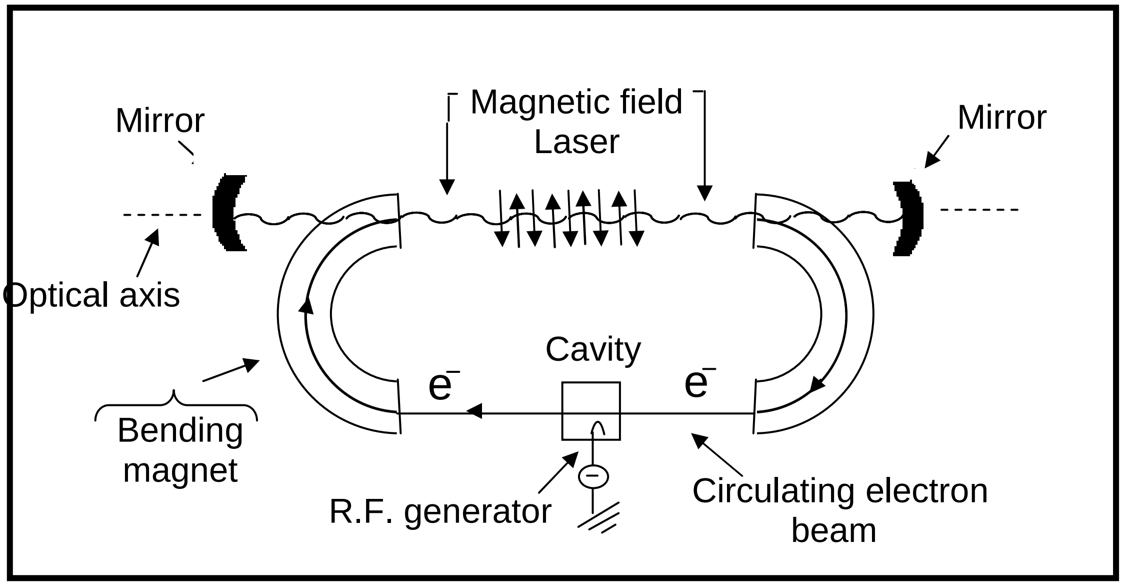

Figure (1.1): Configuration of the first free electron laser operated at Stanford in 1977 [3].

1.2 History of Free Electron Lasers

The history of FELs dates back to as early as 1951, when Hans Motz at Stanford proposed the concept of the undulator, on which the device is based. Between 1957 and 1964, a free electron maser was demonstrated, called a "ubitron" (for undulating beam interaction), that produced peak powers of 150kW at a wavelength of 5mm. FEL research then languished until the 1970, when it resumed from two different but complementary approaches relying on stimulated Compton and Raman scattering, respectively. The Compton scattering regime occurs when the electron current is sufficiently small ;the Raman scattering regime occurs when the electron current is sufficiently large [4].

The FEL in its present form was proposed by John Madey at Stanford in 1976, when it was shown, using an electron beam from a linear accelerator (linac), that the device could amplify radiation from a CO_2 laser [5]. Madey had predicted that an undulator working as an amplifier at optical wavelengths could replace the active medium between two mirrors of an optical cavity. Madey had derived the undulator gain as a consequence of stimulated bremsstahlung radiation based on relativistic quantum, mechanics, where he derived the FEL gain equation using planks constant, $\hbar$. Although $\hbar$ does not appear in the final equation, Madey thought that he had discovered the first truly quantum device! About two years after the Stanford FEL was demonstrated, Colson derived the FEL gain based on pure classical mechanics device [6, 7].

The increases of the interaction efficiency radiation field and electron beam, which was first theoretically derived by Kontradenko and saldin in 1980 and 4 years later by Bonifacio, Pellegrini and Narducci [8].

The UCSB FEL, in 1985 demonstrated single mode operation with a very narrow spectral band width [9].

Even early FELs generally emitted picosecond laser pulses. Experimental and theoretical prograss made in the 1990 have enable FELs to produce much shorter pulses.

In 1992, Pellegrini proposed using a recently developed RF photocathode electron gun coupled to the two-mile Stanford linear accelerator to produce lasing in the hard-X-ray region. The facility was dubbed the linac coherent light source (LCLS). The realization that such a sourse might provid the technology for a new generation of X-ray source led many groups to start development of SASE device in visible to vacuum altraviolet (VUV) spectral range to demonstrate the concept and develop the technologies necessary to build linear light source [10].

After the FEL experiments in 1993, in order to satisfy the needs for the FEL experiments and a chieve high power FEL output, many modifications have been carried out in the beam conditioning section and RF input way, resulting in the increase of the current entered the wiggler and microwave output.

In 1999, a BNL/ANL/UCLA collaboration showed lasing to saturation using a subharmonics seed laser. This technique, called high gain harmonic generation (HGHG), relies on the fact that FELs emit harmonics of the lasing wavelength.

In September 2000, a group at Argonne national lab (ANL) became the first to demonstrate saturation in a visible SASE FEL .

A specially intense blue light shone for the first time on 2002 at BNL'S deep ultra-violet free electron laser (DUV-FEL)[11].

In Iraq, the FEL has a small potential, where all its studies are purely theorical. In 2004, Z.L.Hussein studied the effects of collective fields (space charge and radiation fields) on the output of helical free electron laser amplifier [12].

1.3 Physical Basis of Free Electron Laser

The electrons oscillate inside the undulator and emit spontaneous emission, this radiation acts back on the undulating electrons and bunches them, after that the bunched electrons emit coherent stimulated radiation and amplify the co-propagation electromagnetic wave [13].

1.4 Basic Components of the FEL Amplifier

The three major components of the FEL amplifier device are the accelerator, electron beam and undulator. The accelerator is a system of electrodes for making a beam of electrons. In its simplest form it has a heated filament cathode, and an anode with a hole through which some electron pass. It may also have other electrodes to form the electrons into narrow beam and to control the number of electrons in the beam.The accelerators used to provide the electron beam, are of many types: electrostatic, induction line, radio – frequency (rf) linac, pulsed diode, or strong rings. Some of their basic characteristic, their energy range, and the FEL wavelengths for which they are more commonly used are given in table 1. The choice of electron accelerator also plays a key role in FEL operation [1, 14]. An electron accelerator can produce a beam of relativistic electrons with energy ranging from MeV to GeV, and peak currents ranging from few hundreds of milliamp. to few tens of kiloamp. this electron beam passes through a magnetic device called an undulator, which produces a transverse magnetic field (i.e. perpendicular to the direction along which electron beam is injected) that is static in time, but varies sinusoidally in space. Also helical magnets have certain well depths and limit energy levels [15]. The amplitude of this magnetic field (B_0) is typically a few kilogauss, and the undulator period (λ_0) is typically a few centimeters.

Undelator magnets are of two main types (helical or planar) [16]. In the first case the magnetic field vector rotates around the axis as a function of axial distance, in the second case its direction is fixed, and its amplitude oscillates along the axis (as in fig.(1.2)).

Table1: particle accelerators for FELs [1].

	Energy	Peak current	Pulse length	Wavelength
Electrostatic	1-10Mev	1-5A	1000-2000ns	mm to 0.1mm
Induction linac	1-50MeV	1-10KA	10-100ns	cm to microns
Storage ring	0.1-10GeV	1-1000A	30-1000ps	1 micron to nm
RF linac	0.01-25GeV	100-5000A	0.1-30ps	100 micros to 0.1nm

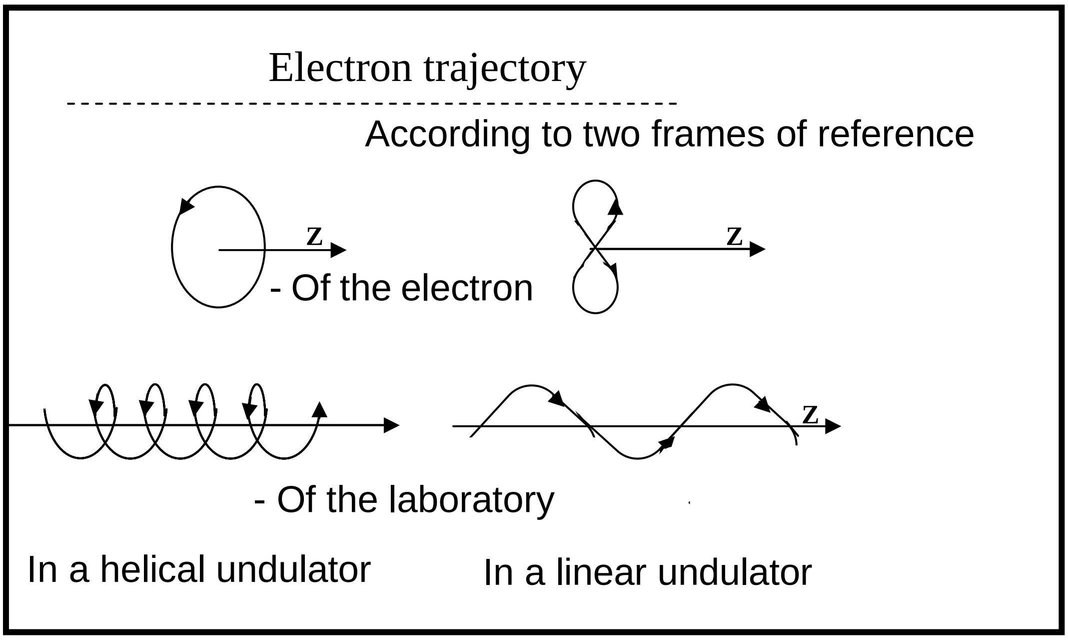

Figure (1.2): Comparison of the trajectories of electrons in a helical undulator of a free electron laser.

1.5 Types of Free Electron Laser

As with vacuum – tube devices, FEL devices can be divided into two classes [17]: amplifiers and oscillators (as in fig. 1.3).

FEL amplifiers is the simplest device. An external radiation field from an external master oscillator seeds the FEL and gets amplified by interaction with the electron beam.The basic working principle of an FEL can be explained best by this device.

The FEL oscillator can be considered as an FEL amplifier with feedback. The radiation in an FEL oscillator grows from fluctuations of the electron beam density. For an FEL oscillator in the optical wavelength range the feedback is carried out by means of an optical resonator which also defines the radiation modes which can be excited in the resonator.

An electron beam passes through an undulator, the initial random field of spontaneous radiation becomes amplified in intensity and enhanced in coherence characteristics. This process is called self-amplifier spontaneous emission (SASE) [18].

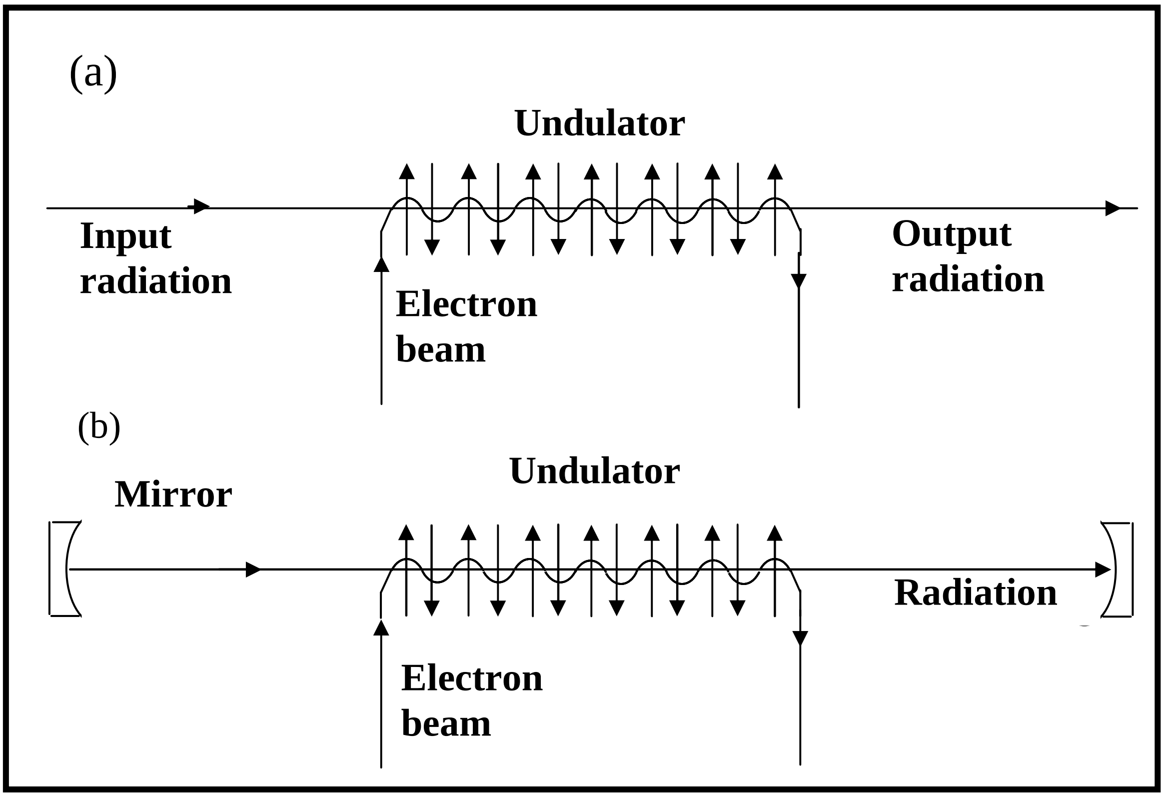

Figure (1.3): Free electron laser configurations
(a) Amplifier (b) Oscillator

1.6 Properties of the Free Electron Laser

Because of the dependence of the radiation wavelength on the undulator period, magnetic field, and electron beam energy-quantities that can be easily and continuously changed-the FEL is a tunable device that can be operated over a very large frequency range . Since the FEL wavelength is not tied to an existing atomic or molecular transition, the resonant wavelength can be tuned from millimeters to nanometers . So FELs have over conventional lasers that emit only a single wavelength beam when their medium is excited to produce light [19].

The efficiency of the energy transfer from the beam kinetic energy to the electromagnetic wave is between 0.1 to a few percent for most FELs, but it can be quite large, up to about 40%, for specially designed system. The beam energy not transferred to the electromagnetic wave remains in the beam and can be easily taken out of the system, to be disposed of, or recovered elsewhere. This fact suggests that high-average power FELs can be designed without the problem, common in atomic and molecular lasers, of heating the lasing medium [20].

The time structure of the laser beam mirrors that of the electron beam. Depending on the accelerator used, one can design system that are continuous-wave (cw) or with pulses as short as picoseconds or sub picoseconds. Tunability, high efficiency, and time structure make FEL a very attractive source of coherent electromagnetic power. In some wavelength regions, like the X-ray, the FEL is unique [21].

Another attractive characteristic of such laser is the high-quality beam emitted by free electrons when they move through a magnetic field . The quality of the electron beam for FELs is measured by the brightness,which is proportional to the electron bunch charge divided by the six-dimensional phase-space volume (the product of the transverse dimensions, the transverse velocities, the bunch length and the energy spread) occupied by electrons in the bunch.

At last there is a brilliance that depending on the status of the art of the electron beam technology, the FEL brilliance can be larger, in some spectral regions (in particular in VUV-X), by many order of magnitude than the brilliance of the existing sources(laser and synchrotron radiation) . Higher brilliance will allow to increase spatial resolution without loosing spectral resolution.

1.7 Aim of the Project

The aim of the project is to study the trajectory of electrons inside the magnetic field through the study of the effects of many parameters such as: undulator magnetic field, undulator period, and Lorentz factor that have an important role to know the behavior of the helical undulator for free electron laser amplifier.

Chapter Three

Simulation, Results, and Discussion

3.1 Introduction

An overview description of the trajectory for uniform electrons beam passing along the axis of a helical magnet for amplifier free electron laser was given here.

First an explanation for computer programs and how makes it to serve the work. The important stage is deal with the electrons behavior and discuss these results with or with out laser beam inside the undulator, through the study to the effect of many parameters like Lorentz parameter, undulator parameter …etc. this have important role to known the efficiency of device and give the best applicable result..

A one-dimensional logarithm was presented for the simulation of the free electron laser undulator that explained in chapter two.

3.2 Input Parameters

In order to study the trajectory of electron we have to restrict our project to a certain parameters. These parameters are chosen from the Stanford 76 model [33]:

-The parameters of electron beam are:

Peak current (I)	70mA
Initial relativistic factor(γ)	48

-The parameters set for applied electromagnetic radiation:

Emission wavelength(λ)	11 μm
Average Power laser	10 MW

The electric field ($\vec{E}$) could be calculated as:

$$\vec{E} = \sqrt{2\mu c S_{av}}$$

where

μ = vacuum permeability.

c = the speed of light.

S_{av} = power of the laser per unit area.

The parameters of the helical undulator are:

+

Undulator length(l_0)	5.120m
Undulator period(λ_0)	.032m
Peak undulator magnetic field(B_0)	.240 T
Undulator parameter(K)	.72

3.3 Programming Steps

The program that has been used was written using Matlab (6.1). The flowchart of the program can be shown in figure (3.1).

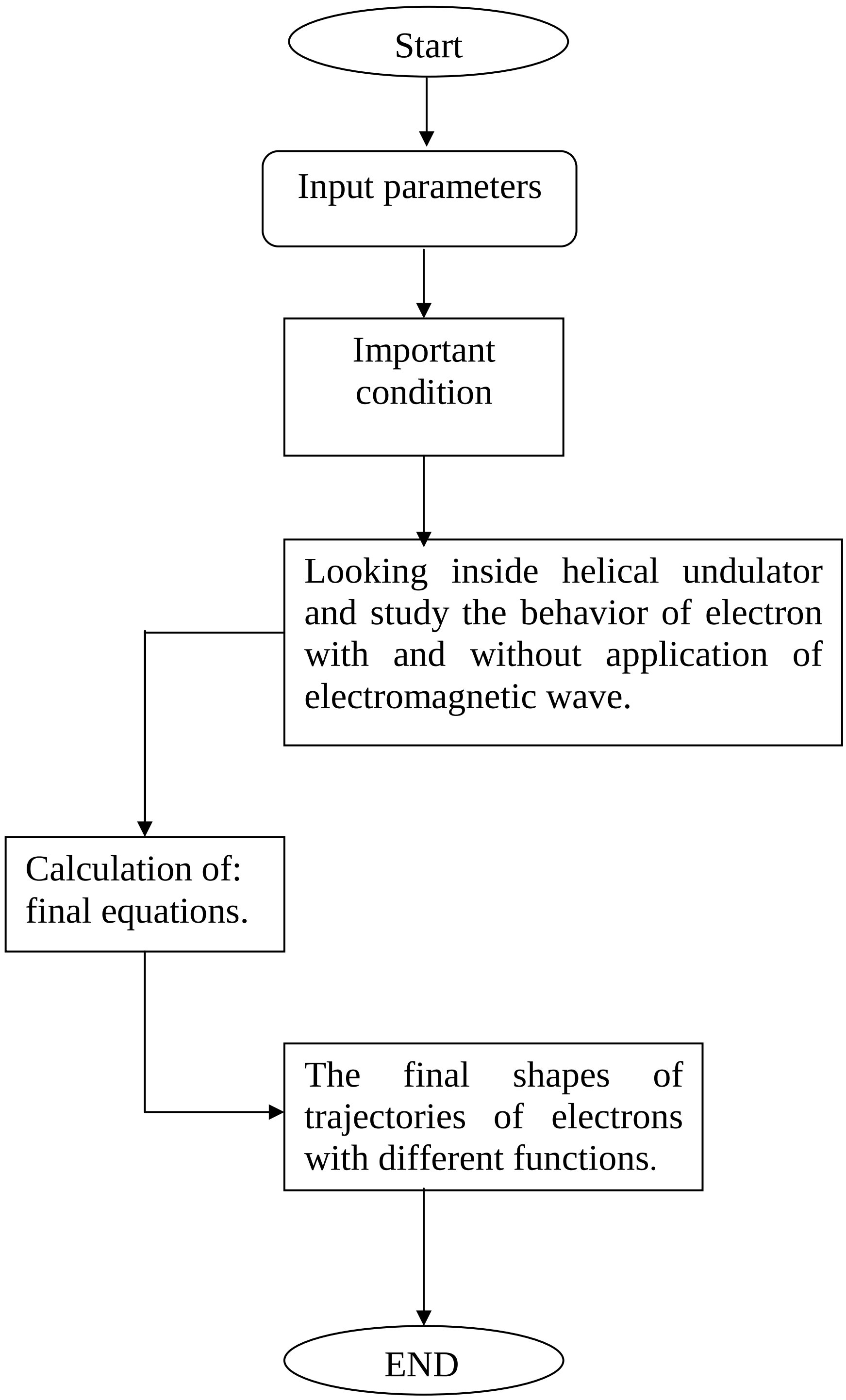

Figure (3.1): Main work steps of program

3.4 The Shape of Trajectory in 3D

In a free electron laser, the active medium is not made of atoms or molecules; it consists of a beam of free electrons that is sent through an undulator magnet which have a field varies sinusoidal with distance. As seen before, the free electrons passing parallel to the z-axis having two components one in x- direction and the other in y-direction, which represent by equations [(2.11) and (2.12)] respectively. If we put these two equations in a computer program along undulator length z=.32m (see Fig.(3.2)),then the program gives results for electrons trajectory in three dimensions represented as a helix about the z-axis with ten undulator period (Fig.(3.3)).

The helical field is shown by a series of rotating arrows (each representing the transverse magnetic field vector) and the electron beam axis is shown as a straight line through the center of the helical field.

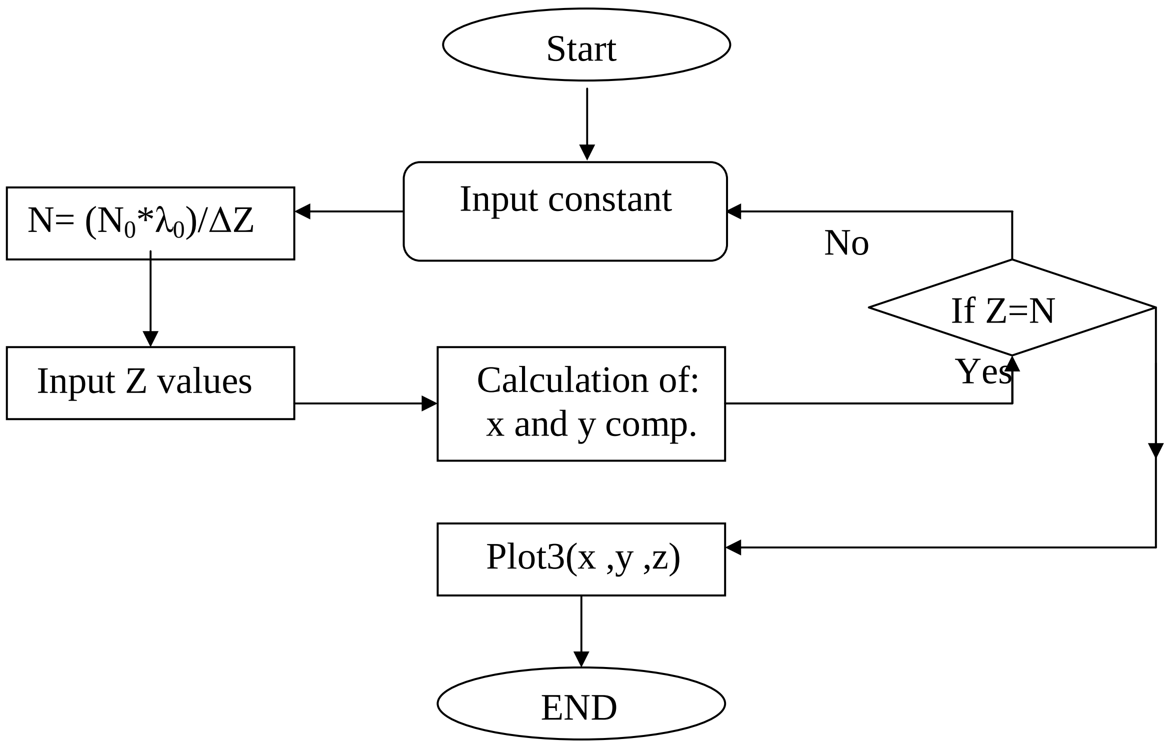

Figure (3.2): Flowchart for electron motion inside helical undulator

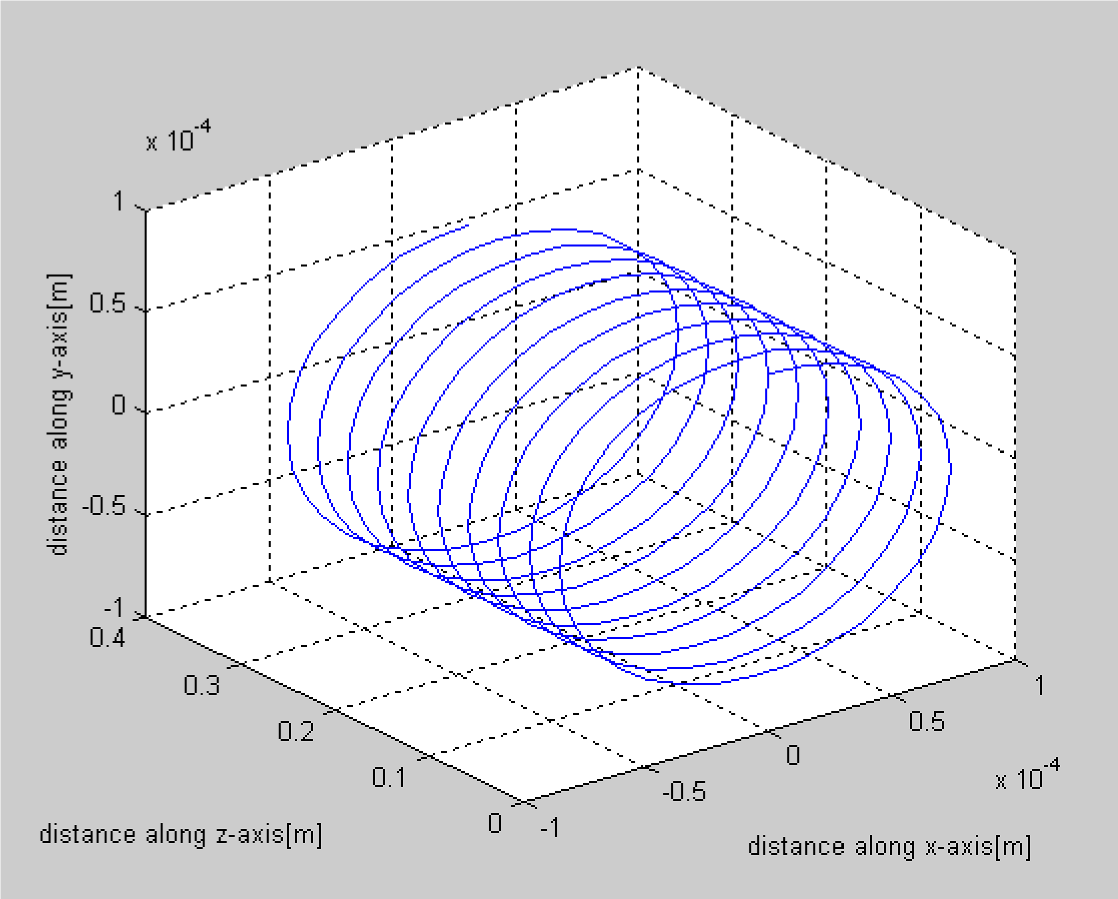

Figure (3.3): Electron trajectory inside the helical undulator.

3.5 The Shape of Trajectory in 2D

The electron moves with a constant longitudinal velocity (v_z) whilst describing a circle, with radius ($r = K\lambda_0 / 2\pi\gamma$). The radius of electron trajectory should have a limit values inside the helical undulator and the exact trajectory of the electron as a function of time will be depend on it .The electrons motion may be represented in two components that gives a circular shape with radius r , Fig. (3.4).

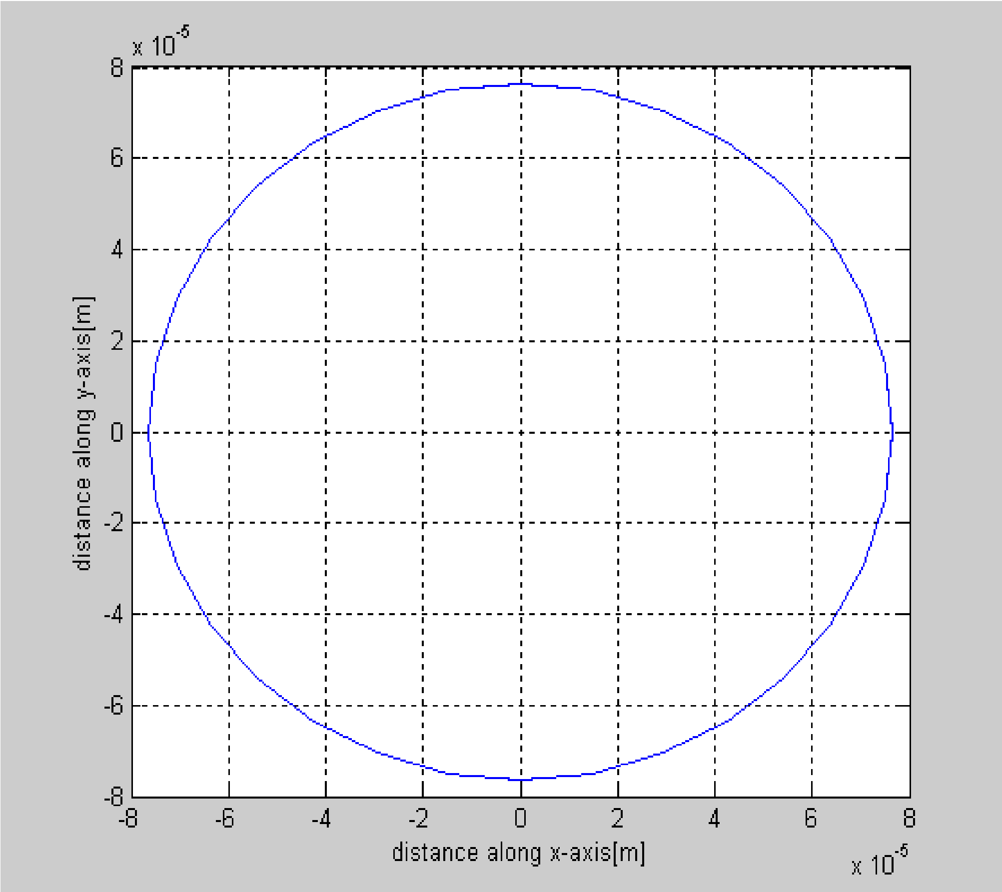

Figure (3.4): The electron motion in two coordinates that give the circle shape.

3.6 Motion of Electron without Application of the Electric Field

3.6.1 Velocities of electron

The electron moves in the helical undulator with two velocities:

1. Transverse electron velocity $v_{\perp}$

It represents the component for the two electron velocities, one in x-axis and the other in y-axis, as seen in equation (2.10).The amplitude of this velocity is $c\theta_s$, so its able to control on the amplitude of this velocity inside the helical undulator by two parameters: undulator parameter(K) and Lorentz factor (γ), as see later.

2. Longitudinal electron velocity v_z

Its also called (z) velocity (eq.(2.34)), v_z is near to the velocity of light with large γ and small K .

There are many parameter affects on the electron velocity like magnetic field, Lorenz factor and undulator parameter. The study of this influences give an imagination of the electron behavior inside the undulator before applying the electric field.

The figures (3.5,3.6); shows the variation of electron velocity in x-direction represented by cosine function, the maximum value of electron velocity at zero point, and sin function in y- direction out off phase π. In this case the number of periods (N_0) is equal to (10).

In the two figures above the values of velocity are small compared with the speed of light that means small transverse electron velocity while the longitudinal velocity is very near to the speed of light which is the important condition for FEL operation.

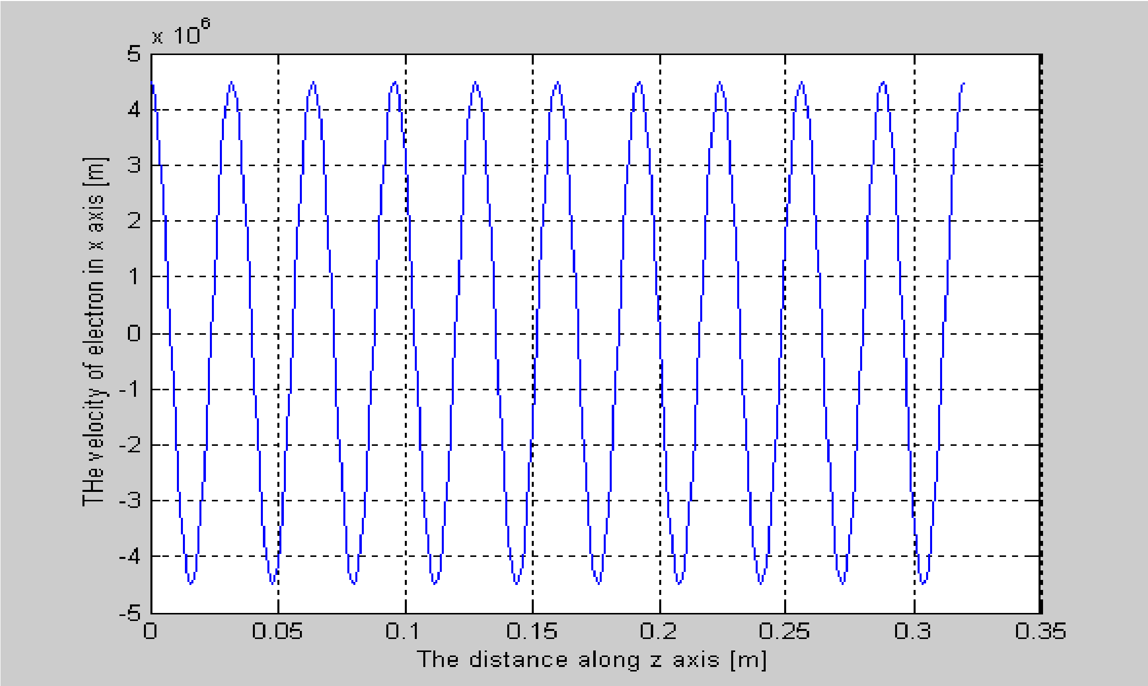

Figure (3.5): The variation of electron velocity in x-axis with the distance along z-axis.

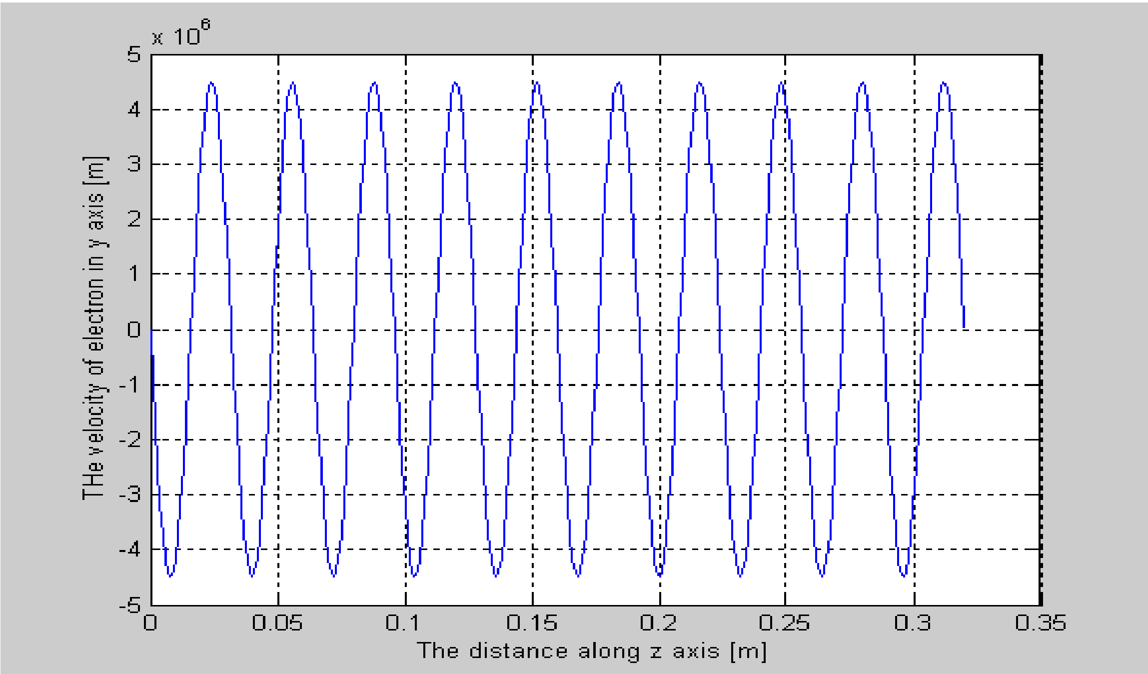

Figure (3.6): The variation of electron velocity in y-axis with the distance along z-axis.

3.6.2 Effect of the amplitude of the undulator magnetic field.

The amplitude of the undulator magnetic field affects on the electrons radius (r) inside the undulator by the undulator parameter (K).In Fig. (3.7); normalized amplitude of the vector proportional to the product of amplitude of undulator magnetic field and undulator period was ($K = 93.37\lambda_0 B_0$).In Fig.(3.8). the amplitude of the undulator magnetic fields is proportional with the radius of electron and there will be a linear relationship between them, i.e. the increasing in the magnetic field of undulator lead to increasing in the electron radius.

If $r = K\lambda_0 / 2\pi\gamma$ and by using equation above the result is:

$$r = 93.37\lambda_0{}^2 B_0 / 2\pi\gamma$$

So

$$r / B_0 = 93.37\lambda_0{}^2 / 2\pi\gamma$$

This value represents the slope of the straight line in (Fig. (3.8)). So is able to obtain the ratio of radius of electron trajectory due to the undulator magnetic field at any point inside the helical undulator. By using this graph and compare it with the theoretical value, we get an important role in FEL application.

Since ($r = V_\perp / \omega$)and ($V_\perp$)depend on(K / γ),so (r)depends on K and γ as well as λ_0.Therefore r depends on B directly so as $V_\perp$.The undulator magnetic field will be affects the velocities of electron in different ways. Increasing in magnitude of the magnetic field lead to increasing in the value of amplitude of transverse electron velocity and also decreasing the value of the longitudinal electron velocity as seen in Fig.(3.9) and Fig.(3.10) at the same condition(γ=48, λ=0.032m).Since the energy is conserved, that means any increasing in $V_\perp$ causes a decreasing in v_z .

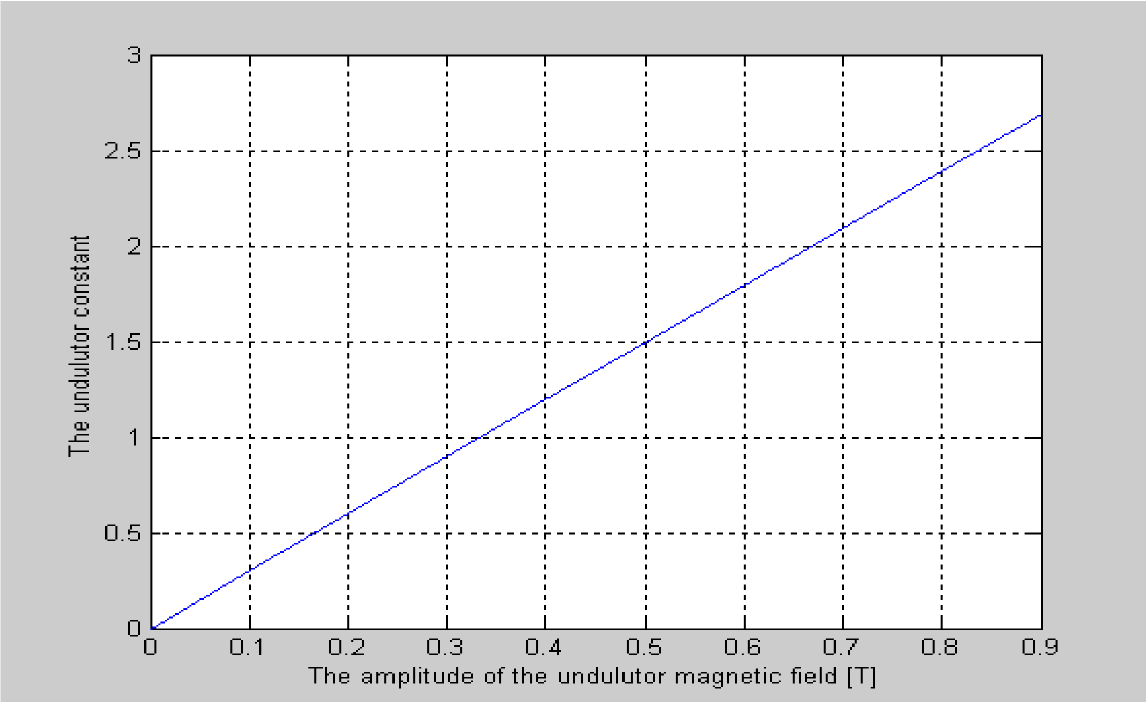

Figure (3.7): Relationship between undulator period and the amplitude of the undulator magnetic field with λ_0 =0.032m.

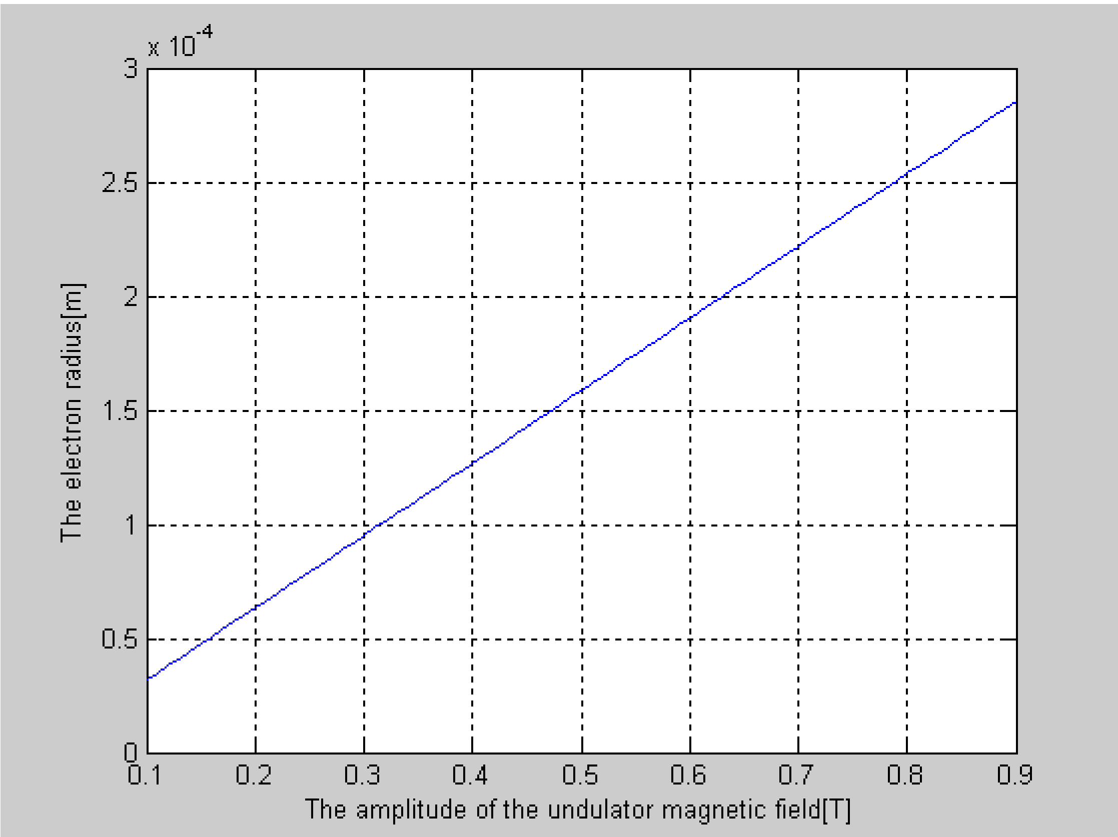

Figure (3.8): The effect of the amplitude of the magnetic field on the electron radius.

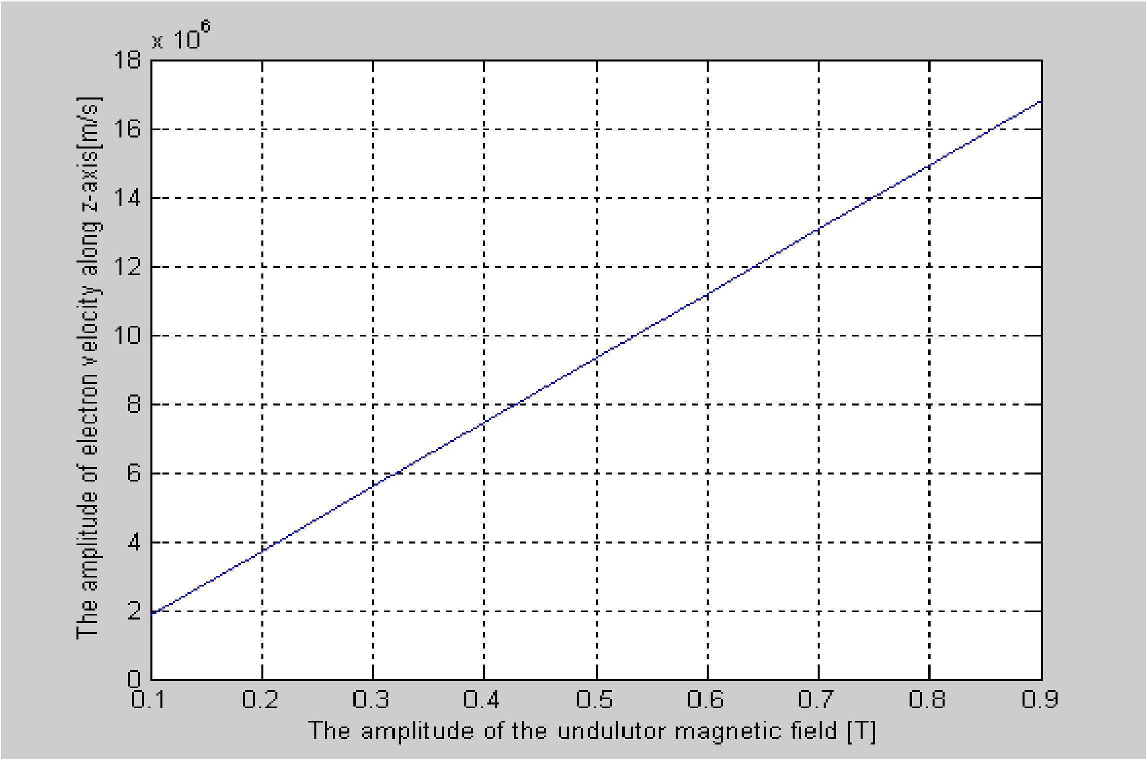

Figure (3.9): The effect of the amplitude of the magnetic field on the amplitude transverse electron velocity.

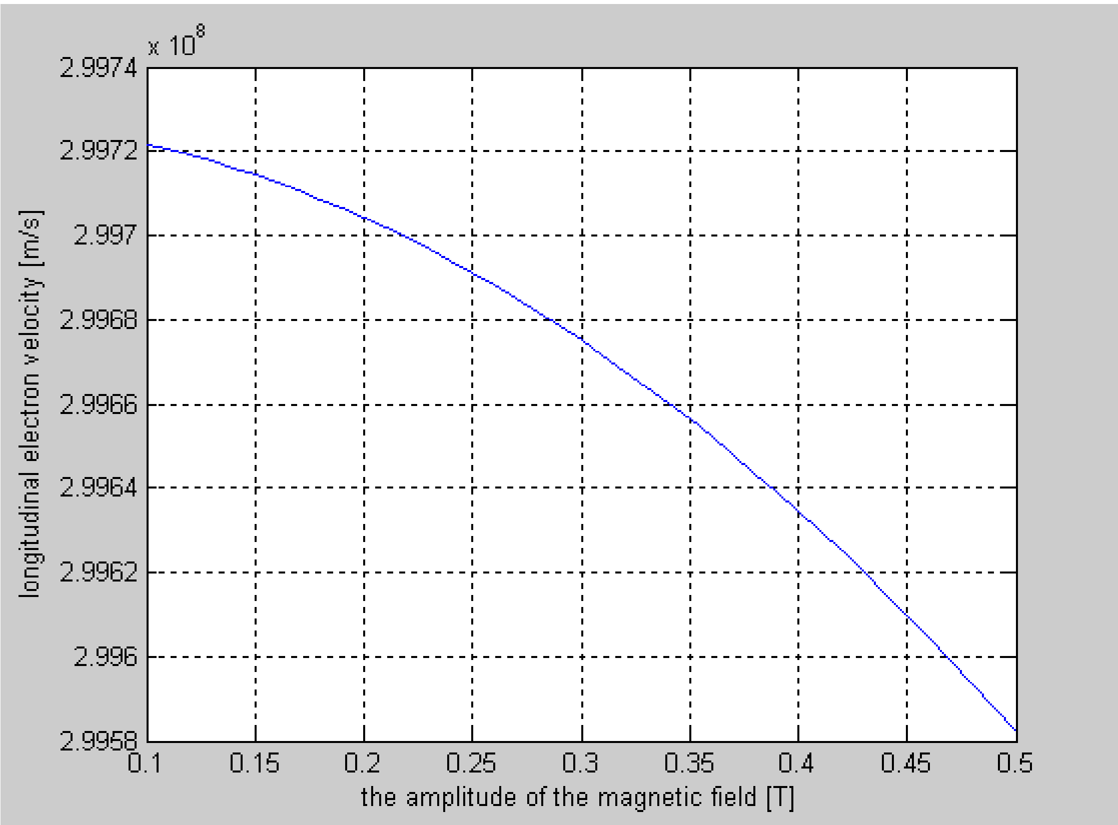

Figure (3.10): The effect of the amplitude of the magnetic field on the longitudinal electron velocity

3.6.3 Effect of the Lorentz factor.

When traversing the undulator, the electrons are subject to acceleration and radiate electromagnetic wave. This spontaneous emission is fundamental to the operation of FEL. One relativistic electron traversing an undulator magnetic emits radiation in a narrow cone, with angular aperture of order ($1/\gamma$).The aperture ($1/\gamma$) of the synchrotron radiation cone can be compared with the angle between the electron trajectory and the undulator axis, (K/γ). If K is less than 1, the emitted radiation is contained within the synchrotron radiation cone, and the emission is predominantly in a single line. If K is higher than 1, the synchrotron radiation cone sweeps an angle larger than its aperture. In this case, the spectrum is rich in harmonics and approaches the synchrotron radiation spectrum when K is very large when $K>>1$, the magnet is usually referred to as a wiggler, reserving the name undulator for the case K less than or in the order of one.

As seen, the Lorentz factor relation is ($\gamma^2 = 1/1-\beta^2$) where (β) is the ratio of the velocity of electron due to the velocity of light ($\beta = v/c$), because the important condition v ≈ c the amount of ($1/\gamma^2$) should be very small so the values of γ should be limit according to this condition.

Because r proportional with (K/γ) and so as $V_\perp$ then γ will effect in same way on r and $V_\perp$, Fig.(3.12) show that there are inverse proportional between γ and r also according to the mathematical relation between them and so as in Fig.(3.13).According to the law of conservation energy and because ($\varepsilon = (1/2)mV^2$), V should be conserved, V represent the total velocity of electron inside the undulator ($V^2 = V_\perp{}^2 + v_z{}^2$) so the effect of Lorentz factor on velocities of electron take the form: when γ is cause a decreasing in $V_\perp$ at same time its causes increasing in v_z as seen in (Fig.(2.13)and Fig.(2.14)).

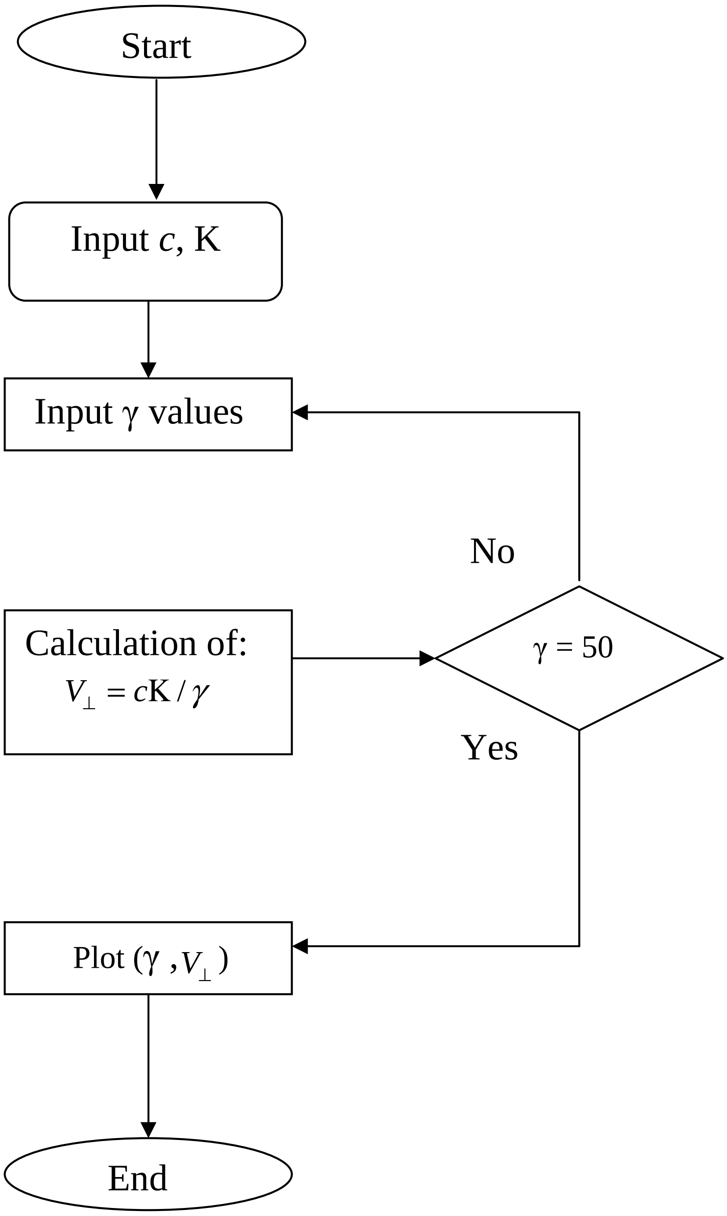

Figure (3.11): the flowchart of the influence of Lorentz factor on the transverse electron velocity.

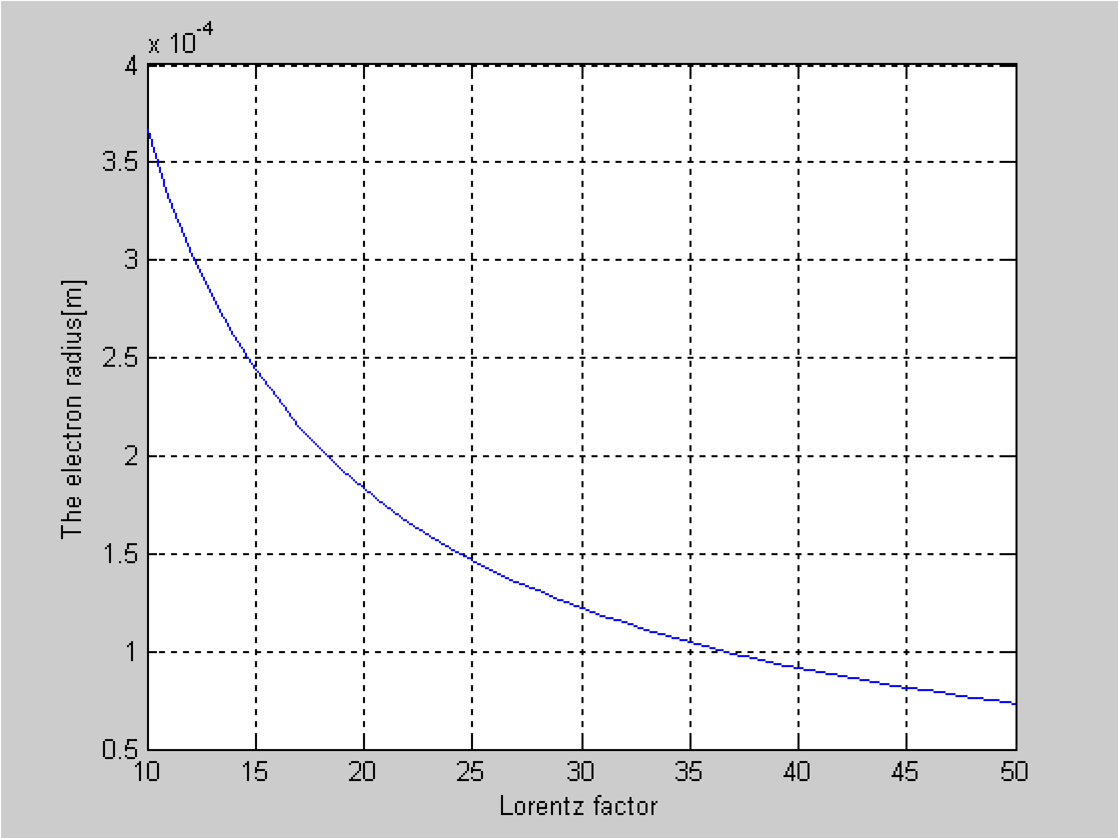

Figure (3.12): The effect of Lorentz factor on the electron radius.

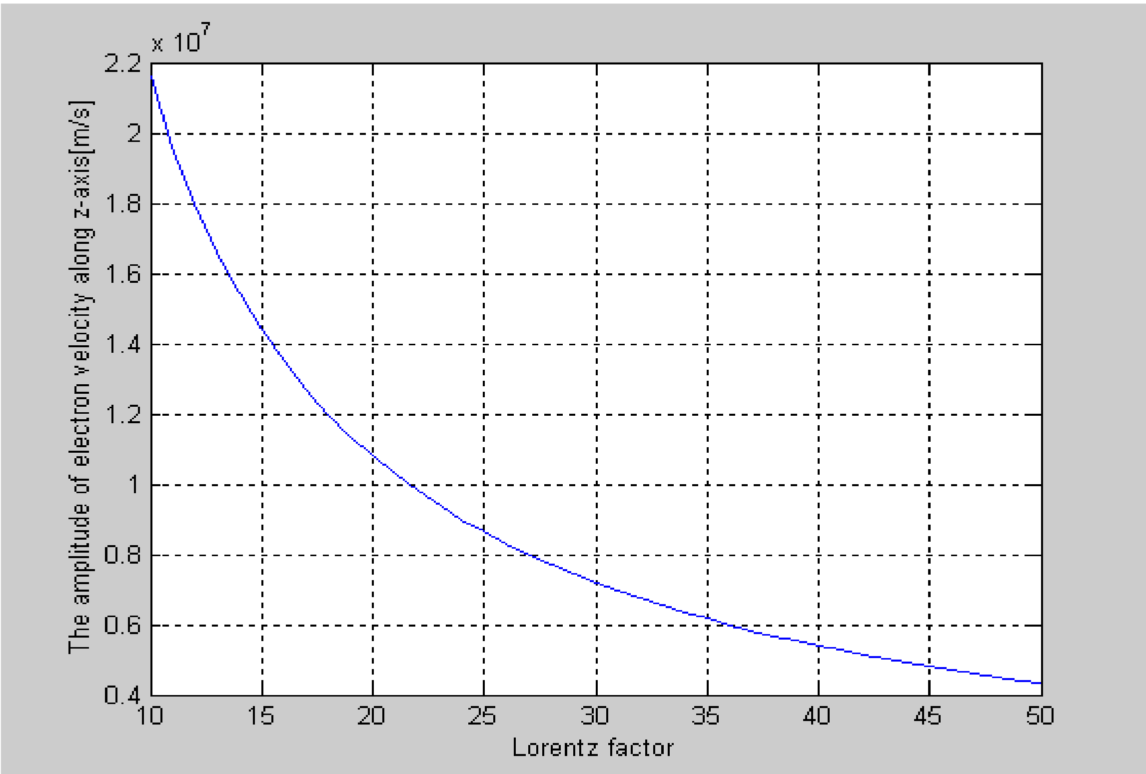

Figure (3.13): The effect of Lorentz factor on the amplitude of transverse electron velocity

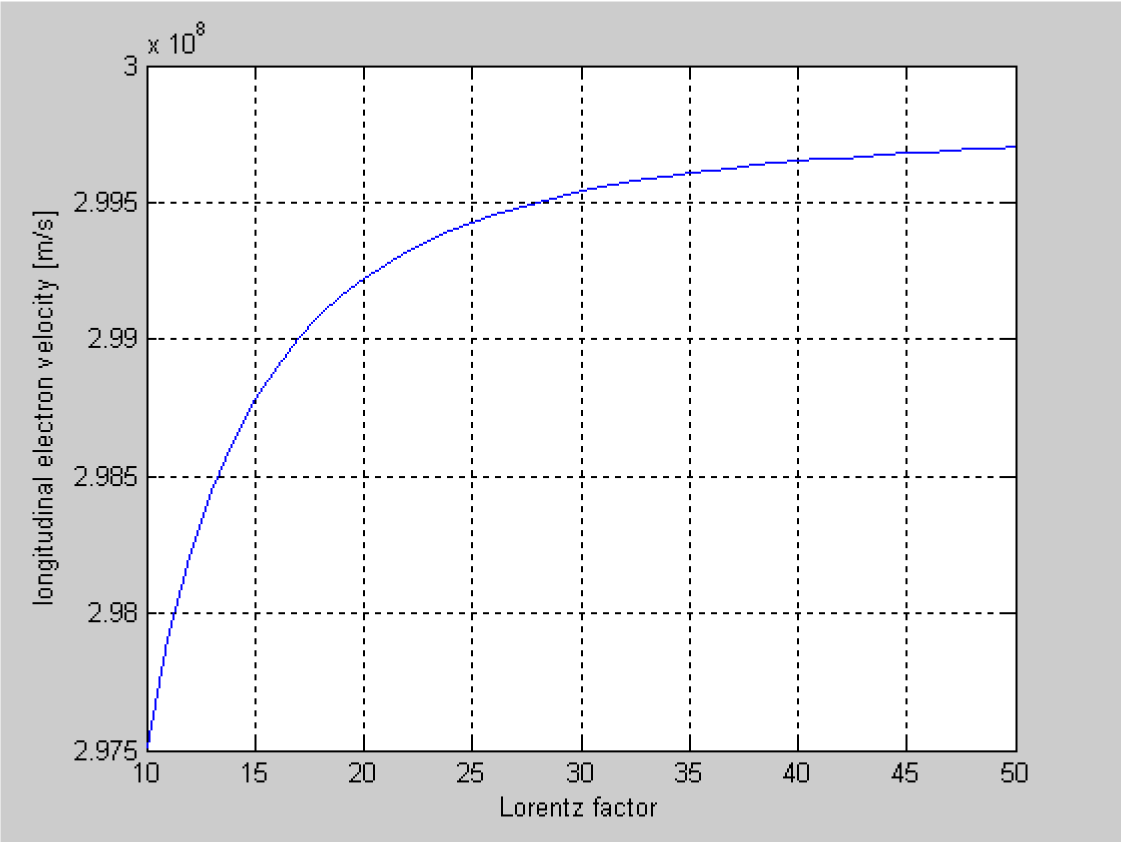

Figure (3.14): The effect of Lorentz factor on the longitudinal electron velocity

3.6.4 Effect of the undulator period

The undulator period is very important parameter that gives the limitation for undulator length. For example, let the undulator period (λ_0 =3.2 cm) and the total number of period (N_0=160) the undulator length will be equal to (5.12 m), when λ_0 increases the undulator length increases also.

Since r proportion with $\lambda^2{}_0$,so Fig.(3.16) show a non linear relationship between them and also any increasing in λ_0 lead to increasing in r ,as a result, $V_\perp$ be increase also (Fig.(3.17)) and v_z be decrease in same time according to the law of conservation energy (Fig.(3.18)).

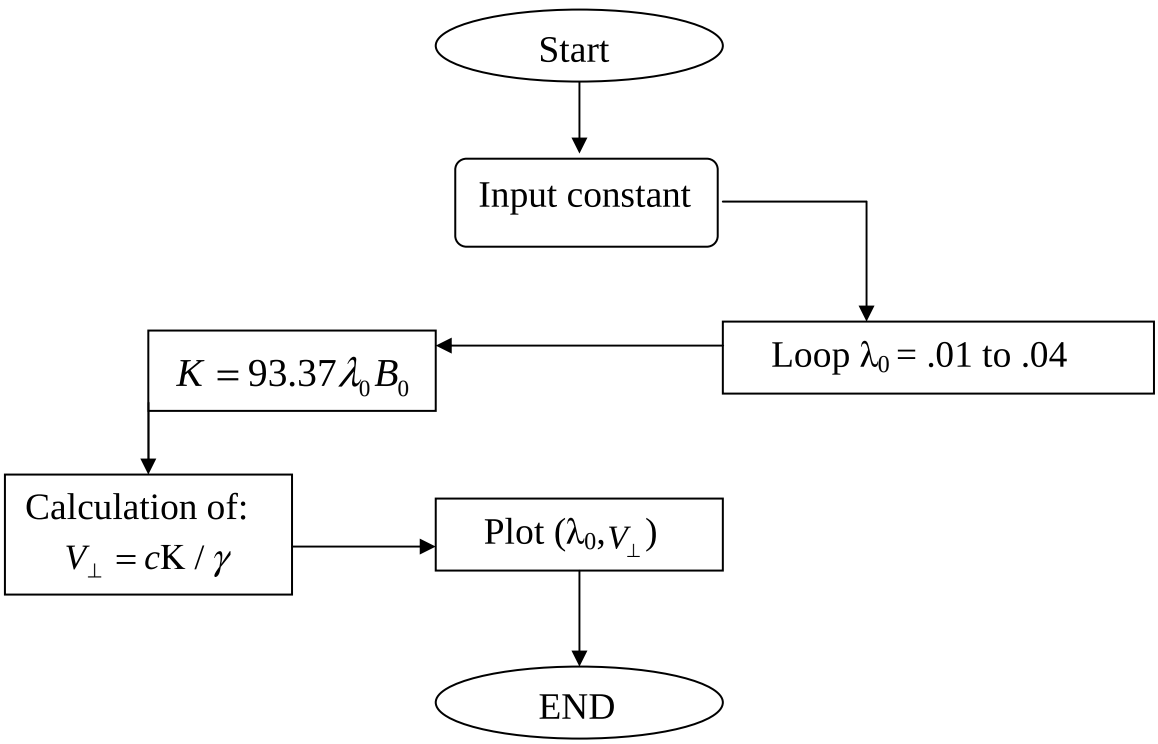

Figure (3.15): Flowchart of the influence of undulator period on the velocity of electron.

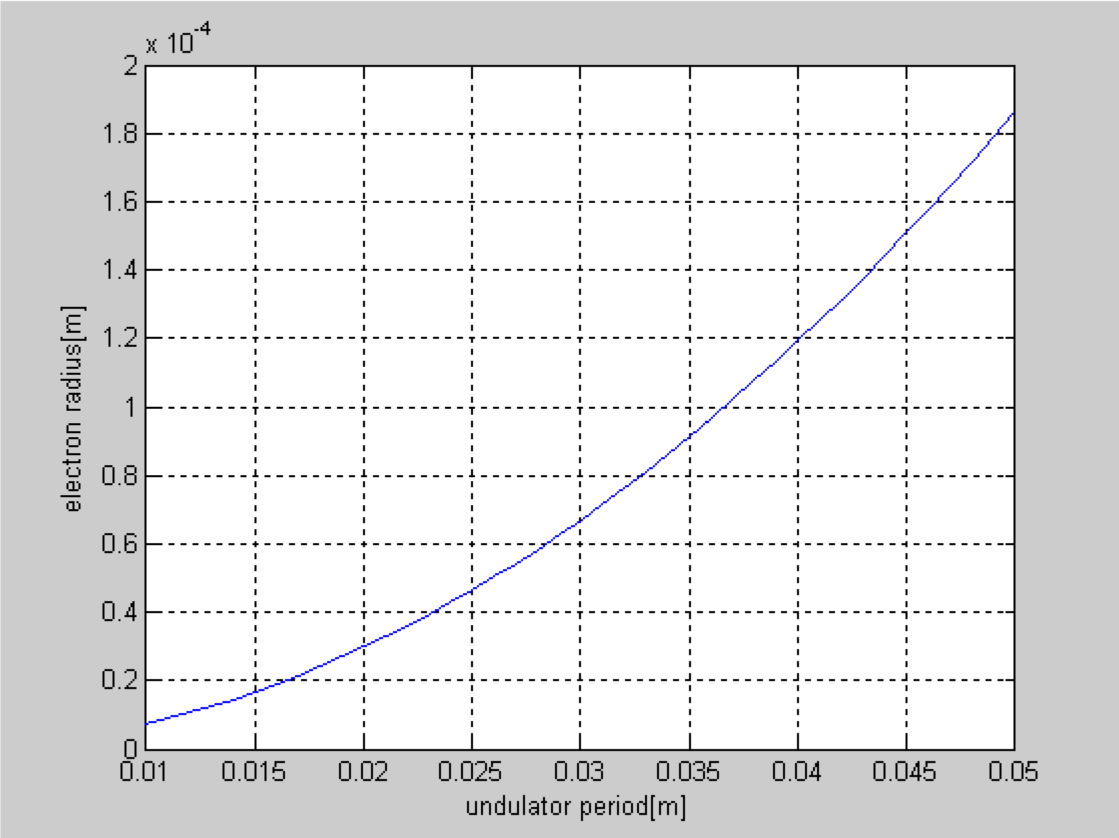

Figure (3.16): The effect of the undulator period on the electron radius

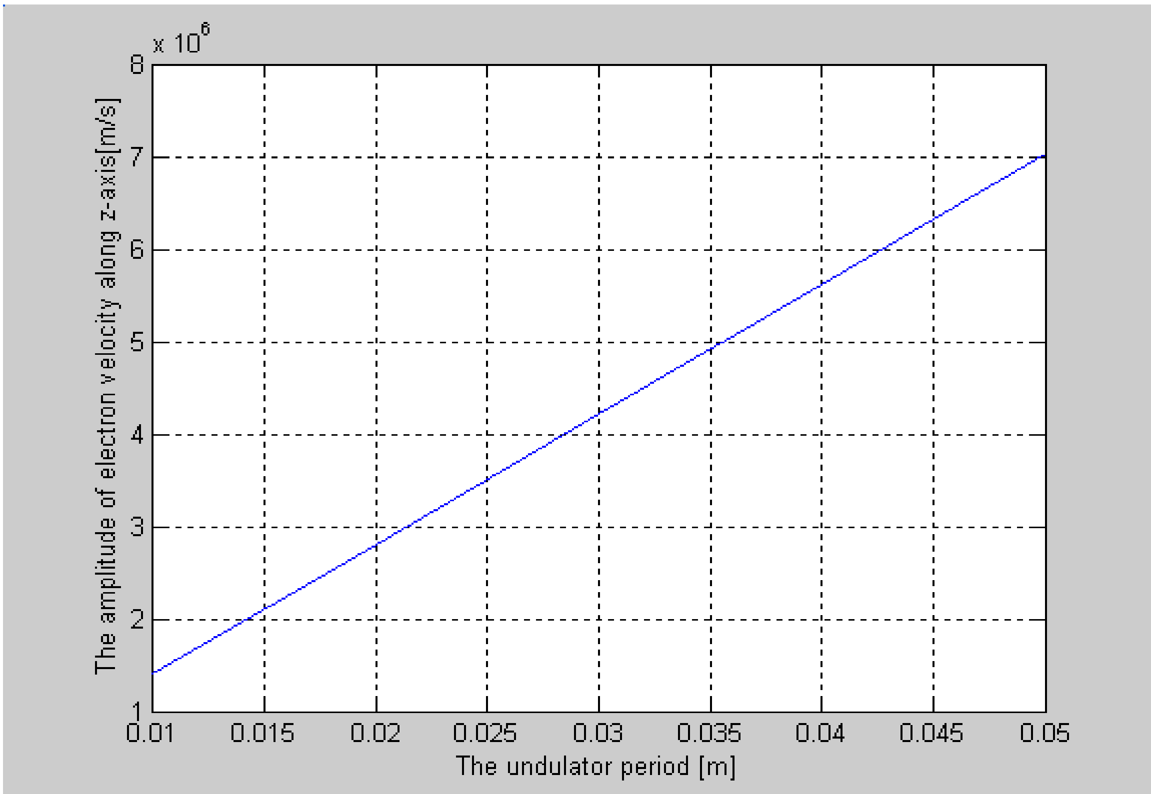

Figure (3.17): The effect of the undulator period on the amplitude of transverse electron velocity

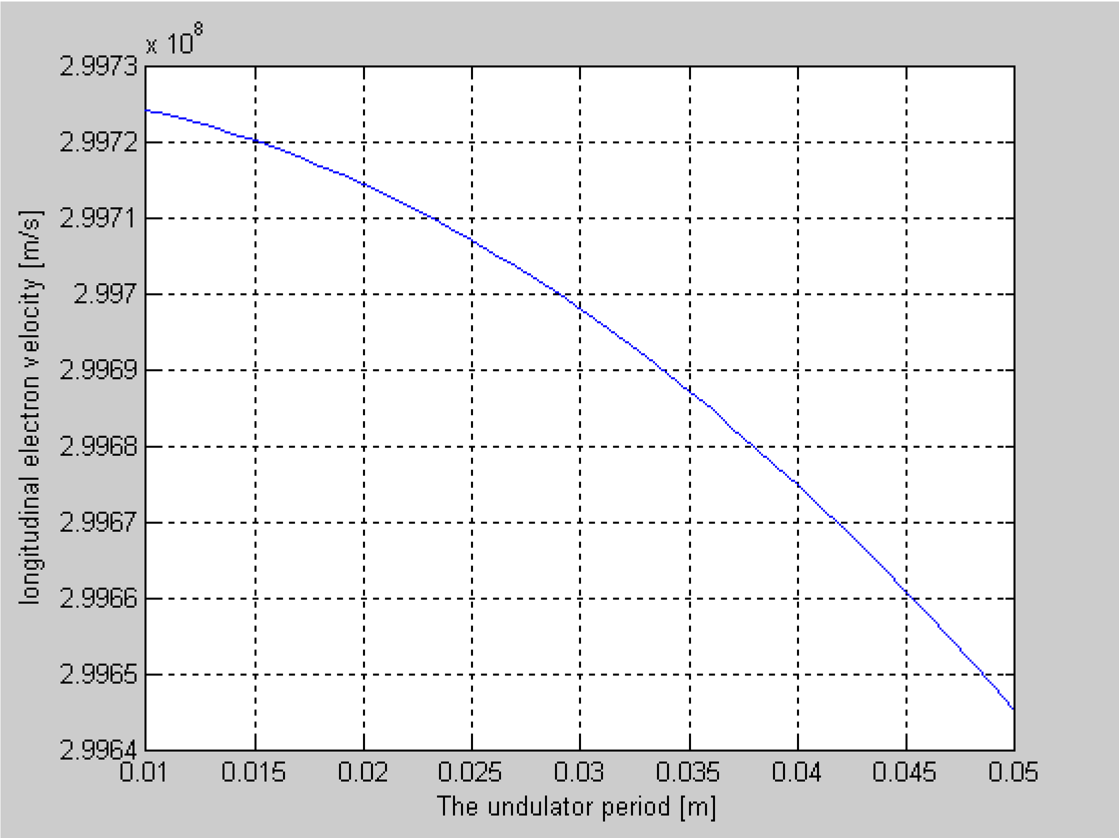

Figure (3.18): The effect of the undulator period on the longitudinal electron velocity

3.7 Electron Motion in a Helical Field after the Application of Laser

A laser field is superimposed on the oscillating electron.Because the electric field of the laser radiation is transverse; it can do work on the transversely oscillating electron and cause them to accelerate or decelerate [35]. So the direction of the longitudinal electron velocity is very important when electric field is applied inside the helical undulator.

The longitudinal electron velocity as given in equation (2.38) in chapter two, with small charge that the quantity $(k_0 z + kz - wt)$ replaced by the ponderomotive phase (ψ). Near the resonance, the argument of the sine term oscillates rapidlly between +1 and -1, and its contribution averages is zero. Different electrons show different phases ψ and therefore experience the longitudinal electron velocity with different magnitudes and direction, leads to bunching of the electrons as shown in (fig.(3.19)).

As given previously, λ_0 is proportional to K, in equation(2.34) v_z depend on K with constant γ but in equation(2.38) v_z depend on K with other parameter called optical vector potential (K_s), which comes from the application of the electric field, K_s represent as increasing factor on v_z, so the longitudinal electron velocity after the application of the electric field (fig.(3.20)) be larger than the longitudinal electron velocity befor the application of the electric field (fig.(3.18)) .

Since B_0 depends on K, and λ_0 depend on K, so B_0 and λ_0 affects on v_z, in the same way under the same condition,as shown in figure (Fig.(3.20) and Fig.(3.21)).

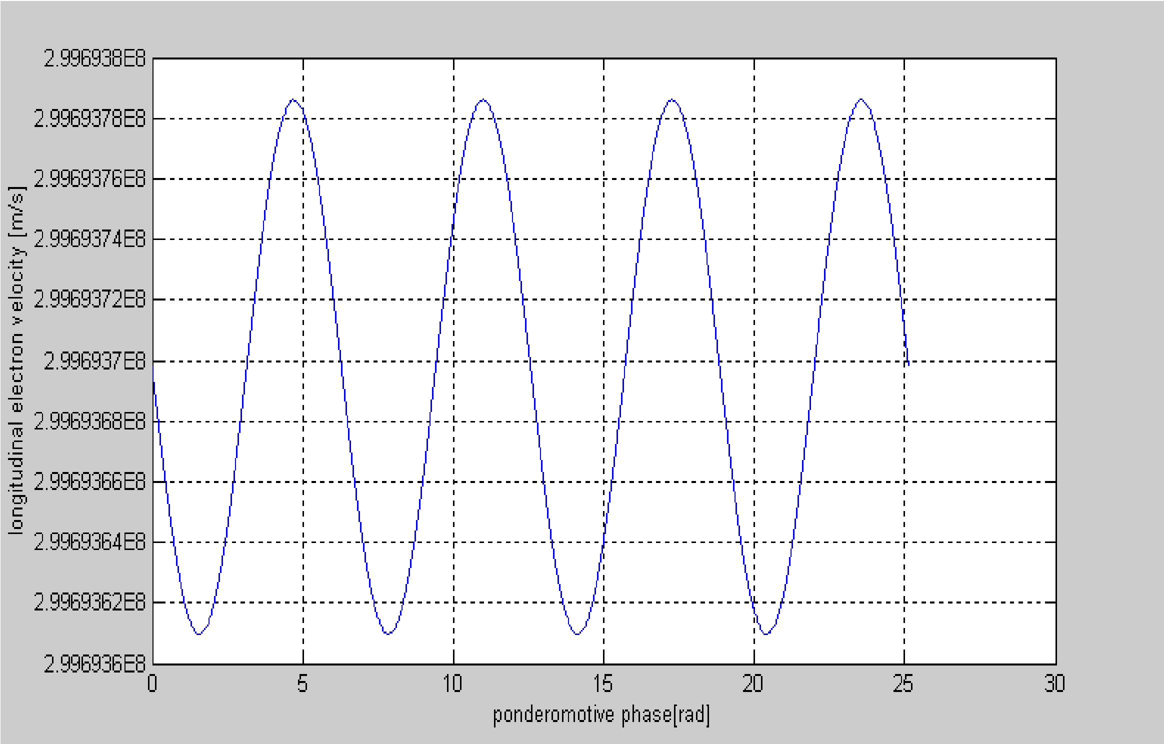

Figure (3.19): The relationship between the penderomotive phase and longitudinal electron velocity E=27.45e+03V/m and B=0.240T.

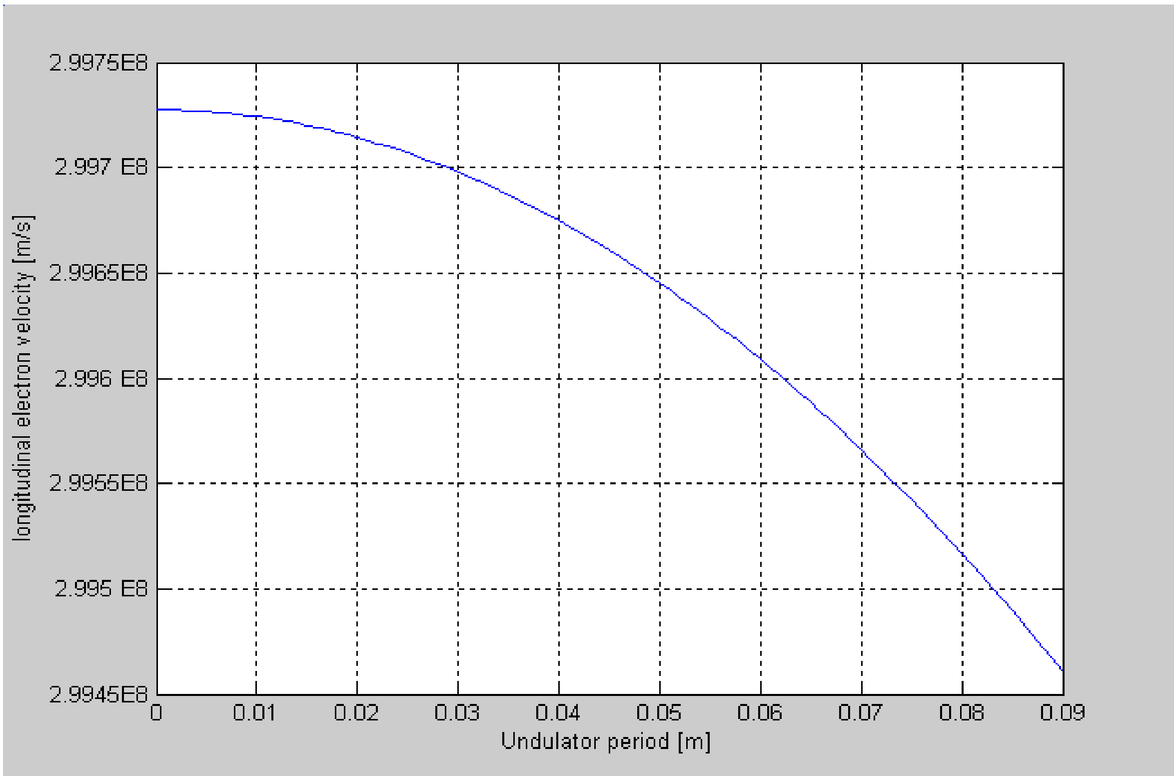

Figure (3.20): the effect of the undulator period on the longitudinal electron velocity with E=27.45e+03V/m and B=0.240T.

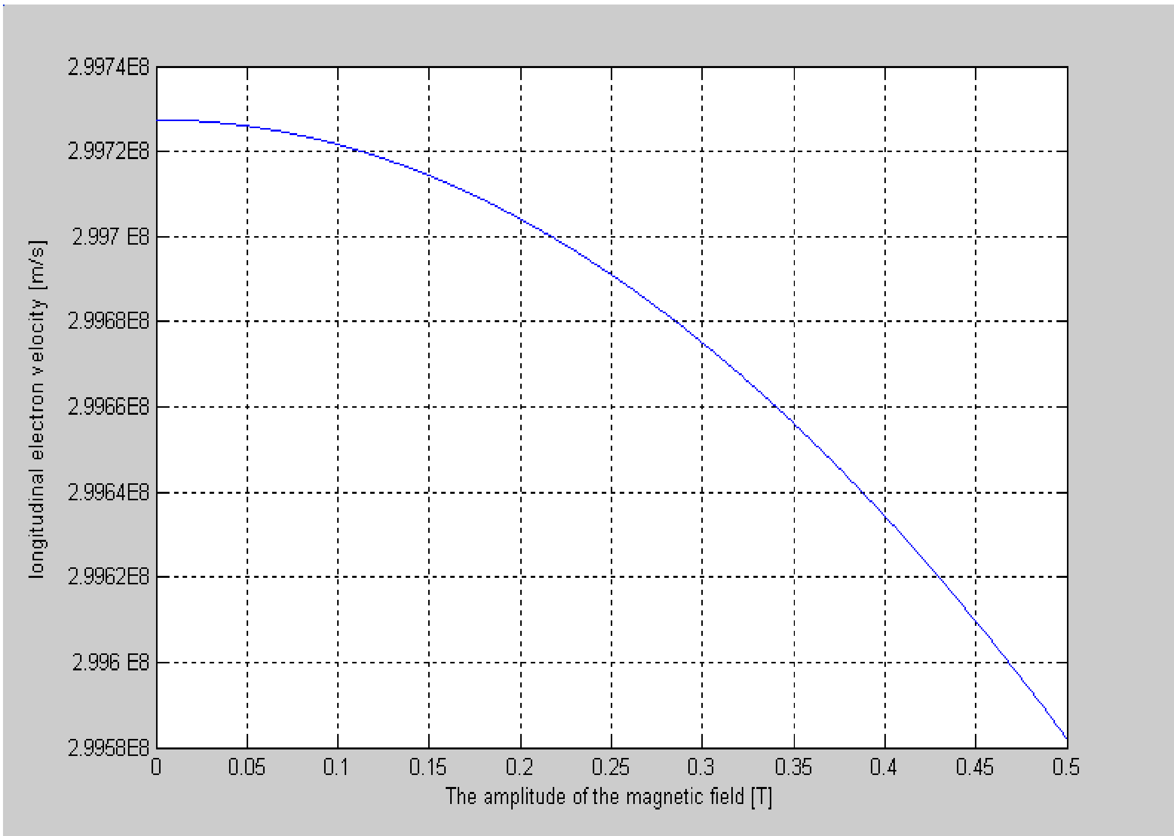

Figure (3.21): The effect of the magnetic field on the longitudinal electron velocity with E=27.45e+03V/m

3.8 Transverse Electron Velocity as a Function of Time and Position.

When the electrons passing through the undulator, the magnitude and direction of its velocity changes according to how the electron far from the electric field of the incident electromagnetic wave. When the electron very close to the electric field, there are an interchange of energy between them so the electrons have different energies that gives different velocities for electrons. These velocities can be found using Fig.(3.23),in this figure a maximum region when electron have maximum transverse velocity and minumum region when transverse electron velocity in minumum value, could be seen.

Each electron have different position at different times inside the undulator ,the transverse electron velocity at certian point be found by limit the position of electron with calculate the time that its used to reach at this point.

Equation (2.29) used as a main equation in computer program which gives Fig.(3.23).

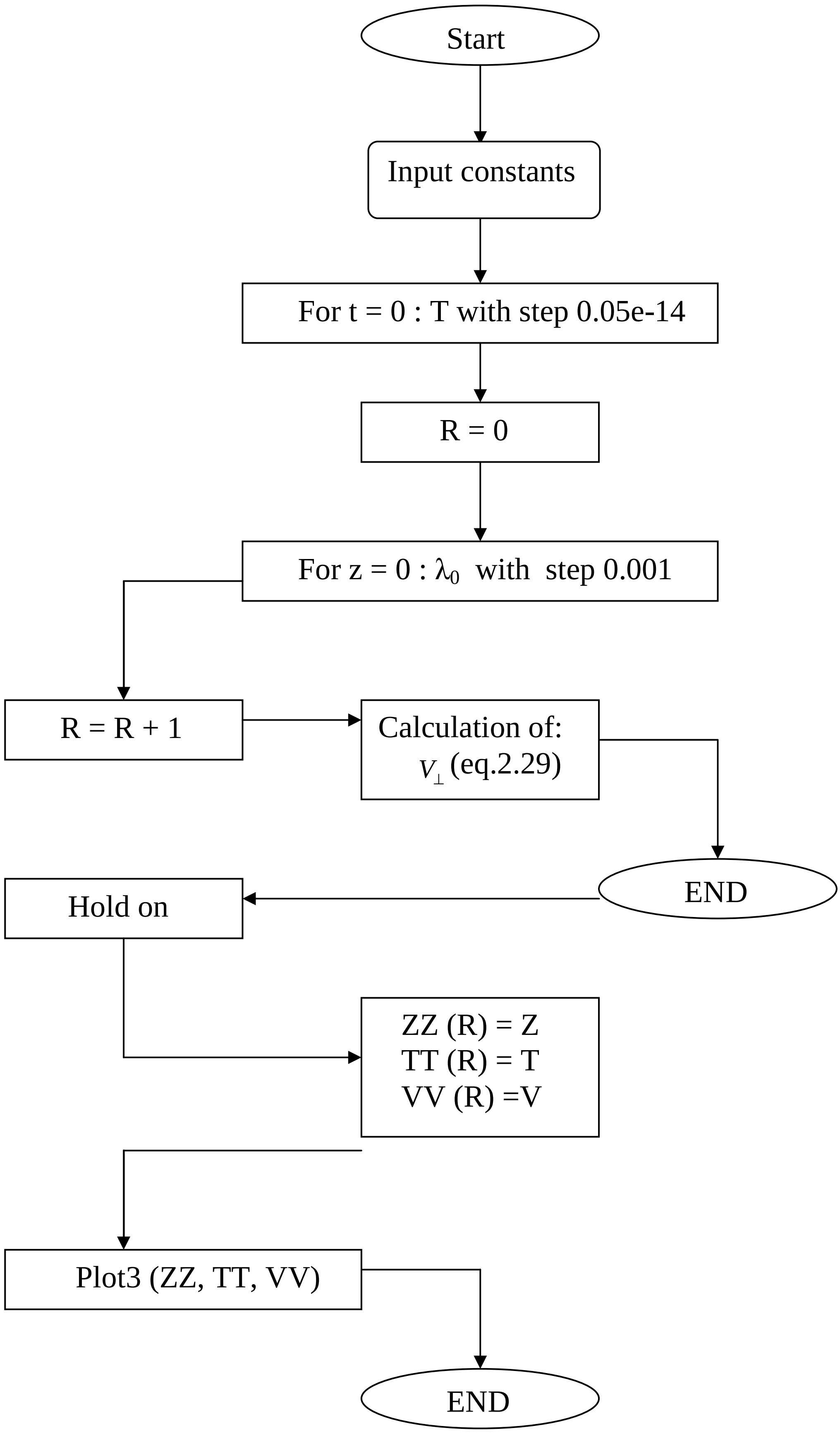

Figure (3.22): The flow chart of electron velocity as a function of time and position.

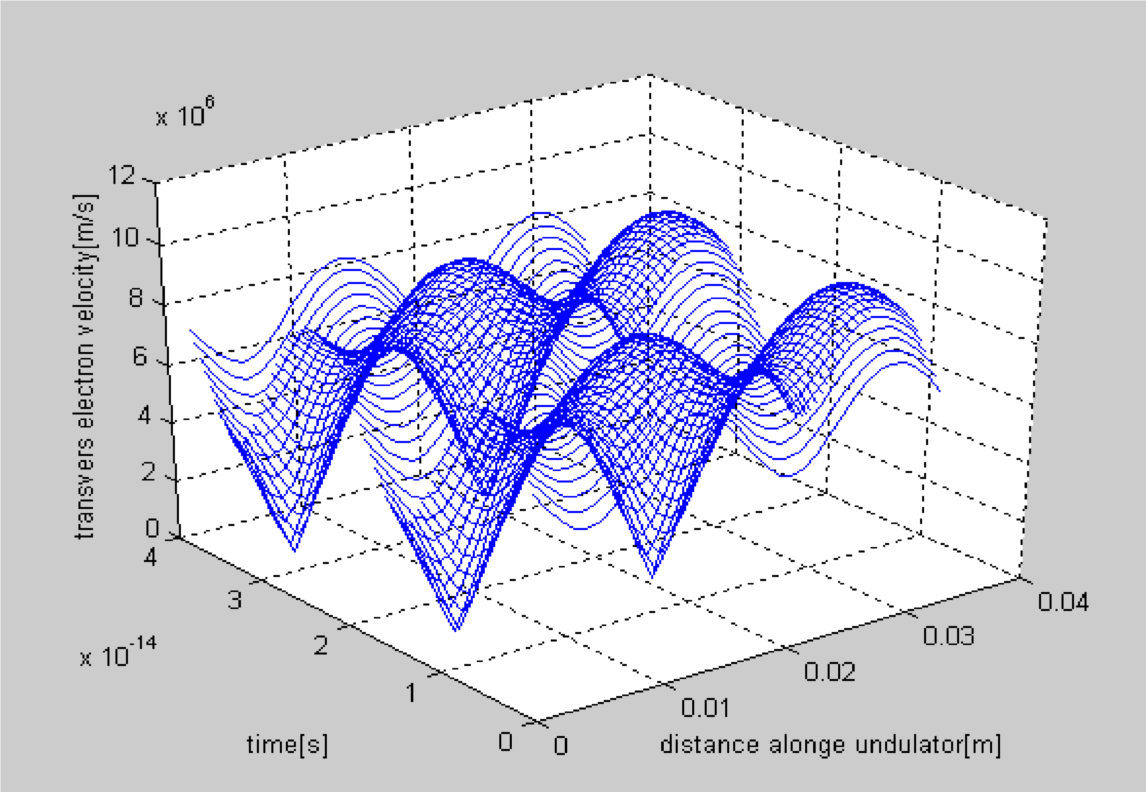

Figure (3.23): The electron velocity as a function of time and distance when K_s=0.940

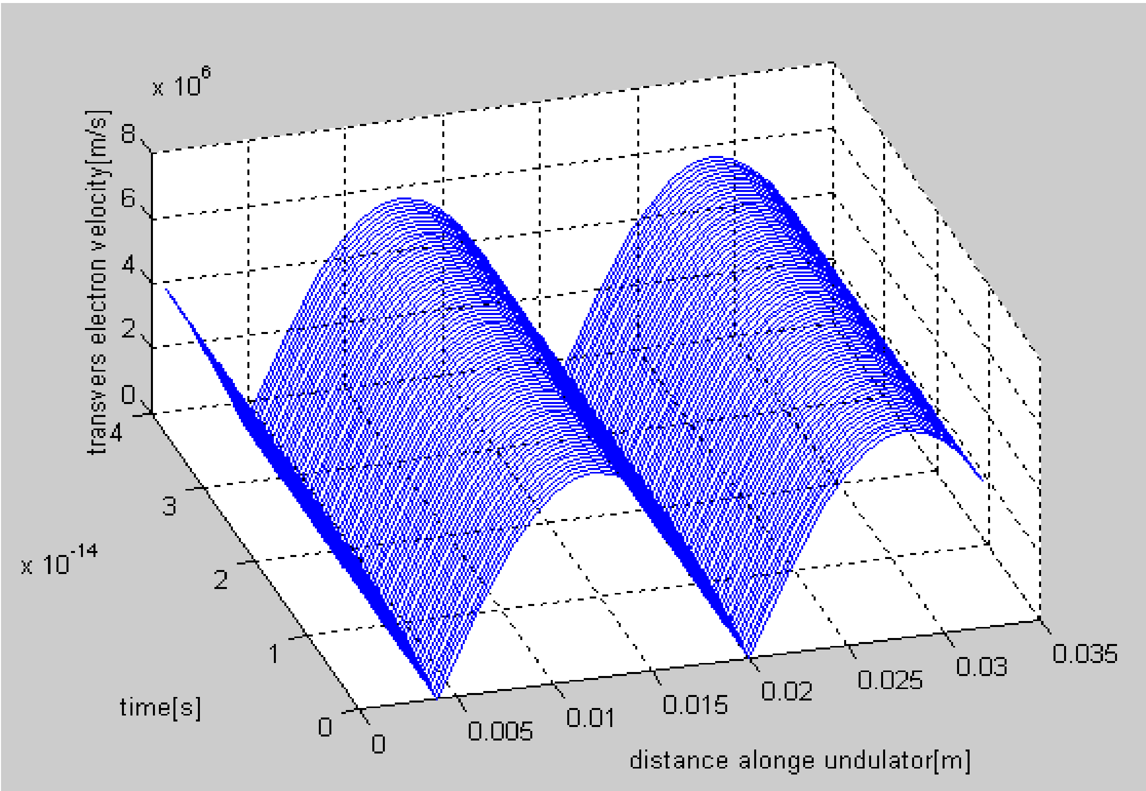

Figure (3.24): The electron velocity as a function of time and distance when K_s=9.40e-10

3.9 Energy Variation of Electron

The electron was accelerated or decelerated by the laser electric field depending on the phase of the optical field with respect to the electron oscillation. Electrons which are oscillating in phase with the optical field are accelerated, while those half an optical wavelength a head or behind are decelerated [34]. So its able to find the electron energy by using the phase of electron with limiting its position inside the helical undulator (Fig.(3.25)).

Equation (2.24) was used as a main equation for computer program to gives the shape below.

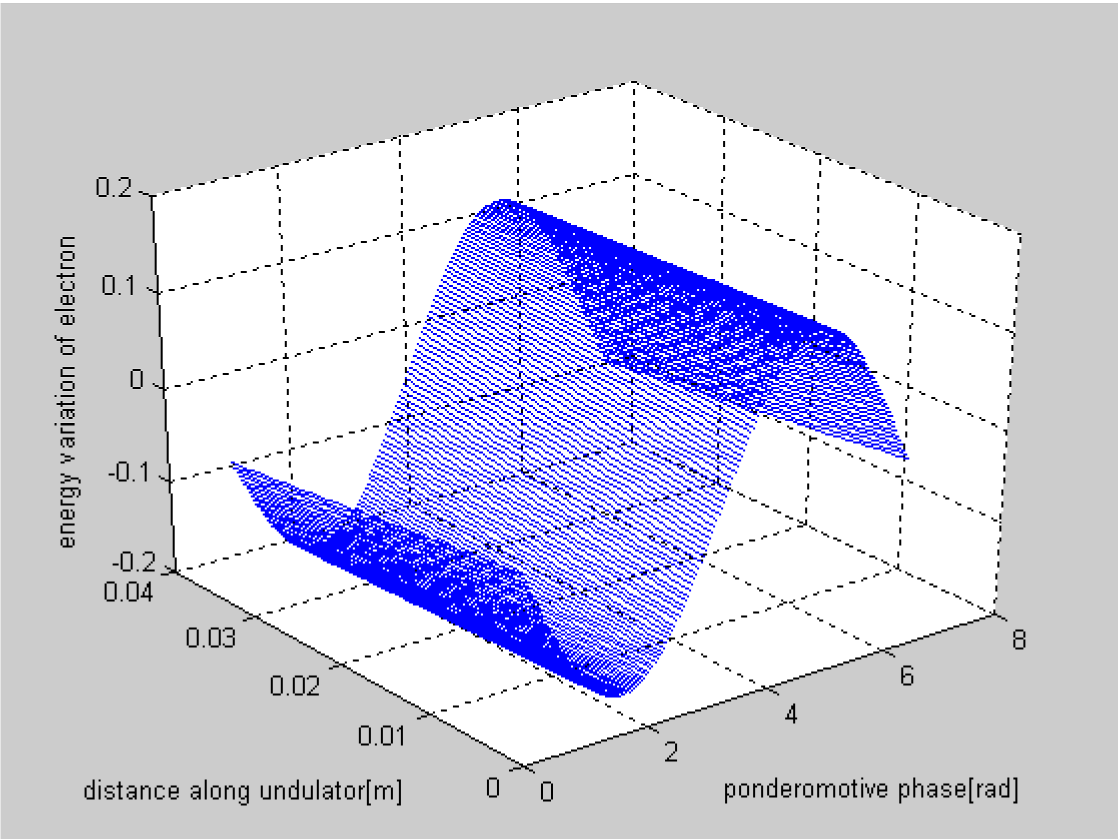

Figure (3.25): The electron energy inside the helical undulator.

Examination Committee Certificat

We certify that we have read the thesis entitled "**The Trajectory of Electron in Free Electron Laser Helical Undulator**"and as an examination committee, examined the student **Zina Mahmoode Al-Daghistani** on its contents, and that in our opinion it is adequate for the partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

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List of symbols

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$\vec{F}$	Lorentz force (N)
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k	Wave number of the input wave (m^{-1})
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m_0	Rest mass of the electron (kg)
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$\vec{R}$	Exact trajectory of electron
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t	Time (s)
$V_\perp$	Transverse electron velocity (m/s)

X	Distance along x-axis (m)
Y	Distance along y-axis (m)
Z	Distance along undulator (m)
β	Electron velocity normalized to the velocity of light
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γ	Lorentz factor
ε_0	Electron rest energy (eV)
ε	Electron energy (eV)
μ	Vacuum permeability (H/m)
θ_S	Electron rotating angle
λ_0	Undulator period (m)
λ	Radiation wavelength (μm)
$\vec{v}$	Total electron velocity (m/s)
v_x	Electron velocity along x-axis (m/s)
v_y	Electron velocity along y-axis (m/s)
v_z	Longitudinal electron velocity (m/s)
ψ_0	Entrance phases (rad)
ψ	The ponderomotive phase (rad)
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Supervisor's Certification

We certify that this thesis was prepared under our supervision at the "AL-Nahrain University"as a partial requirement for the degree of Master of Science in Physics.

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Date: **/ 4 / 2006**

الاهداء

رسول الله ماذا يقول الشعر فيك ويكتب

خلق كماءِ المزن صاف وقلب اطيب

ينساب صوتك في الاثير كانه

صوت العنادل...بل ارق واعذب

قرآن ربك خير كنز حزته

هو حرزك الباقي الذي لايسلب

الى الحبيب المصطفى(صلى الله عليه وسلم) اهدي ثمرة جهدي

زينة

خلاصة

نظرية البعد الواحد استخدمت لدراسة اداء مكبر ليزر الالكترون الحر عندما تملك حزمة الالكترونات توزيع منتظم للكثافة في مسار عمودي على محور المموج اللولبي مع اطوار ابتدائية منتظمة توزع بين فترة (2π-٠). لقد تم استخدم برنامج matlab 6.1 في بناء برامج الدراسة.

في هذا العمل تمت دراسة لحركة الالكترون داخل المموج اللولبي واستنادا الى معادلات الحركة للالكترون فقد تم التركيز على العوامل الثلاثة التالية:

١- المجال المغناطيسي

٢- فترة المموج

٣- عامل لورنتز

حيث ان الزيادة في المجال المغناطيسي تؤدي الى الزيادة في نصف قطرمسار الالكترون وكمية السرعة المستعرضة للالكترون وبنفس الوقت تؤدي الى النقصان في كمية السرعة الطولية للالكترون. اما في حالة فترة المموج نلاحظ تكرار الظواهر اعلاه في حالة المجال المغناطيسي مع اختلاف السلوك الموجي للالكترون بسبب اختلاف المعادلات المستخدمة فى هذه الحالة. واخيرا الحالة الثالثة فان الزيادة في عامل لورنتز تؤدي الى النقصان في نصف قطرمسار الالكترون وكمية السرعة المستعرضة للالكترون وفي نفس الوقت تؤدي الى الزيادة في كمية السرعة الطولية للالكترون.الظواهر السابقة تمت مناقشتها حسب قانون حفظ الطاقة داخل المموج اللولبي.

كذلك تمت المناقشة لدراسة تاثير طور الالكترون، المجال المغناطيسي للمموج وفترة المموج بوجود المجال الكهربائي للموجة الكهرومغناطيسة الداخلة(مثل موجة الليزر المراد تكبيرها) على كمية واتجاه السرعة الطولية للالكترون. حيث وجد ان هنالك زيادة في كمية السرعة الطولية للالكترون تحت تاثير نفس العوامل سواء ان كان مجال مغناطيسي او فترة المموج في هذه الحالة مقارنة مع الحالة الاولى التى تكون بغياب المجال الكهربائي.

Republic of Iraq
Ministry of Higher Education
And Scientific Research
AL-Nahrain University
College of Science

The Trajectory of Electron
in
Free Electron Laser Helical Undulator

A Thesis

Submitted to the College of Science
Al-Nahrain University
in partial fulfillment of the requirements for
the Degree of Master of Science in
Physics

By

Zina Mahmoode Al-Daghistani

(B.Sc.2000)

Supervisor

Dr. Mohammed I. Sanduk

A.D. **2006** | *A.H.* **1427**

جمهورية العراق
وزارة التعليم العالي
والبحث العلمي
جامعة النهرين
كلية العلوم

مسار الإلكترون داخل المموج اللولبي لليزر الكترون الحر

رسالة
مقدمة الى كلية العلوم، جامعة النهرين كجزء من متطلبات نيل شهادة الماجستير في الفيزياء
من قبل

زينة محمود الداغستاني
بكالوريوس ٢٠٠٠

المشرف
د.محمد عزالدين الصندوق

ربيع الاول ١٤٢٧
نيسان ٢٠٠٦

Contents

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Supervisor's Certification

We certify that this thesis was prepared under our supervision at the "AL-Nahrain University"as a partial requirement for the degree of Master of Science in Physics.

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Chapter Two

Theory of Free Electron Lasers

2.1 One-Dimensional Theory of the FEL Amplifier

The field equations and equations of motion should be solved simultaneously. In principle, modern supercomputers allow one to perform direct simulation of the FEL process. The results of such simulation depend on a large number of problem parameters. They provide the possibility of obtaining a numerical answer for a specific set of input data, but hardly help to understand the FEL physics. A deeper insight into FEL physics can be obtained only by introducing some simplifying assumptions about the properties of the electron beam and of the electromagnetic field.

The one-dimensional model is an important one from the methodological point of view. There are four assumptions were used in the one-dimensional, steady-state model of the FEL amplifier:

1. The electron beam has a uniform density distribution in the direction perpendicular to the undulator axis ;
2. The electrons move along identical trajectories parallel to the undulator axis;
3. The amplified wave is a monochromatic plane wave ;
4. The electron beam is infinitely long.

This model allows one to study the ideal mechanism of amplification.

Theoretical analyses of the helical undulator FEL experiments have, hitherto, been able to treat the collective regime only in the limit of an idealized one-dimensional undulator field which is valid only as long as the electron–beam radius is much shorter than the undulator period [22].

Calculation of the small-signal gain of free electron laser is usually done using one-dimensional model, the small-signal regime of the free electron laser is analyzed for single frequency, uniform-undulator operation, taking diffraction into account [23].

2.2 Trajectories of Electron inside Helical Undulator

Consider a helical undulator in the frame of reference moving with a matched beam's axial velocity at the undulator entrance. The heart of the FEL is the undulator which produces a magnetic field that is static in time, but varies sinuoidally in space [24]:

$$\vec{B} = B_0\{\vec{e}_x \cos(k_0 z) - \vec{e}_y \sin(k_0 z)\} \tag{2.1}$$

where

$k_0 = 2\pi / \lambda_0$.

B_0 is the amplitude of the undulator magnetic field;

λ_0 is the undulator period;

and e_z are unit vector along the x, y

The direction of the pondermotive force (Lorentz force) is given by the vector cross-product between the transverse electron velocity and the radiation magnetic field. When the phasing is correct, this direction opposes the axial streaming, yielding electron deceleration and where the electrons experience the Lorentz force, execute transverse oscillations and emit synchrotron radiation in the forward direction [25] .

$$\vec{F} = m\vec{a} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{2.2}$$

With charge (-e) and electron rest mass (m_e), in this case Lorentz force reduce to

$$\vec{F} = \gamma m_e \vec{a} = -e\left(\vec{v} \times \vec{B}\right) \tag{2.3}$$

where $\vec{a}$ is the electron accelerator, γ is the relativistic factor which equal to $(1-\beta_z{}^2)^{-1/2}$ [26].

Let electron move with velocity:

$$\vec{v} = \vec{e}_x v_x + \vec{e}_y v_y + \vec{e}_z v_z \tag{2.4}$$

By using eq. (2.3) with eq. (2.1) and, eq. (2.4), the x and y components of this force are:

$$m_e \gamma \frac{dv_x}{dt} = ev_z B_y = -eB_0 \sin(k_0 z) v_z \tag{2.5}$$

$$m_e \gamma \frac{dv_y}{dt} = ev_z B_x = -eB_0 \cos(k_0 z) v_z \tag{2.6}$$

integrating by dt gives expression for v_x and v_y

$$v_x = \frac{dx}{dt} = \theta_S c \cos(k_0 z) \tag{2.7}$$

$$v_y = \frac{dy}{dt} = -\theta_S c \sin(k_0 z) \tag{2.8}$$

Where $\theta_s = K/\gamma$ is represents the maximum angle between the electron trajectory and the undulator axis. For relativistic beams, this quantity is much less than 1 and K is the undulator parameter [27].

$$K = \frac{eB_0 \lambda_0}{2\pi m_e c}$$

The electron velocity in this field, in units of c, is

$$\vec{\beta} = \theta_s \{ \vec{e}_x \cos(k_0 z) - \vec{e}_y \sin(k_0 z)\} + \vec{e}_z \beta_z \quad (2.9)$$

Notice that in the case of a helical undulator the axial component of the velocity, β_z, remains constant. Let $\beta = V_{\perp} / c$, $V_{\perp}$ is the transverse electron velocity [28].

$$V_{\perp} = \theta_s c \{ \vec{e}_x \cos(k_0 z) - \vec{e}_y \sin(k_0 z)\} \quad (2.10)$$

Further integration of equations (2.7) and equation (2.8) give x and y

$$x = \theta_s \frac{\lambda_0}{2\pi} \sin(k_0 z)$$

$$y = \theta_s \frac{\lambda_0}{2\pi} \cos(k_0 z) \quad (2.12)$$

So:

$$R(t) = \{ \vec{e}_x \theta_S \frac{\lambda_0}{2\pi} \sin(k_0 z), \vec{e}_y \theta_S \frac{\lambda_0}{2\pi} \cos(k_0 z), \vec{e}_z z \} \quad (2.13)$$

This equation gives the exact trajectory of electron as functions of time while the electron passing through the helical undulator for amplifier FEL where the radius of electron trajectory is [29]:

$$r = \theta_S \lambda_0 / 2\pi \quad (2.14)$$

2.3 Mechanism of Free Electron Laser

2.3.1 Spontaneous emission

As the electrons pass through the undulator field they undergo forced transverse oscillations. Like any oscillating charge moving with a relativistic speed, the electron radiate in the forward direction in a narrow cone with a semi-angle of ≈1/γ, where γ is the energy of the electron in units of its rest mass energy, which is typical of bresstrahlung radiation.

The frequency of radiation can be easily calculated. In the rest frame of the electron, the undulator rushes towards it with a speed $v_z(\approx c)$, where v_z is the z-component of the velocity of the electron beam. In this frame, the undulator has a period λ_0/γ, where γ is Lorentz factor corresponding to v_z. The magnetic field of the undulator is transformed into an electromagnetic wave of wavelength λ_0/γ. The electron oscillates in the electric field of this wave, and classically, radiates at the same wavelength λ_0/γ [illegible]. Transforming back to the laboratory frame, the radiation frequency is Doppler [illegible] wavelength of the emitted radiation is $\lambda = \lambda_0/2\gamma_z^2$, here γ represent of the " Longitudinal " kinetic energy of the particle. One would prefer to write the expression in term of γ that represents the total kinetic energy of the particle. So the wavelength of the emitted radiation can finally be written as,

$$\lambda = \frac{\lambda_0}{2\gamma^2}\left(1+K^2\right) \qquad (2.15)$$

Which is called the resonance equation that it's comes from resonance condition.

Typically, the longitudinal distribution of the electron is random within a distance λ. The radiation from individual electrons therefore adds incoherently and is termed as spontaneous emission. The electron radiate only for a finite time, since the length of the undulator is finite, and so the radiation spectrum give by $1/N_0$ (where N_0 is the number of undulator periods).

2.3.2 Stimulated emission

In a FEL, as in an atomic laser, there is a phase correlation between emitting electrons. This correlation is obtained by modulating the longitudinal beam density on the scale of the radiation wavelength, a process called bunching (fig.(2.2)) . Electron bunches propagate down the undulator, they are bathed in the same light they generate. As they wiggle back and forth through the magnets and interact with the electron field of this electromagnetic wave, some gain energy and some loss energy, depending upon their phase relationship with the electromagnetic wave and the magnetic field [30].

Let $\vec{E}$ is the vector of the electric field of the wave:

$$\vec{E} = E_0(\vec{e}_x \cos(kz - wt + \psi_0) + \vec{e}_y \sin$$

Where

E_0 =Amplitude of the input wave electric field,

ψ_0 =Entrance phases, assumed to uniformly distribute between 0 and 2π;

$k = 2\pi / \lambda$ The wave number of the radiation field;

$w = 2\pi f$ The angular frequency .

The energy exchange between the electron and the electromagnetic wave is due to the transverse component of the electron velocity. The rate of electron energy change is:

$$\frac{d\varepsilon}{dt} = m_e c^2 \frac{d\gamma}{dt} = -eV_\perp \vec{E} \qquad (2.17)$$

By using equation (2.10) and equation (2.16), equation (2.17)) becomes:

$$\frac{d\varepsilon}{dt} \cong -e\theta_S E\{\cos(k_0 z)\cos[(kz - wt + \psi_0)] - \sin(k_0 z)\sin[(kz - wt + \psi_0)]\}$$

$$\frac{d\varepsilon}{dt} = -e\theta_S E\cos(k_0 z + kz - wt + \psi_0) \qquad (2.18)$$

where $k_0 = 2\pi/\lambda_0$

The term $(k_0 + kz - wt)$ plays an important role in the study of the interaction between the electron and the radiation, and called the ponderomotive phase, Also it's equal to the angle between the transverse velocity of the particle, $V_\perp$ and the vector of the electric field, $\vec{E}$. For an e[illegible] and the wave, the scalar product $(eV_\perp.\vec{E})$ should be kept nearly constant a long the whole undulator length, i.e. a synchronism should be provided. For $w \approx w_R$, i.e. near resonance, the z-component of the electron velocity is $w/(k+k_0)$, as can be derived from the resonance condition.

Different electrons see different phases ψ and therefore experience the ponderomotive force with different magnitudes and directions, and this leads to bunching of electrons. The distance between successive bunchlets is $2\pi/(k+k_0)$, which is approximately λ. All the electrons in a bunchlet radiate at the same phase, and successive bunchlets radiate with a phase difference of 2π.Hence this radiation develops coherence, and is termed as stimulated emission.

Detailed information about electron bunch dynamics in the interaction region is very important for development of high- efficiency FEL-Oscillators and amplifiers. Numerical simulation is the most convenient method for such investigation [31].

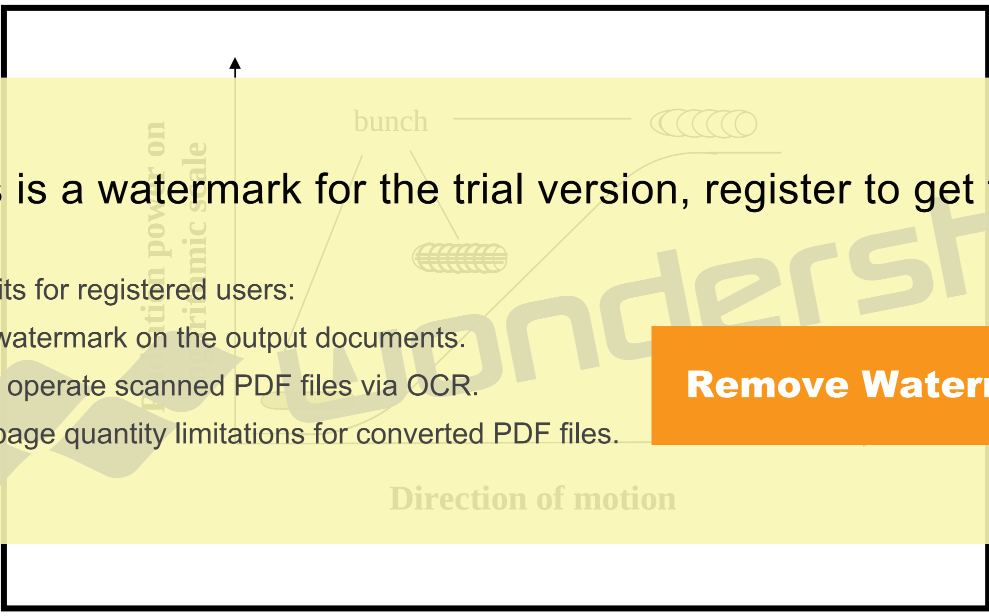

Fig (2.2): the bunch of electrons with increasing density modulation.

2.4 Energy of Electron

All the electrons before the interaction with the electromagnetic wave have equal initial energy, ε_0 , but after the interaction these electrons will no longer have the same energy as each electron will acquire a different energy (+ve or –ve) depending on the interaction location along the undulator and on the initial phase between the electron and the input electromagnetic wave [32].

As shown in the section (2.3) the rate of electron energy change is given by:

$$\frac{d\varepsilon}{dt} = -e\theta_S E\cos\psi \qquad (2.19)$$

where the phase (ψ) determines whether an electrons energy increases or decrease in time.

Eq.(2.17) into eq.(2.19) we get:

$$\frac{d\gamma}{dt} = -\frac{e\theta_s E}{m_e c}\cos\psi$$

$$K_s = \frac{\lambda e E_0}{2\pi m_e c^2} \qquad (2.21)$$

with the optical vector potential

And $w = 2\pi f$

Then

$$\frac{d\gamma}{dt} = -K_s\theta_s w\cos\psi \qquad (2.22)$$

Remembering that $\psi = (k + k_0)z - wt + \psi_0$ and $z = v_z t$

$$\frac{d\gamma}{dt} = -K\theta_s w\cos((k + k_0 - w/v_z)t + \psi_0) \qquad (2.23)$$

Integrating by dt gives expression for γ

$$\gamma = \frac{-\theta K_s w}{\left(k + k_0 - \dfrac{w}{v_z}\right)} \sin\left[\left(k + k_0 - \frac{w}{v_z}\right) z + \psi_0\right]$$

$$\gamma = \frac{-\theta_s K_s w}{(k + k_0 - wt)} \sin\psi \qquad (2.24)$$

Equation (2.24) represents the beam energy measured in the rest energy units $\varepsilon = m_e c^2 \gamma$ [33].

2.5 Transverse Electron Velocities after Application of the Electric Field

The electron has new transvers velocity

Using eq.(2.1),eq.(2.2)and eq.(2.16), the x and y components of this new active force on electron is :

$$m_e \gamma \frac{dv_x}{dt} = -e\{E_0 \cos(kz - wt + \psi_0) + v_z B_0 \sin(k_0 z)\} \qquad (2.25)$$

$$m_e \gamma \frac{dv_y}{dt} = -e\{E_0 \sin(k_0 z - wt + \psi_0) + v_z B_0 \cos(k_0 z)\} \qquad (2.26)$$

By integration there two equations:

$$\gamma v_x = \vec{e}_x cK \cos(k_0 z) + \vec{e}_y cK_s \sin(kz - wt + \psi_0) \qquad (2.27)$$

$$\gamma v_y = -\vec{e}_x cK \sin k_0 z + \vec{e}_y cK_s \cos(kz - wt + \psi_0) \tag{2.28}$$

So

$$V_\perp = (c/\gamma)\sqrt{K^2(\cos k_0 z - \sin k_0 z)^2 + K_s^2(\sin(kz - wt) - \cos(kz - wt))^2} \tag{2.29}$$

Equation (2.29) represents the equation of transverse electron velocity after the interaction with electromagnetic field.

2.6 Longitudinal Electron Velocity

The velocity of electron along the axis of undulator, will be study for two cases:

1. The longitudinal velocity of electron [illegible] field.

As seen in section (2-2)

$$v_x = \frac{dx}{dt} = \theta_S c \cos(k_0 z) \tag{2.30}$$

$$v_y = \frac{dy}{dt} = -\theta_S c \sin(k_0 z) \tag{2.31}$$

$$V^2 = v_x^2 + v_y^2 + v_z^2$$

$$v_z^2 = V^2 - v_x^2 - v_y^2 \tag{2.32}$$

According to Lorentz factor definition

$$v^2 = c^2((\gamma^2 - 1)/\gamma^2)$$

$$v^2 = c^2(1 - 1/\gamma^2) \tag{2.33}$$

From eq. (2.30), eq. (2.31), and eq.(2.33), eq.(2.32) becames:

$$v_z^2 = c^2\left(\frac{\gamma^2 - 1}{\gamma^2}\right) - \frac{K^2 c^2}{\gamma^2}$$

$$v_z^2 = c^2\left(1 - \frac{1 + K^2}{\gamma^2}\right)$$

$$v_z = c\left(1 - \frac{1 + K^2}{\gamma^2}\right)^{1/2} \tag{2.34}$$

For $v_z \approx c$, γ is large therefore that v_x and v_y according to eq. (2.31) and eq. (2.32) are both small compared with c.

2. The longitudinal velocity of electron after the interaction with electromagnetic wave.

As seen in section (2-5)

$$\gamma v_x = \vec{e}_x cK \cos(k_0 z) + \vec{e}_y cK_s \sin(kz - wt + \psi_0) \quad (2.35)$$

$$\gamma v_y = -\vec{e}_x cK \sin(k_0 z) + \vec{e}_y cK_s \cos(kz - wt + \psi_0) \quad (2.36)$$

Let

$$V_\perp^2 = v_x^2 + v_y^2$$

$$V_\perp^2 = \frac{c^2}{\gamma^2}(K^2 + K_s^2 + 2KK_s \sin(k_0 z + kz - wt + \psi_0)) \quad (2.37)$$

Where $t = z/v_z$

By using eq.(2.34) and eq.(2.37), eq.(2.32)

$$v_z^2 = c^2\left(\frac{\gamma^2 - 1}{\gamma^2}\right) - \frac{c^2}{\gamma^2}(K^2 + K_s^2 + 2KK_s \sin(k_0 z + kz - wt + \psi_0))$$

$$v_z^2 = c^2\left[1 - \frac{1}{\gamma^2} - \frac{(K^2 + K_s^2 + 2KK_s \sin(k_0 z + kz - wt + \psi_0))}{\gamma^2}\right]$$

$$v_z = c\left[1 - \frac{(1 + K^2 + K_s^2 + 2KK_s \sin(k_0 z + kz - wt + \psi_0))}{\gamma^2}\right]^{1/2} \quad (2.38)$$

This equation represents the longitudinal electron velocity with the z- direction due to interaction with the electromagnetic wave [33].