STUDYING THE PARAMETERS AFFECTING THE PRESSURE DROP ACROSS THE PACKED BED

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by

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Certification

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Abstract

Semi-empirical equations for fluid flow through packed bed have been achieved, depending on Buckingham π theorem. Two types of fluids have been used (water and air) separately (single phase flow). Several types and kinds of packing materials with different sizes have been used in the packed bed, and each had been studied separately.

Different parameters affecting the pressure drop of fluid flow through packed bed have been studied. These parameters are fluid velocity, bed porosity, bed diameter, sphericity, particle diameter, packing height and wall effect.

A certain semi-empirical equations for fluid flow through packed bed have been achieved for a certain shape and type of packing system called singular equation (mono size spherical particle system, mono size non spherical particle system, binary sized spherical particle system, ternary sized spherical particle system, quaternary sized spherical particle system, quinary sized spherical particle system and multi-sized spherical particle system). There were eleven singular equations have been written, six of them for water flow and five for air flow through packed beds.

A general semi-empirical equation has been acheived that can be used for all shapes and types of packing systems.

The results of calculations from both singular equations and general equation were comparable. The results of all calculations for fluid flow through packed bed have been compared with many documented experimental literatures. This comparison gave a very good agreement, and has been represented in tables and curves. The results from Ergun equation

using similar conditions have been represented in the curves for the sake of comparison.

Porosity empirical formulas had been achieved for all the equations used in the calculations. The calculation results of these formulas have been compared with Furnas equation of porosity and with experimental results taken from documented literature data; the comparisons show a very good agreement between the porosity formulas and experimental results.

The minimum fluidization velocity is an indication for the fluidization point, therefore; a semi-empirical equation based on Leva equation had been modeled to evaluate the minimum fluidization velocity to calculate the fluidization point.

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Notations

Symbols		Notations
A	=	The bed cross sectional area (m).
a	=	Representation of packing and fluid characteristics at laminar flow.
B	=	The permeability coefficient for the bed
b	=	Representation of packing and fluid characteristics at turbulent flow.
D_R	=	Diameter of the bed (m).
d_p	=	Diameter of the particle (m).
d_t	=	Diameter of tube (m).
d_e	=	Equivalent diameter of the pore channels (m).
dp_{eff}	=	Effective particles diameter (m).
d_{pi}	=	Diameter of particle i in mixture (m).
e	=	Porosity of the bed.
f_w	=	Correction factor.
f	=	Friction factor for packed bed.
g	=	Acceleration due to gravity, 9.81 (m/s^2).
K	=	Dimensionless constant whose value depends on physical properties of the bed and fluid.
K_C	=	Kozeny's constant.
L	=	The height of packing in the bed (m).
l	=	Thickness of the bed (m).
Δp	=	Pressure drop through packed bed, Pa ($kg/m.s^2$).
R	=	Reduce of horizontal pipe.

Re_{mf}	=	Reynold number at minimum fluidization velocity.
Re_p	=	Modified Reynolds number for packed bed.
Re	=	Reynolds number.
Δr	=	An annulus thickness of element.
S	=	Specific surface area of the particles (m^2/m^3).
S_p	=	Surface area of a particle (m^2).
S_B	=	Specific surface area of the bed (m^2/m^3).
S_c	=	Surface of the container per unit volume of bed (m^{-1}).
τ_{rz}	=	Shear stress.
u	=	Superficial velocity (m/s).
u_1	=	Average velocity through the pore channels (m/s).
u_{mf}	=	Minimum fluidization velocity (m/s).
u_t	=	Terminal velocity (m/s).
V	=	Volume of the fluid flowing through bed in time t.
V_p	=	Volume of a particle (m^3).
x_i	=	The weight fraction of particle i.

Greek Symbols

ε	=	Porosity of the bed.
ε_{mf}	=	Minimum fluidization porosity of the bed.
μ	=	Fluid viscosity (kg/m.s).
Φ	=	Sphericity.
ρ	=	Density of fluid (kg/m^3).
ρ_p	=	Density of particle (kg/m^3).
ρ_b	=	Bulk density (g/cm^3).
ρ_t	=	True density (g/cm^3).

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Chapter One

Introduction

Fluid flow through packed bed has many important applications in chemical and other processes in engineering fields such as fixed-catalytic reactor, adsorption of a solute, gas absorption, combustion, drying, filter bed, distillation, extraction, wastewater treatment and the flow of crude oil in petroleum reservoir [1].

A typical packed bed setup is a cylindrically-shaped column filled with packing materials. The column can vary in diameter, height, and material. The packing material can vary in shape, roughness, and particle size [2,3]. The most important factor in concerning the bed from a mechanical perspective is the pressure drop required for the liquid or the gas to flow through the column at a specified flow rate [4].

The pressure drops in packed beds at different fluid velocities can be modeled using an equation developed by Sabri Ergun [5]. The pressure drops in this model applies to a broad spectrum of fluids and packing materials, but it does not predict pressure drop behavior after the point of fluidization because of bed expansion and changes in packing void fraction [6]. Ergun’s equation does not take in consideration wall effects, which represents pipe like flow around the edges of the column [7,8]. Ergun believed that the pressure drop over the length of the packing was dependent upon rate of fluid flow, viscosity and density of the fluid, closeness and orientation of packing, size, shape, and surface of the packing material [9].

The advantage of using packed column rather than just tank or other reaction vessel is that the packing affords a large contacting surface area for fluids [10]. Usually increased surface area provides a high degree of turbulence in the fluids which are achieved at the expense of increased capital cost and/or pressure drop, and a balance must be made between these factors when arriving at an economic design [7].

The aim of this work is to:

I. Writing a semi-empirical equation for fluid flow through packed beds, and studying the effect of different parameters on pressure drop of fluid flow through packed beds, like fluid velocity, height of packing, type of packing materials, particles size, bed porosity and bed diameter. The semi-empirical equations includes:
   a. A general semi-empirical equation that can be used for all types of packing systems for water and air flow through packed beds.
   b. A partial semi-empirical equation (singular) for each type of packing systems.

II. Writing porosity formulas fitted to the semi-empirical equations.

III. Writing a semi-empirical equation to evaluate the minimum fluidization velocity, in order to determine the working range of the written equations, this is in the fixed region of the fluid flow diagram.

Chapter Two

Literature Survey and Theoretical Background

2.1 Introduction

A packed bed is simply a vertical column that is partially filled with small media varying in shape, size, and density. A fluid (usually air or water) is passed thought this column from the bottom and the pressure is measured by two sensors above and below the packed bed. This packed bed becomes "fluidized" when the fluid flows at such a high velocity that the closely packed particles are freed and the space between the packing increases and the particles appear to float and oscillate slightly in the column so the mixture behaves as though it is a fluid [11]. The pressure drop in packed and fluidized beds depends on the type of packing, the bed void fraction, properties of the fluid, column diameter, and also the flow rate of fluid [12].

2.2 Packed Beds

The flow of fluid through bed composed of stationary granular particles is a frequent occurrence in the chemical industry and therefore expressions are needed to predict pressure drop across beds due to the resistance caused by the presence of the particles [13].

Packed systems in industry may be divided into the following classes:

1. Fixed beds
    a. Solid- gas system.
    b. Solid- liquid systems.
2. Moving beds.
3. Solid- liquid- gas system.

Typical example of solid-gas fixed-bed systems are the catalytic reactors which were used by the Germans in the Fischer-Tropsch synthesis retorting of oil Shale, roasting of ores, combustion of coal and coke in fuel beds, and blast furnace operations [14].

The most important solid-liquid fixed-bed applications are water filtration, flow of oil through sand strata, coal washing, and leaching [15].

Moving beds are employed in the FCC (fluidized catalytic cracking) process and CFBC (Circulating Fluidized Bed Combustion) [16].

The solid-liquid-gas system comprises fractionating towers, absorbers, scrubbers, and many other kinds of chemical engineering equipment [14]

2.3 Fluidized Beds

A fluidized bed is a packed bed through which fluid flows at such a high velocity that the bed is loosened and the particle-fluid mixture behaves as though it is a fluid [17].

Fluidization has been an important part of chemical industry since the early 1900s through processes such as petroleum cracking beds, polymer production, aquatics, and the food industry. Fluidization is a process where solid particles are transformed into a fluid like state through suspension in liquid or gas media. First references to fluidization behavior date all the way back to 1556, but the use of fluidization in a more practical sense did not occur until the nineteenth century [18].

2.4 Specific Surface Area

The specific surface area of a particle is used through most of the equations or formulas of fluid flow through packed bed, and it is defined as follows:

$$S = \frac{S_p}{V_p} \qquad \text{… (2.1)}$$

Where S is specific surface area of a particle in m^{-1}, S_p is the surface area of a particle in m^2 and V_p is the volume of a particle in m^3. Therefore for spherical particle:

$$S = \frac{\pi\ d_p^{\ 2}}{\pi\ \left(d_p^{\ 3}/6\right)} = \frac{6}{d_p} \qquad \text{… (2.2)}$$

Where d_p is the particle diameter in m [19,7].

2.5 Void Fraction

The void fraction is the voided volume between packing particles in a column. It can be defined as the ratio of the empty volume to the total volume of the bed [20], i.e:

$$\varepsilon = \frac{Volume\ of\ voids\ in\ a\ bed}{total\ volume\ of\ the\ bed} \qquad \text{… (2.3)}$$

Other names given to the void fraction are porosity, fractional voidage, or simply voidage. The liquid in a packed bed usually fills this voided volume. For spherical packing, geometric analysis predicts that the void fraction will be constant with consistent packing methods, regardless of the diameter of the spheres [21].

The porosity has a great effect on the properties of packed beds. There is no doubt that any small change in porosity of the bed leads to a big change in pressure drop across the bed. **Leva in 1951** [22] found that a 1% decrease in the porosity of the bed produced about an 8% increase in the pressure drop, whilst **Carman in 1956** [23] reported a higher value, 10% increase in the pressure drop for every 1% decrease in porosity [24].

2.6 Shape Factor

For fluid flow through packed beds many particles of irregular shapes usually used. To treat this problem the particles are considered as spheres by introducing a factor called sphericity Φ which allows calculation of an equivalent diameter [25].

The sphericity of a particle is the ratio of the surface area of this sphere having the same volume as the particle to the actual surface area of the particle, as shown below:

$$\Phi = \frac{a_{sphere}}{a_{particale}} = \frac{6/d_p}{S_{particle}/V_{particle}} \qquad \ldots (2.4)$$

For a sphere, the surface area $S_p = \pi d_P^2$ and the volume is $V_p = \pi d_p^3/6$. Table 2.1 below shows the shape factor for different packing geometries [21].

Table 2.1 Shape factor for different particles [26, 27]

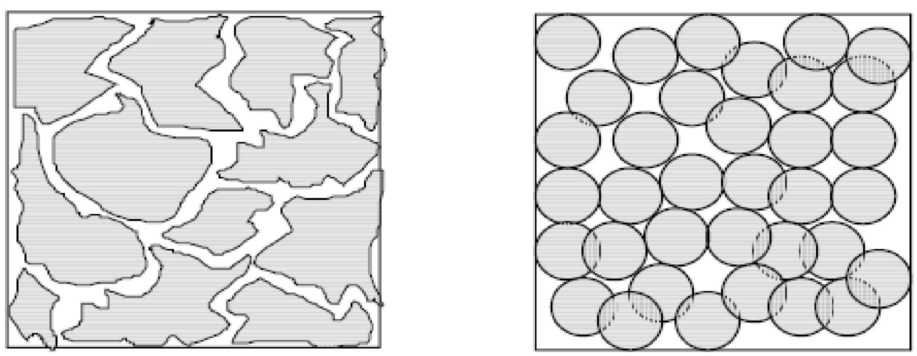
Material	*Shape Factor*	*Material*	*Shape Factor*
Spheres	1.0	Flint sand	0.65
Cubes	0.81	Crushed glass	0.65

Cylinders, d_p=L (length)	0.87	Coal dust	0.73
Berl saddles	0.3	Mica flaskes	0.28
Rasching rings	0.3	Rounded sand	0.83
Sands, average	0.75	Ottawa sand	0.95

The shape factor is difficult to evaluate when dealing with small irregular shapes. The particle shape affects the packed bed resistance in two ways [24]:

i) The fluid paths in beds of irregular particles are more tortuous than those in similar beds of spheres (Fig.2.1).

ii) It have voids differing in both size and shape from those of similar beds consisting spheres.



a. irregular particles b. regular particles

Figure 2.1 Different shapes of particles [28]

2.7 Parameters Affecting Fluid Flow Through Packed Beds

The variables affecting resistance to flow through a packed bed can be classified into two basic categories [24]:

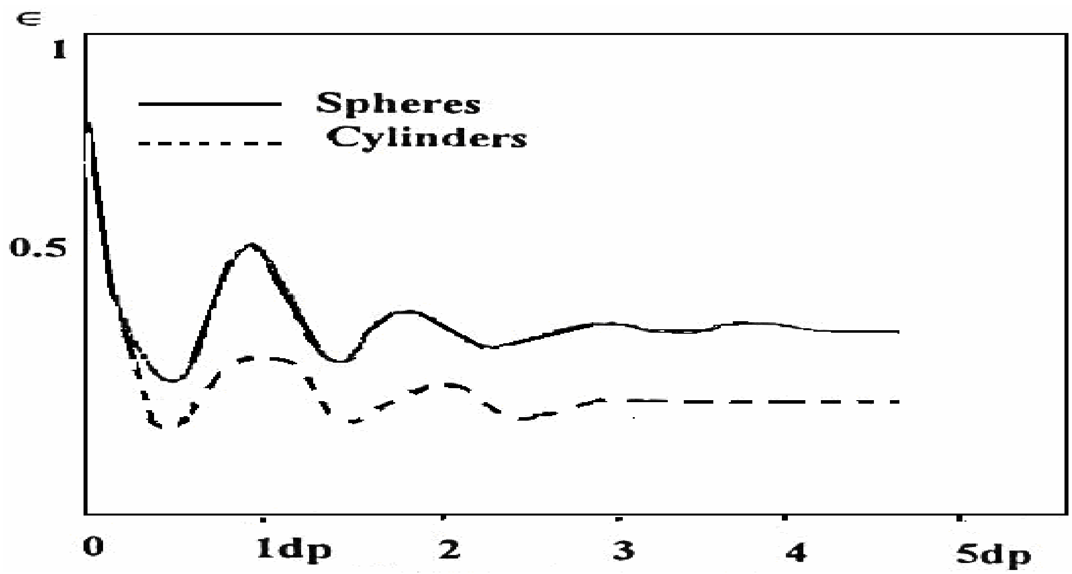
1. Parameters related to the fluid flowing through the bed such as viscosity, density, and rate of fluid flow.
2. Parameters related to the nature of the bed are numerous and to be considered as shape and size of the particles, container walls effects, porosity of the bed, surface roughness of the particle, and orientation of particles.

2.8 Container Walls Effects

The walls retaining a packed bed affect the resistance of the fluid flow in two ways [24]:

i) The particles adjacent to the walls pack more loosely than those more remote from them thus increasing the porosity of the zone near the walls. Figure 2.2 shows the fluctuation of porosity in a bed of spheres and cylinders.

ii) They create an additional surface area providing additional resistance to flow.



Distance from the wall

Figure 2.2 The Fluctuation of porosity in a bed of spheres and cylinders [29]

To decrease wall effects, the particle diameter should be small in comparison with the column diameter in which the packing is contained [30]. **Furnas** in 1931 [31] studied the wall effect and found that when the ratio of the column diameter to the particle diameter is greater than 10:1, the wall effect can be neglected. **Carman** in 1937[33] and **Coulson** in 1949[34] made no correction for the change in porosity near to the wall. They used the mean porosity and for low rates added half the area of the walls to the surface area of the particles. **Graton and Fraser** in 1953 [32] showed that the porosity of the bed is greater in the layers next to the wall, which lead to increase the fluid permeability there. A wall effect correction factor *fw* for velocity though packed bed has been determined experimentally by **Coulson** [7] as:

$$f_w = \left(1 + \frac{1}{2}\frac{S_c}{S}\right)^2 \qquad \ldots (2.5)$$

Where: S_c is the surface of the container per unit volume of bed.

S is the specific surface area of the particles.

Numerous investigators including **Carman** in 1937, **Sullivan and Hertel** in 1940**, Coulson** in 1949**,** and **Leva** in 1959 have stated that at high flow rates the wall effect is negligible. On the contrary, **Dudgeon** in 1964 believed that the wall effect was independent of the flow rate, but his work has been subjected to considerable criticism by **Franzini** in 1967 [24].

2.9 Theoretical Basis of Fluid Flow through Packed Beds and Previous Work

The first experimental work on the subject was carried out by **Darcy** in 1830 [35] in Dijon when he examined the rate of flow of water from the local fountains through beds of sand of various thicknesses. He states an empirical linear relationship between the flow rate and pressure gradient such that “the volume rate of flow is directly proportional to the pressure drop and inversely proportional to the thickness of the bed.” This relation, often termed Darcy’s law, has subsequently been confirmed by a number of workers and can be written as follows:

$$u = K\frac{(-\Delta P)}{l} \qquad \ldots (2.6)$$

where $-\Delta P$ is the pressure drop across the bed, l is the thickness of the bed, u is the average velocity of flow of the fluid, defined as (1/A)(dV /dt), A is the total cross sectional area of the bed, V is the volume of fluid flowing in time t, and K is a constant depending on the physical properties of the bed and fluid.

The linear relation between the rate of flow and the pressure drop leads one to suppose that the flow was streamline, this would be expected because the Reynolds number for the flow through the pore spaces in a granular material is low, since both the velocity of the fluid and the width of the channels are normally small. The resistance to flow then arises mainly from viscous drag. Equation 2.6 can then be expressed as:

$$u = B\frac{(-\Delta P)}{\mu l} \quad \ldots (2.7)$$

where μ is the viscosity of the fluid and B is termed the permeability coefficient for the bed, and depends only on the properties of the bed. The value of the permeability coefficient is frequently used to give an indication of the case with which a fluid will flow through a bed of particles or a filter medium. The values of B apply only to the laminar flow region for various types of packing [36].

Hagen in 1839 [37], carried out the first carefully documented friction experiments in low-speed tube laminar flow, from which the **Hagen-Poiseuille law** [38] arose, this law experimentally derived in 1838 from the Darcy's law, formulated and published in 1840 and 1846. Poiseuille's law or the Hagen-Poiseuille law is a physical law concerning the voluminal laminar stationary flow of Newtonian fluid through a cylindrical tube with constant circular cross-section [39].

Considering a horizontal pipe of radius R and length L with an annulus element of thickness Δr as shown in figure 2.3 below [41]:

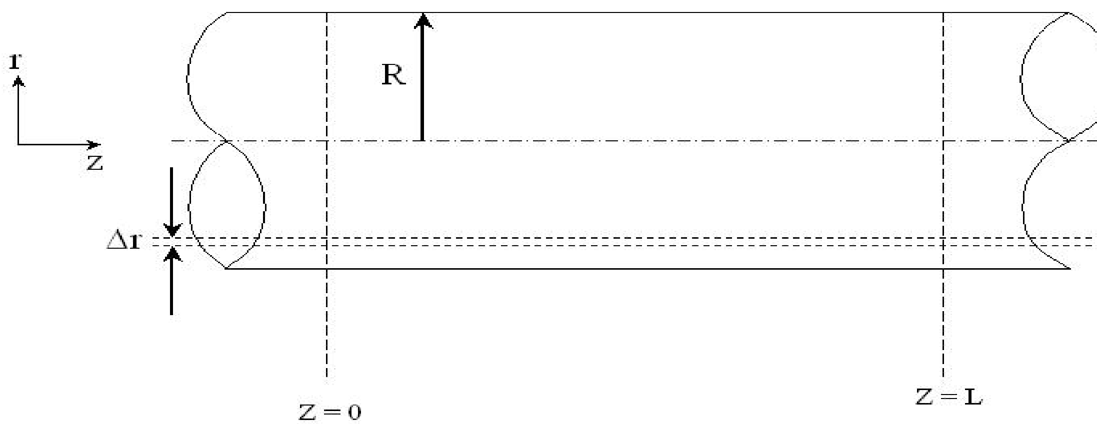


Figure 2.3 Schematic diagrams for a pipe

The momentum balance on the increment is as follows:

$$rate\ of\ momentum\ in - rate\ of\ momentum\ out + sum\ of\ forces\ acting\ on\ system = accumulation \quad \dots (2.8)$$

1. Rate of momentum in across cylindrical surface = $(2\pi\ r\ L)\ \tau_{rz}|_r$
2. Rate of momentum out across cylindrical surface = $(2\pi\ r\ L)\ \tau_{rz}|_{r+\Delta r}$
3. Rate of momentum in across annular surface at z = 0 is

   $(2\pi\ r\ \Delta r\ u_z)(\rho\ u_z)|_{z=0}$
4. Rate of momentum out across annular surface at z = L is

   $(2\pi\ r\ \Delta r\ u_z)(\rho\ u_z)|_{z=L}$
5. Pressure force acting on system = $(2\pi\ r\ \Delta r)(P_0 - P_L)$

Where P_0 and P_L is the fluid pressure at z=0 and at z=L, respectively.

For horizontal pipe the gravitational force is neglected.

Substitution of the above five terms into the general momentum balance equation (2.8):

$$(2\pi\ rL)\left(\tau_{rz}|_r - \tau_{rz}|_{r+\Delta r}\right) + (2\pi\ r\Delta r u_z)\ (\rho u_z)|_{z=0} - (2\pi\ r\Delta r u_z)\ (\rho u_z)|_{z=L} + (2\pi\ r\Delta r)\ (P_0 - P_L) = 0 \quad \dots (2.9)$$

Since the velocity is constant along the z – axis the net of the momentum across the annulus is zero arranging and dividing equation (2.9) by ($2\pi\ L\ \Delta r$) give the following[40]:

$$\frac{d}{dr}\left(r\tau_{rz}\right)=\frac{\left(P_0-P_L\right)}{L}r \qquad \text{… (2.10)}$$

integrating equation (2.10) as follows:

$$\tau_{rz}=\frac{\left(P_0-P_L\right)}{2L}r+\frac{C_1}{r} \qquad \text{… (2.11)}$$

using the boundary condition at r = 0, τ_{rz} = 0 which leads to make the shear stress to reach infinity therefore C_1 must be zero

$$\tau_{rz}=\frac{\left(P_0-P_L\right)}{2L}r \qquad \text{… (2.12)}$$

the shear stress is defined as follows:

$$\tau_{rz}=-\mu\frac{du_z}{dr} \qquad \text{… (2.13)}$$

substituting equation (2.13) into (2.12) and arranging

$$\frac{du_z}{dr}=\frac{\left(P_0-P_L\right)}{2\mu L}r \qquad \text{… (2.14)}$$

Integrating equation (2.14)

$$u_z=\frac{\left(P_0-P_L\right)}{4\mu L}r^2+C_2 \qquad \text{… (2.15)}$$

B. C. at r = R, u_z = 0

$$C_2=-\frac{\left(P_0-P_L\right)}{4\mu L}R^2 \qquad \text{… (2.16)}$$

Substituting equation (2.16) into (2.15) and arranging

$$u_z=\frac{\left(P_0-P_L\right)}{4\mu L}R^2\left(1-\left(\frac{r}{R}\right)^2\right) \qquad \text{… (2.17)}$$

Equation (2.17) is the velocity distribution inside pipe as a function of the radius.

The average velocity is obtained by the following expression [41] as follows:

$$u = \frac{\int_0^{2\pi}\int_0^R r\, u_z\, dr\, d\theta}{\int_0^{2\pi}\int_0^R r\, dr\, d\theta} \quad \ldots (2.18)$$

Substitution of equation (2.17) into (2.18) and integrating gives

$$u = \frac{\Delta P d_t^2}{32\, \mu L} \quad \ldots (2.19)$$

Rearranging equation (2.19) gives

$$\frac{\Delta P}{L} = \frac{32\, \mu u}{d_t^2} \quad \ldots (2.20)$$

Where equation (2.20) is the Hagen-Poiseuille equation [39].

Considering a unit volume packed bed, the volumes occupied by the voids and the solid particles are ε and (1-ε) respectively, where **ε** is the void fraction or porosity of the bed. Let S is the surface area per unit volume of the solid material in the bed. Thus the total surface area (S_B) in a packed bed of unit volume is (1 - ε) S.

An equivalent diameter d_e for flow through the bed can be defined as four times the cross-sectional flow area divided by the appropriate flow perimeter. For random packing, this is equal to four times the volume occupied by the fluid divided by the surface area of particles in contact with the fluid. Thus, the equivalent diameter is:

$$d_e = \frac{4\varepsilon}{(1-\varepsilon)S} \quad \ldots (2.21)$$

If the free space in the bed is assumed to consist of a series of tortuous channels, then equation (2.19) for flow through a bed may be rewritten by the substitution of the equivalent diameter:

$$u_1 = \frac{\Delta P}{32\, \mu L}\left(\frac{16\, \varepsilon^2}{(1-\varepsilon)^2 S^2}\right) \quad \ldots (2.22)$$

The average velocity through the pore channels (u_1) is defined as the superficial velocity (u) divided by the porosity of the bed [42]. Therefore equation (2.22) will be as follows:

$$u = \frac{\Delta P}{2\mu L}\left(\frac{\varepsilon^3}{(1-\varepsilon)^2 S^2}\right) \qquad \text{… (2.23)}$$

Replacing equation (2.23) by the following equation:

$$u = \frac{\Delta P}{K_c \mu L}\left(\frac{\varepsilon^3}{(1-\varepsilon)^2 S^2}\right) \qquad \text{… (2.24)}$$

Where K_C values [37,7] are given in the following figure:

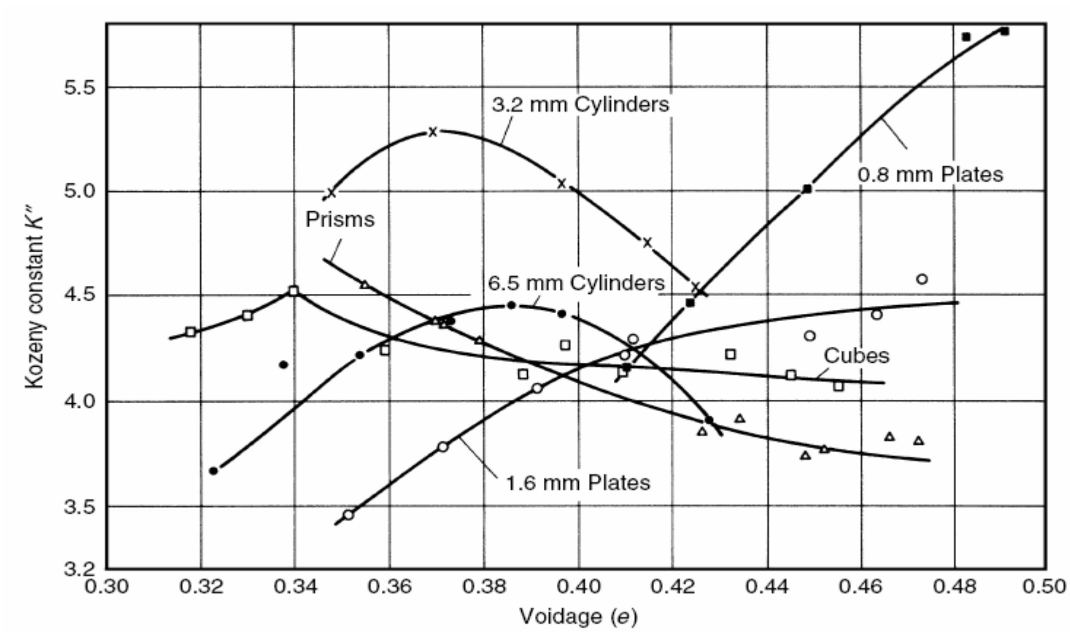


Figure 2.4 Variation of **Kozeny's** constant K_C with porosity for various shapes [7]

Forchheimer in 1901 [43] proposed a quadratic equation for the non-linear flow region:

$$\frac{\Delta P}{L} = a u + b u^2 \qquad \text{... (2.25)}$$

where u is the fluid velocity, ΔP is the pressure drop, L is the length of the medium, and ***a***,***b*** are factors which depend on both fluid and porous medium properties. The expression for *a* and *b* has been studied by many investigators (e.g., **Forchheimer** in 1930 [44]**, White** in 1935[45], **Carman** in 1937[33], **Ergun** in 1952 [46], **Schneebeli** in 1955 [47], **Irmay** in 1964 [48]; **Ward** in 1964 [49]; **Blick** in 1966 [50]; **Ahmad** in 1967[51]; **Scheidegger** in 1974 [52]). The most widely used expression for *a* and *b* is that given by **Ergun** in 1952 [46].

Forchheimer [43] includes the kinetic effect of fluid which is not included in the models for small-Reynolds-number flows. For this reason, he suggested that a term representing the kinetic energy of fluid should be included, i.e. the resulting equation is of the form:

$$\frac{\Delta P}{L} = \frac{\mu}{B} u + a\ \rho\ u^2 \qquad \text{… (2.26)}$$

The origin of the terms in equation (2.26) indicates that the linear term represents a flow resistance due to viscous shear. The quadratic term represents the kinetic energy losses**.**

Carman and Kozeny [53] derived an expression for pressure drop under viscous flow as:

$$\frac{\Delta p}{L} = \frac{150\ (1-\varepsilon)^2\ u\ \mu}{\varepsilon^3\ \O^2 d_P^2} \qquad \text{... (2.27)}$$

Burke and Plummer [54] derived an expression for change in pressure at turbulent flow resulting from kinetic energy losses as:

$$\frac{\Delta p}{L} = \frac{1.75(1-\varepsilon)\rho u^2}{\varepsilon^3\ \O d_P} \qquad \text{... (2.28)}$$

Ergun in 1952 [46] proposed a semi-empirical equation by adding the Carman-Kozeny equation for purely laminar (viscous) flow through a porous medium modeled as an assembly of capillaries, to the Burke-Plumner equation derived for the fully turbulent limit in a capillaric medium.

$$\frac{\Delta P}{L} = 150 \; \frac{\mu u}{\phi^2 d_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75 \; \frac{\rho u^2}{\phi d_p} \frac{(1-\varepsilon)}{\varepsilon^3} \quad \dots (2.29)$$

Where ΔP, ε, ρ, d_p, Φ, u, L, and μ are the pressure drop, void fraction of the bed, density of the fluid, particle diameter, sphericity of the particle, fluid velocity, height of the bed, and the fluid viscosity respectively. Ergun equation is a unique among many equations because it covers any flow type and condition (laminar, transitional and turbulent) [6].

For beds consisting of a mixture of different particle diameters, the effective particle diameter (dp_{eff}) can be used instead of d_p (in equation 2.29) as:

$$dp_{eff} = \frac{1}{\sum_{i=1}^{n} \frac{x_i}{d_{pi}}} \quad \dots (2.30)$$

Where x_i is the weight fraction for particle of size d_{pi} [55,21].

Davidson and Turk in 1954 [56] studied the flow of hydrogen, nitrogen and argon through packed beds of lamp-and carbon-blacks at atmospheric pressure and very low gas velocities. These materials can be packed to cover an appreciable range of porosities and so permit examination of the relationship between porosity and permeability over a wide range. They found that the porosity functions suggested by Kozeny, Carman and Arnell are not applicable to such beds; the permeability of the bed is related to the fourth power of the bed density.

The Ergun equation was developed for incompressible packed beds composed of uniform spherical particles. Despite this, the Ergun model has been used in situations where the particle shape was non-spherical and/or the particle size distribution was non-uniform (**MacDonald et al.,** 1979[57]), as well as for wood chips. For non-uniform particle size distributions, the volume-surface mean diameter has been found to best describe flow through the packed bed (**Comiti and Renaud**, 1989[58]).

Ergun used fitted parameters for spheres (A and B) as (150 and 1.75) as represented in equation 2.29 above. In recent years many investigators used Ergun equation to model fluid pressure drop through packed beds of different particles shapes , As shown in table 2.2 below, the values of A and B are significantly greater than those found by Ergun and others for a wide range of particles at bed void fractions above 0.35.However, **Francher** and **Lewis** (**Macdonald et al.,** 1979[57]) also reported much higher values of A and B for beds of consolidated granular material (e.g. sandstone particles) with low void fractions ($\varepsilon < 0.3$) (i.e. A = 750 to 5500. **Francher** and **Lewis** report negative values of B in some cases, as has been found for wood chips (**Lee and Bennington**, 2004[59]). Although there is a monotonic increase in both A and B as the average chip diameter increases there is clearly more to the dependence than this. For example, for the pin chips A is independent of chip diameter while B increases with it. **Quak and Chad** in 2005 [61] used Ergun equation to model fluid pressure drop through packed beds of wood chips (parallelepiped particles) having a range of size distributions. The bed void fraction and the liquid velocity were changed to cover the range of conditions expected in commercial digesters. As in past work with wood chips the average bed voidage was used in the correlations. **Quak and Chad** tests showed that when compacted, the void fraction in the chip column was not uniform, likely due to normal forces generated within the column and the resulting increase in friction between the chips and the vessel wall [61].

Table 2.2 Measured parameters of Ergun equation for different research's using different particles shapes [61].

Particle		Void fraction (-)	Superficial velocity (mm/s)	Ergum parameters		Reference
Shape/Source	d_p (mm)			*A*	*B*	
Parallelepiped (wood chips)						
Pin chips	3.8			165	52	
25% pins/75% accepts	5.7	0.05 – 0.441	0 – 10	2290	27	Quak and Chad (2005)
12.5% pins/87.5% accepts	6.2			2450	28	
Accept chips	6.9			2000	39	
Spheres	-	> 0.4	-	150	1.75	Ergun (1952)
Bradford	0.0501	0.125		2450	-26300	
Bradford	0.0508	0.123		1800	153000	
3rd Verango	0.0710	0.169		738	-347	Francher and Lewis (1933) from Macdonald et al. (1979)
Ceramic A	0.0183	0.370	-	7880	52800	
Robinson	0.0700	0.203		2070	-3570	
Ceramic B	0.0183	0.378		5480	-184000	
3rd Verango	0.2835	0.119		174	15.1	
Robinson	0.0593	0.195		657	-148	
Plates (t/l = 0.102)	1.29	0.46	1.5 – 111	216	12.2	
Plates(t/l = 0.209)	2.21	0.35	1.0 – 75	161	6.69	
Cylinders (l/d = 5.49)	3.048	0.39	0.6 – 47	166	3.20	Comiti and Renaud (1989)
Spheres	1.12	0.36	1.7 – 128	140	1.68	
Spheres	4.99	0.36	0.4 – 28.8	142	1.59	

Syamlal in 1987 [62] derived a formula for the multi-particle drag coefficient from a Richardson-Zaki type velocity-voidage correlation and a formula for the single-particle drag coefficient. The formula was based on two parameters only, the Reynolds number and the void fraction. The formula was compared with the Ergun equation for a void fraction range of 0.5-0.6 and correctly reduced to a formula for the single-particle drag coefficient, for void fraction of 1.0. The minimum fluidization velocity calculated from the formula compared well with experimental data for Reynolds numbers greater than 10.

Stichlmair, Bravo and Fair in 1989[63] developed a generalized model for the prediction of pressure drop and flooding in packed columns in which gas and liquid flow counter currently. The model has been validated for a wide variety of packing, both random and structured. A single mathematical expression is used to describe all flow regimes: dry gas, irrigated gas flow below the load point, loading region, and flooding.

Shenoy et. al in 1996[64] developed a theoretical model for the prediction of velocity and pressure drop for the flow of a viscous power law fluid through a bed packed with uniform spherical particles. The model was developed by volume averaging the equation of motion. A porous microstructure model based on a cell model is used. Numerical solution of the resulting equation is effected using a penalty Galerkin finite element method. Experimental pressure drop values for dilute solutions of carboxymethylcellulose flowing in narrow tubes packed with uniformly sized spherical particles are compared to theoretical predictions over a range of operating conditions. The extra pressure drop due to the presence of the wall is incorporated directly into the model through the application of the no-slip boundary condition at the container wall. The extra pressure drop reaches a maximum of about 10% of the bed pressure drop without wall effect. The wall effect increases as the ratio of tube diameter to particle diameter decreases, as the Reynolds number decreases and as the power law index increases.

Grulovic and Zdanski in 1999[65] proposed a method for the determination of the terminal velocity of non-spherical particles and compared with experimental data. The method is based on particulate expansion data of fluidized bed and variational model for calculating fluid-particle interphase drag coefficient.

Lee in 2002[66] measured the pressure drop through column of industrial white spruced chips (produced with a chipping head ring) as a function of chip size distribution and the extent of delignification, he also studied how flow resistance which is depended on porosity is affected by the kappa number of chips, which affect their flexibility, and chip size distribution, the compaction forces applied to the column, and the liquid superficial velocity. The chips bed was compressible and inelastic.

Basu et. al. in 2003[11] studied the effect of various velocity range on the packed bed column and took their observations of the packing height and pressure drop in the column their observations were recorded for each trial consisting of a certain initial bed height , column width, and packing material. The two columns used had diameters of 3.5 and 6 inches; the packing materials used were half-inch plastic marbles and pea gravel. They found that the pea gravel followed Ergun equation best.

Ibrahim and Hashim in 2004[67] studied the pressure drop during single fluid phase flow to determine the pressure drop characteristic of the porous media (glass ballotini particulate beds) for coalescence. It is also served to check the reproducibility of the packing technique, and to detect any foreign particulate matter or re-arrangement of the individual particles in the coalesce bed. Furthermore their data provided a basis for comparison with that during two-phase flow with coalescence.

Vivas et. al in 2005[68] studied the macroscopic flow through a two dimensional porous medium model by numerical and experimental methods. The objective of their research is to develop an empirical model by which the pressure drop can be obtained. In order to construct the model, they used a series of blocks as an idealized pressure drop device, so that the pressure drop can be calculated. The range of porosities studied is between 28 and 75 per cent. They found that the pressure drop is a combination of viscosity and inertial effects, the later being more important as the Reynolds number is increased. The empirical equation obtained in this investigation was compared with the Ergun equation.

Hellström and Lundström in 2006[69] suggested a model for flow through porous media taking into consideration the inertia-effects. They used the empirically derived Ergun equation that can describe the response of several porous media but does not reveal the real mechanisms for the flow. In order to increase the understanding of such flows they performed a micro-mechanically based study of moderate Reynolds number flow between parallel cylinders using a Computational Fluid Dynamics approach. Main results are that the Ergun equation fits well to simulated data up to Reynolds number of 20, that inertia-effects must be taken into account when Reynolds number exceeds 10 and that results from stationary simulations replicate time resolved ones at least up to Reynolds of 880.

Chung and Long in 2007[70] studied how the pressure drop of a packed bed is related to the flow rate of the fluid coming into the column , they used particles such as pea gravel, marbles and glass marbles. From the data collected the pressure drop was determined which was then compared to the pressure drop predicted by the Ergun equation**.** They found that Ergun equation better modeled the marbles due to their more fixed void fraction.

2.10 Minimum Fluidization Velocity

The basic of the theory for prediction of the minimum fluidization velocity is that the pressure drop across the bed must be equal to the effective weight per unit area of the particles at the point of incipient fluidization, this expressed mathematically as follows [17]:

$$\frac{\Delta P}{L} = \left(\rho_p - \rho\right)\left(1 - \varepsilon_{mf}\right) g \qquad \ldots \ (2.31)$$

equation (2.29) can now be used for small extrapolation for packed beds to calculate the minimum fluidization velocity at which fluidization begins as follows [71]:

$$\frac{1.75\ d_p^2 u_{mf}^2 \rho^2}{\phi \varepsilon_{mf}^3 \mu^2} + \frac{150\ (1-\varepsilon_{mf}) d_p u_{mf} \rho}{\phi^2\ \varepsilon_{mf}^3 \mu} - \frac{d_p^3 \rho\ (\rho_p - \rho) g}{\mu^2} = 0 \qquad \text{… (2.32)}$$

defining a Reynolds number as:

$$Re_{mf} = \frac{d_p\, u_{mf}\, \rho}{\mu} \qquad \text{… (2.33)}$$

so that equation (2.32) will be as follows:

$$\frac{1.75\,\mathrm{Re}_{mf}^2}{\phi\, \varepsilon_{mf}^3} + \frac{150\ (1-\varepsilon_{mf})\mathrm{Re}_{mf}}{\phi^2\ \varepsilon_{mf}^3} - \frac{d_p^3\, \rho(\rho_p - \rho) g}{\mu^2} = 0 \qquad \text{… (2.34)}$$

when $\mathrm{Re}_{mf} < 10$ (small particles) , the first term can be dropped as follows:

$$u_{mf} = \frac{(\rho_p - \rho) g\, d_p^2\, \varepsilon^3}{150\, \mu (1-\varepsilon)} \qquad \text{… (2.35)}$$

and when $\mathrm{Re}_{mf} > 1000$ (large particles), the second term dropped out[60].

Leva in 1959 [22] made a semi-empirical equation for the prediction of minimum fluidization velocity for gas fluidization as shown below:

$$u_{mf} = \frac{0.0093\ d_p^{1.82} (\rho_p - \rho)^{0.94}}{\mu^{0.88} \rho^{0.06}} \qquad \text{… (2.36)}$$

Wen and Yu in 1966 [72] produced an empirical correlation for u_{mf} for gas fluidization the Wen and Yu correlation is often taken as being most suitable for particles larger than 100 μm, whereas the correlation of **Baeyens** in 1974 [73], shown below in equation (2.37), is best for particles less than 100 μm.

$$u_{mf} = \frac{d_P^{1.8} (\rho_P - \rho)^{0.934}\, g^{0.934}}{110\, \mu^{0.87} \rho^{0.066}} \qquad \text{… (2.37)}$$

Figure 2.5 below shows that particles remains packed in the bed as long as the gravitational force is greater than the force exerted by the upward fluid flow from the bottom of the column. However, when the force exerted by the upward fluid flow equals the gravitational force, the bed will start to expand [74].

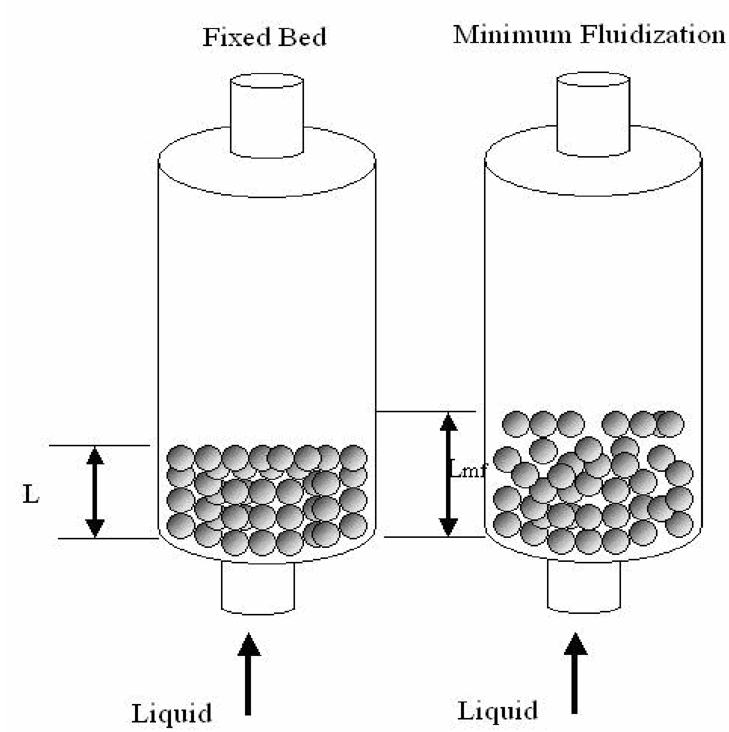


Figure 2.5 Pictorial Representation of the Fluidization Process [18]

Figure 2.6 below illustrate the regions of pressure drop when velocity increased to and beyond minimum fluidization velocity [75].

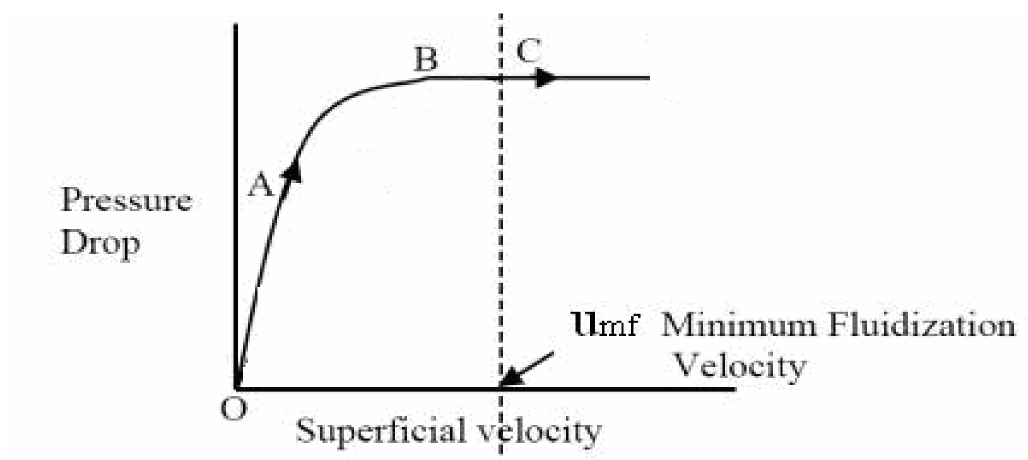


Figure 2.6 Pressure drop versus superficial velocity

At first, when there is no flow, the pressure drop zero, and the bed has a certain height. As the superficial velocity increased tracing the path ABC, at first, the pressure drop gradually increases while the bed height remains fixed. This is a region where the Ergun equation for a packed bed can be used to relate the pressure drop to the velocity which is the laminar fluid flow region. When the point B is reached, the bed starts expanding in height while the pressure drop levels off and no longer increases as the superficial velocity is increased. This is when the upward force exerted by the fluid on the particles is sufficient to balance the net weight of the bed and the particles begin to separate from each other and float in the fluid. As the velocity is increased further, the bed continues to expand in height, but the pressure drop stays constant [73].It is possible to reach large superficial velocities without having the particles carried out with the fluid at the exit. This is because the settling velocities of the particles are typically much larger than the largest superficial velocities used. The point C is defined as the **minimum fluidization velocity (u_{mf})** [71].

When the fluid velocity is increased beyond u_{mf}, three types of particle behavior are possible: slugging fluidization, turbulent fluidization, and entrained flow. These three regions are shown in figure 2.7 below:

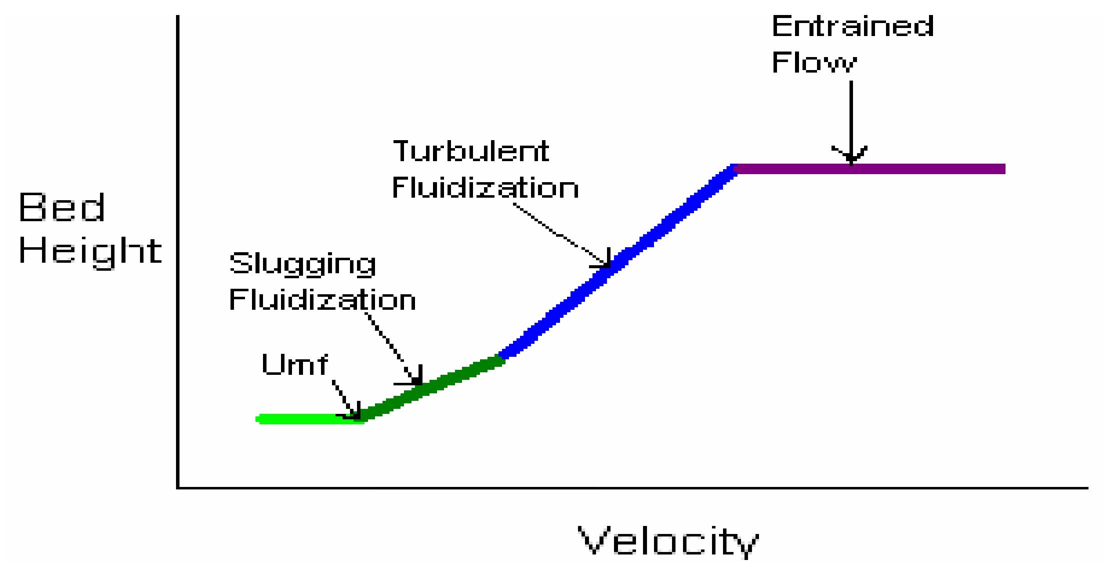


Figure 2.7 Division of three different phases of fluidization [75]

During the onset of slugging fluidization, the pressure drop initially increases with increases in fluid velocity. Slugging behavior is characterized by a continually oscillating bed height at constant fluid velocities [79]. In addition, the bottom layer of the packed particle undergoes turbulent fluidization. As the water velocity continues to increase even more, the pressure drop begins to decrease as the system makes the transition from slugging behavior to turbulent behavior. Turbulent behavior is characterized by a constant bed height at constant fluid velocities with continuous agitation and mixing. While undergoing turbulent behavior, the pressure drop remains constant with increasing fluid velocities [76].

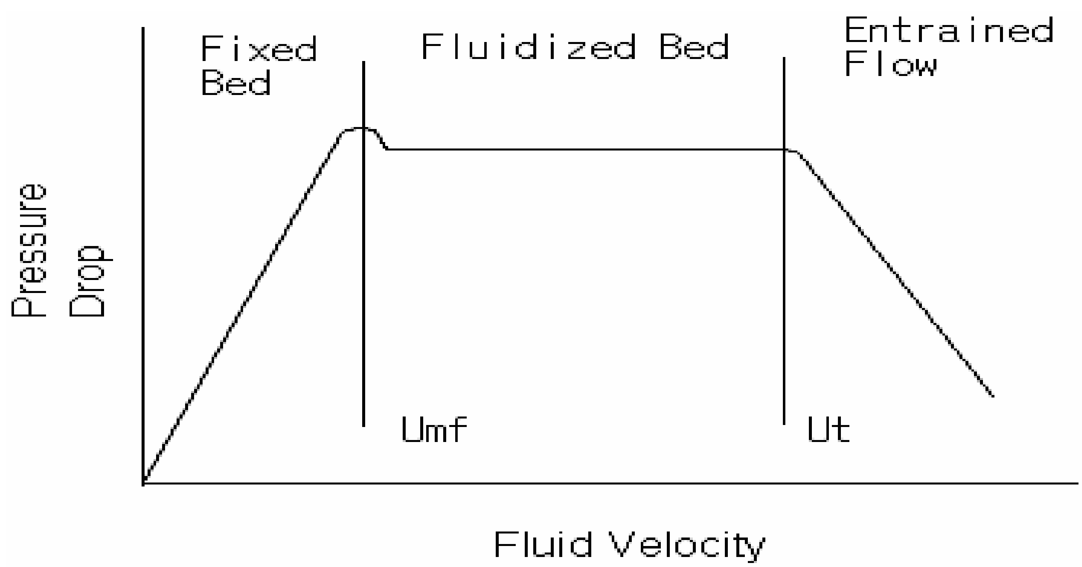


Figure 2.8 The pressure drop across a fluidized bed as a function of fluid velocity

Figure 2.8 [60] above shows that the pressure drop increases with increasing velocity up until u_{mf} as the fluidization enters the entrained flow phase, when the fluid velocity is increased beyond u_t (terminal velocity), the pressure drop decreases as there is decreased resistance to fluid flow [74].

2.11 Friction factor

The most important issue for mechanical perspective for liquid or gas flow through packed bed depends on the pressure drop and friction [75].

Ergun in 1952[46,77] derived a correlation for the friction factor in a column as a function of the Reynolds number, by adding the Blake-Kozeny equation for purely laminar (viscous) flow through a porous medium modeled as an assembly of capillaries, to the Burke-Plumner equation derived for the fully turbulent limit in a capillaric medium as follows:

$$f = \frac{150}{\mathrm{Re}_p} + 1.75 \qquad \ldots (2.38)$$

Where f is the friction factor which is a dimensionless value that accounts for the degree energy loss due to viscosity and kinetics, defined as (Bird et. al., 1996) [41]:

$$f = \frac{\Delta \mathrm{P}}{L} \frac{d_P}{\rho\, u^2} \left(\frac{\varepsilon^3}{1-\varepsilon} \right) \qquad \ldots (2.39)$$

While Re_p is the modified Reynolds number for packed bed defined as (Bird et.al., 1996 [41]):

$$\mathrm{Re}_p = \frac{\rho\, u\, d_P}{\mu(1-\varepsilon)} \qquad \ldots (2.40)$$

where: Δp is the pressure drop across the bed, L is the length of the bed, d_p is the equivalent spherical diameter of the packing, ρ is the density of fluid, μ is the dynamic viscosity of the fluid, u is the superficial velocity, and ε is the void fraction of the bed.

Equation 2.38 is plotted in figure 2.9 below, f vs. Re_p [96].

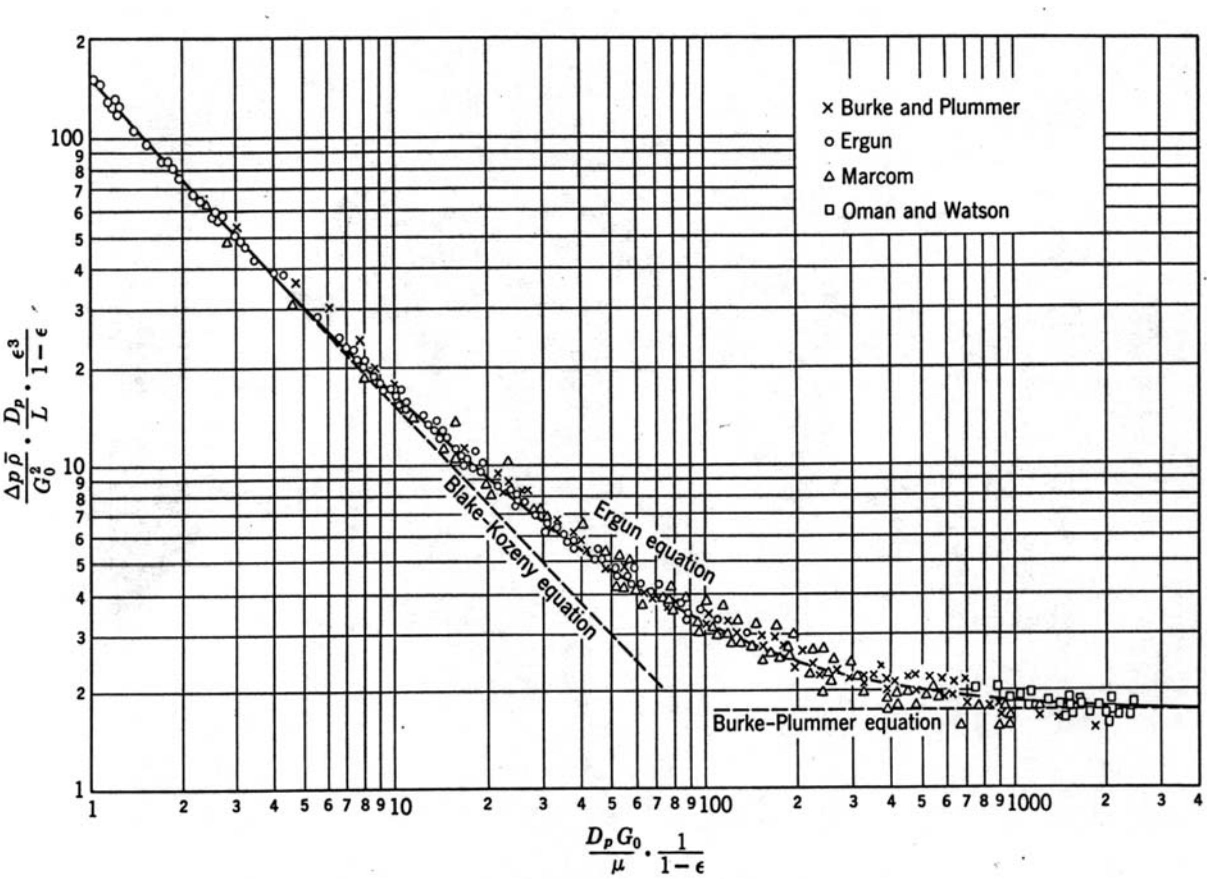


Figure 2.9 Friction factor vs. Reynolds number. This graph illustrates the correlation between Ergun and Blake-Kozeny at $Re_p<10$; Ergun and Burke-Plummer equation at $Re_p>1000$ [40].

As it can be seen from the graph, Ergun's equation does fit both the Kozeny-Carmen and Burke-Plummer models for their respective ranges, through laminar and turbulent flow, Ergun equation also models the frictional losses for Re_p between 10 and 1000 [96].

The friction factor is determined for the entire Reynolds number. For $Re_p < 10$ the flow through packed bed is laminar, the range $10 < Re_p < 1000$ is commonly referred to as transitional whereas flows characterized by $Re_p > 1000$ are considered turbulent [78].

Chapter Three

Fluid Flow Semi-Empirical Equations

3.1 Introduction

This chapter deals with acheiving semi-empirical equations for modeling fluid flow through packed bed. The most important parameter in the equations is the pressure drop. The parameters affecting the pressure drop were taken from Ergun equation. A semi-empirical formula was acheived for the parameters affecting the pressure drop useing Buckingham π theorem [80]. This formula consists of multiplied dimensionless terms raised to certain powers [81]; these powers were evaluated from experimental data taken from literatures with statistical fitting. The semi-empirical equations acheived can be divided into several types according to the packing system, mono size packing system, binary size packing system..., multi-sized packing system.

A semi-empirical equation is acheived for each type of packing referred to as a singular equation. An equation that can be used for all types of packing systems is called general equation. The shape of the singular and general equations are similar, the difference between them is in the constants used in the terms of these equations.

The second factor affecting the fluid flow through packed bed is the porosity of the packed bed. The porosity is included in the pressure drop equation. An empirical formula was acheived to evaluate the porosity for each type of packing using experimental data.

Singular and general semi-empirical equations acheived can be used within the fixed region of the fluid flow diagram; therefore, a semi-empirical equation based on Leva equation [22] was written to evaluate the minimum fluidization velocity, in order to determine the working range of the written equations.

3.2 The Semi-Empirical Equations Model

The semi-empirical equations as written below depend on two main parameters, pressure drop and porosity.

The method of modeling used to derive an expression for the pressure drop was based on curve fitting of the available literatures experimental data by implementing dimensional analysis. This analysis can be summarized as follows:
The pressure drop was assumed to be dependent on fluid velocity (u),packing diameter (d_p),bed length (L),fluid density (ρ),fluid viscosity (μ),porosity (ε),and sphericity (ϕ), and can be written in the following expression:

$$\Delta P = f\left(u, d_p, L, \rho, \mu, \varepsilon, \phi\right) \qquad \dots (3.1)$$

The Buckingham's π theorem [80] was used to write the semi-empirical formula of the fluid flow equation. In this theorem the dimensions of a physical quantity are associated with mass, length and time ,represented by symbols M,L and T respectively, each raised to rational powers[82]. The Buckingham's π theorem [80] forms the basis of the central tool of the dimensional analysis. This theorem describes how every physically meaningful equation involving n variables can be equivalently rewritten as an equation of *n-m* dimensionless parameters, whereas, the number of fundamental dimensions used. Furthermore, and the most important is that it proves a method for computing these dimensionless parameters from the given variables [81]. According to this theorem n=8 and m=3, then this theorem gave us five dimensionless groups.

Table 3.1 The dimensions of parameters used in expression 3.1

Variable	Dimension
Pressure drop	$M L^{-1} T^{-2}$
Fluid velocity	$L T^{-1}$
Particle diameter	L
Bed length	L
Fluid density	$M L^{-3}$
Fluid viscosity	$M L^{-1} T^{-1}$
Porosity	-
Sphericity	-

Selecting the variables particle diameter, fluid velocity, and fluid density.

The particle diameter (d_p) has the dimension L therefore $L = d_p$

The fluid velocity (u) has dimensions $L T^{-1}$ therefore $T = d_p u^{-1}$

The fluid density (ρ) has dimensions $M L^{-3}$ therefore $M = \rho d_p^3$

The first group (π_1) = $\Delta P (M^{-1} L T^2)$

$$\pi_1 = \frac{\Delta P}{\rho u^2} \quad \text{… (3.2)}$$

The second group (π_2) = $L (L^{-1})$

$$\pi_2 = \frac{L}{d_p} \quad \text{… (3.3)}$$

The third group (π_3) = $\mu (M^{-1} L T)$

$$\pi_3 = \frac{\mu}{\rho u d_p} \quad \text{… (3.4)}$$

The fourth group (π_4) = ε

$$\pi_4 = \varepsilon \quad \text{… (3.5)}$$

The fifth group (π_5) = ϕ

$$\pi_5 = \phi \quad \text{… (3.6)}$$

Therefore the equation for the pressure drop dependence on fluid velocity (u), packing diameter (d_p), bed length (L), fluid density (ρ), fluid viscosity (μ), porosity (ε), and sphericity (ϕ) will be as follows:

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p}\right)^{b_2} \left(\frac{\mu}{\rho\, d_p\, u}\right)^{b_3} \varepsilon^{b_4}\ \phi^{b_5} \qquad \text{… (3.7)}$$

While Reynold number is difined as:

$$\text{Re} = \frac{\rho\ d_p u}{\mu} \qquad \text{… (3.8)}$$

Then the equation (3.7) can be written as follows:

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p}\right)^{b_2} \left(\frac{1}{\text{Re}}\right)^{b_3} \varepsilon^{b_4}\ \phi^{b_5} \qquad \text{… (3.9)}$$

Since ($\Delta P/\rho u^2$) describes the fluid flow through packed bed, therefore; equation 3.9 can be considered as an empirical equation of fluid flow through packed bed. Each term of this equation is a dimensionless group, because ($\Delta P/\rho u^2$) is dimensionless number.

3.3 The Porosity Formulas

The porosity is included in equation 3.9 as one of the main parameters. The porosity can be evaluated experimentally using the following equation [83]:

$$\varepsilon = 1 - \frac{\rho_b}{\rho_t} \qquad \text{... (3.10)}$$

Where: ρ_t : is the true density of the particles, (g/cm^3)

ρ_b : is the apparent bulk density, (g/cm^3)

The porosity can be calculated theoretically using Furnas equation (3.11) [31]:

$$\varepsilon = 0.375 + 0.34 \frac{d_p}{D_R} \qquad \text{… (3.11)}$$

Equation (3.11) is one of the simplest expressions that shows the dependence of packed bed porosity on particle diameter (d_p) and bed diameter (D_R).

Equation (3.11) can be modified to add a more accurate empirical formula for the porosity of equation (3.9). The new form of the suggested porosity depends on particle diameter and bed diameter. Experimental data were used to write the new form of the porosity which can be written in the following expression.

$$\varepsilon = \frac{b_1}{\left(b_2 D_R^{\ b_3} + b_4 d_p^{\ b_5}\right)^{b_6}} \qquad \ldots (3.12)$$

The constants of equation (3.12) can be evaluated from experimental data taken from literatures by using statistical fitting.

It will be seen in chapter 4 and chapter 5, that 13 equations of the same shape of equation 3.9 had been written, and used in the present work. The difference between these equations was in the rational powers raised to the different terms. There will be 6 singular equations for water flow through packed bed for the different types of packing (mono spherical particles, mono non spherical particles, binary spherical particles,...etc). There will be 5 singular equations for air flow through packed bed for the different types of packing (mono spherical particles, binary spherical particles,...etc). A general equation for water and a general equation for air flow through packed bed were also written and used. All these equations have there own porosity formulas. These equations of fluid flow and the porosity formulas were dependent on experimental data available from literatures.

3.4 Minimum Fluidization Velocity Semi-Empirical Equation

The acheived semi-empirical equation can be used for fluid flow up to the fluidization point. The minimum fluidization velocity is an indication for the fluidization point, therefore; the minimum fluidization velocity must be evaluated to find the fluidization point.

The experimental method for determining the minimum fluidization velocity is a graphical method by making two straight lines tangents to the pressure drop-velocity curve on either side of the fluidization point, the intersection of these lines denotes where fluidization occurs(u_{mf})[84,85], as shown in figure below:

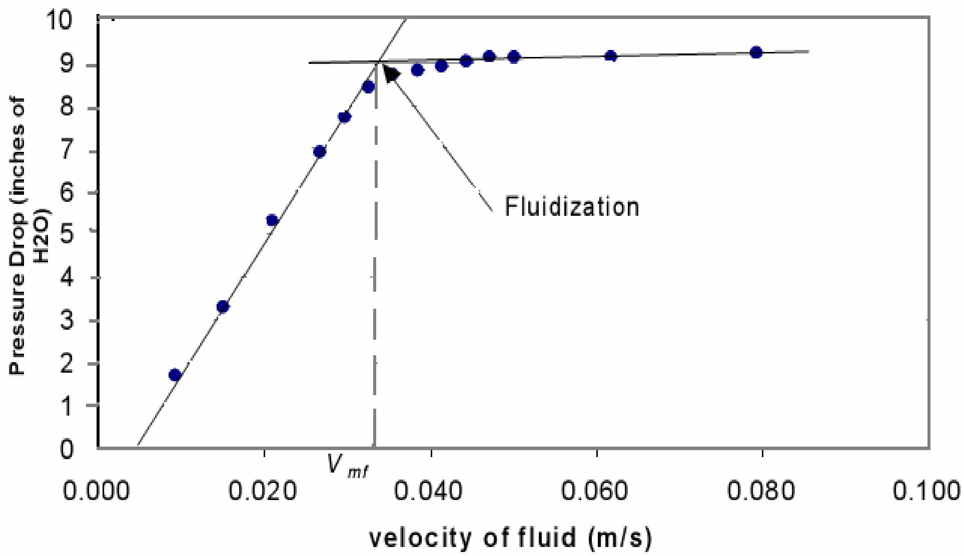


Figure 3.1 Standard plot of fluid velocity vs. pressure drop [110,85].

Leva [22] equation which is a semi-empirical equation for the prediction of minimum fluidization velocity could be used for fluidization in the gas phase only, which is written as follows:

$$u_{mf} = 0.0093 \frac{d_p^{1.82} \left(\rho_p - \rho\right)^{0.94}}{\mu^{0.88} \rho^{0.06}} \quad \ldots (2.36)$$

For the fluidization in liquid phase a semi-empirical equation has been **acheived** based on the same form of **Leva** equation, by using experimental data from literatures for liquid phase, and making statistical fitting for this data. The semi-empirical equation is comparable with the Leva equation, and can be written as follows:

$$u_{mf} = 0.0878 \frac{d_p^{0.0048} \left(\rho_p - \rho\right)^{-0.0532}}{\mu^{0.0279} \rho^{0,1694}} \quad \ldots (3.13)$$

Chapter Four

Results

The present chapter deals with the results of the achieved semi-empirical equations. These results depend on values of velocities, particles diameter, bed length and other parameters taken from experimental work. The results are divided into two main categories, water and air flow through packed bed. Each of the two categories was divided into subcategories. For each type of packing system, a semi-empirical equation was achieved. A general form of an empirical equation was achieved for multi sized of packing systems. Equations for each type of packing had been called singular. The equation of a multi sized had been called general equation.

4.1 The semi-empirical equations constants

Equation (3.9) derived in chapter three was fitted using experimental data obtained from literatures, in order to calculate the different constant in it. This had been done for water and air flow through packed bed of different types of packing. The resulted constants are presented in tables 4.1 and 4.2.

Table 4.1 Constants of equation 3.9 for water flow through packed bed

System type	b1	b2	b3	b4	b5
Mono sphere	230840.9	0.8302	0.8815	-3.0274	0.1
Mono non-sphere	3641.207	0.7670	0.4006	-3.1935	0.93
Binary	41.7922	1.1210	0.4194	-0.2435	0.1
Ternary	0.3343	0.9829	0.1480	-5.0567	0.1
Quaternary	10.9685	0.3339	0.1542	-3.9204	0.1
Quinary	1.6327	1.8577	0.1687	-0.5900	0.72
Generalized for multi sized	55.3456	1.2439	0.3316	-0.3947	0.1

Table 4.2 Constants of equation 3.9 for air flow through packed bed

System type	b1	b2	b3	b4	b5
Mono sphere	783.6491	0.1937	0.2557	-0.3318	1
Binary	3.215083	1.1050	0.2356	-1.6698	1
Ternary	4.9298	1.2343	0.1757	-0.9865	1
Quaternary	5.2649	1.3899	0.2950	-0.2323	1
Quinary	0.5597	2.1127	0.3301	-0.1012	1
Generalized for multi sized	14.1817	0.7736	0.3419	-1.1315	1

The porosity used in equation 3.9 was taken from formula 3.12 after fitting for water and air flow through packed bed. The resulting constants are written in tables 4.3 and 4.4.

Table 4.3 Porosity formula constants for water flow through packed bed

System type	b1	b2	b3	b4	b5	b6
Mono sphere	0.0624	0.2125	0.0566	-0.1803	0.0109	0.4625
Mono non-sphere	0.0302	0.0014	-0.2784	0.0125	-0.097	0.6634
Binary	0.0161	1.8949	0.8813	-0.1864	0.0342	0.9870
Ternary	0.0380	2.0945	-0.0569	-2.4662	0.0099	0.9163
Quaternary	0.1397	0.4855	3.9653	-0.0533	1.7936	0.0869
Quinary	0.4592	1.1199	0.5084	0.0706	0.0147	-0.2369
Generalized for Multi-sized	0.1480	1.6178	6.3578	-0.00028	1.7666	0.0579

Table 4.4 Porosity formula constants for air flow through packed bed

System type	b1	b2	b3	b4	b5	b6
Mono sphere	0.1202	0.1106	-0.3381	-2.4099	0.9253	0.8488
Binary	0.1611	1.6782	-0.0469	-1.9749	0.0191	0.3762
Ternary	0.1834	4.3115	-0.0771	-5.6073	0.0167	0.3228
Quaternary	1.4931	0.0946	-0.6967	0.8098	0.3707	-3.7901
Quinary	0.2279	0.5961	0.3926	-0.7136	0.2336	0.0603
Generalized for Multi-sized	1.0337	0.0014	0.1418	1.8929	1.1610	-0.1970

4.2 Studying the effect of different parameters on the semi-empirical general equation

This section shows the effect of different parameter on pressure drop using equation 3.9 after the substitution of the constants for the achieved general equation of multi-sized particle systems. The system includes all different types of packing systems (mono size spherical particles system, mono sized non spherical particles system, binary sized spherical particles system, ternary sized spherical particles system, quaternary sized spherical particles system, quinary sized spherical particles system).

A certain range for each parameter was taken in this study according to the available experimental data from literatures.

Most of the experimental previous works were studying the effect of different parameters of fluid flow on the pressure drop. So to get good comparison for the model form with the available experimental data, equation 3.9 have been multiplied by (ρu^2).The new form of the equation will be a pressure drop equation.

The important parameters affecting the pressure drop in the equation was found to be particles diameter, porosity and bed length. The fluid velocity used was taken with in the fixed region.

The fluid physical properties used in all fluid flow equations were taken from experiments held at temperature of 32°C for air flow and 25°C for water flow through packed bed.

The effect of the different parameter on pressure drop has been studied and shown in the following subsections.

4.2.1 Water flow through packed bed

The effects of the different parameters are shown in fig.4.1 to 4.3.

4.2.1.1 Effect of particle diameter on pressure drop

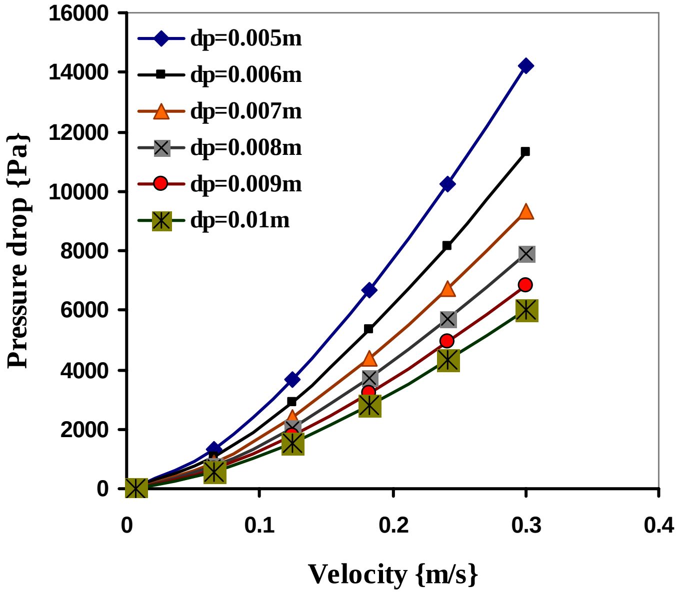


Figure 4.1 Pressure drop vs. velocity for the conditions bed diameter 0.08m, porosity 0.33, bed length 0.1m, at different particle diameters.

4.2.1.2 Effect of porosity on pressure drop

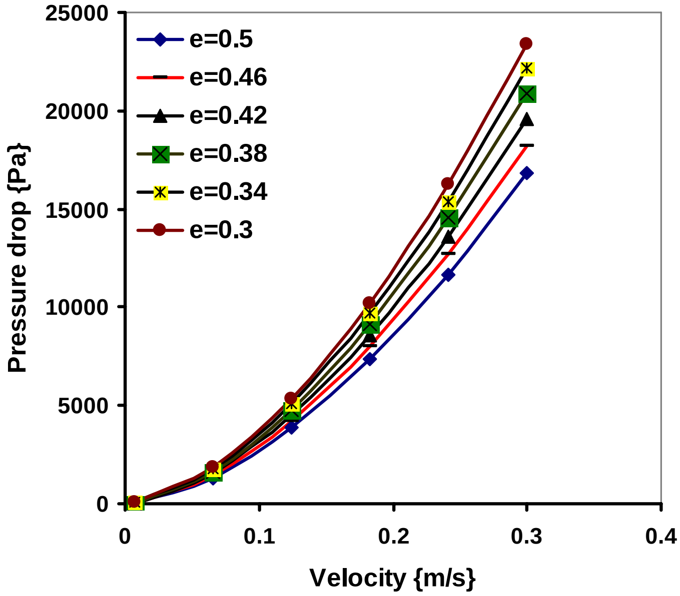


Figure 4.2 Pressure drop vs. velocity for the conditions bed diameter 0.08 m, particles diameter 0.005m, bed length 0.1m, at different porosities.

4.2.1.3 Effect of bed length on pressure drop

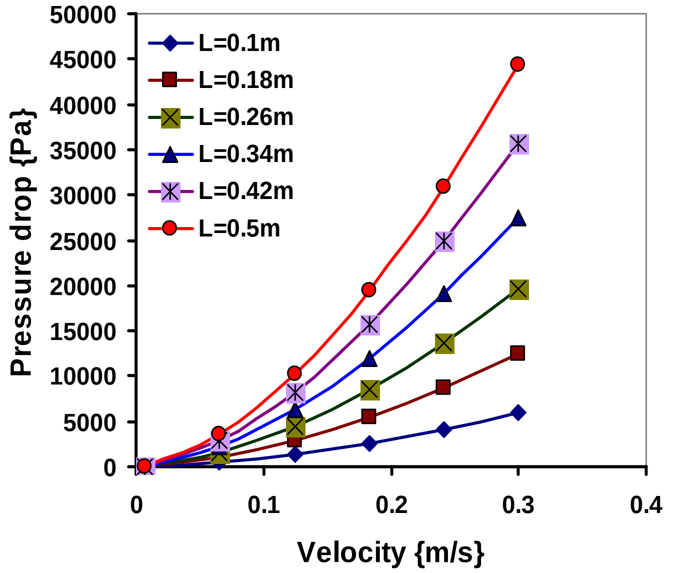


Figure 4.3 Pressure drop vs. velocity for the conditions bed diameter 0.08m, particles diameter 0.01m, porosity 0.33m, at different bed lengths.

4.2.2 Air flow through packed bed

The effects of different parameters are shown in figures 4.4 to 4.6.

4.2.2.1 Effect of particle diameter on pressure drop

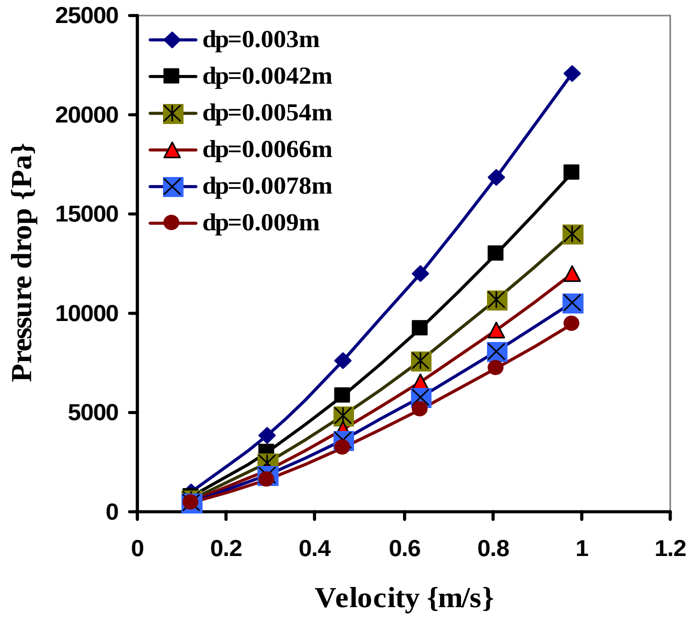


Figure 4.4 Pressure drop vs. velocity for the conditions bed diameter 0.07m, porosity 0.46m, bed length 0.1m, at different particle diameters.

4.2.2.2 Effect of porosity on pressure drop

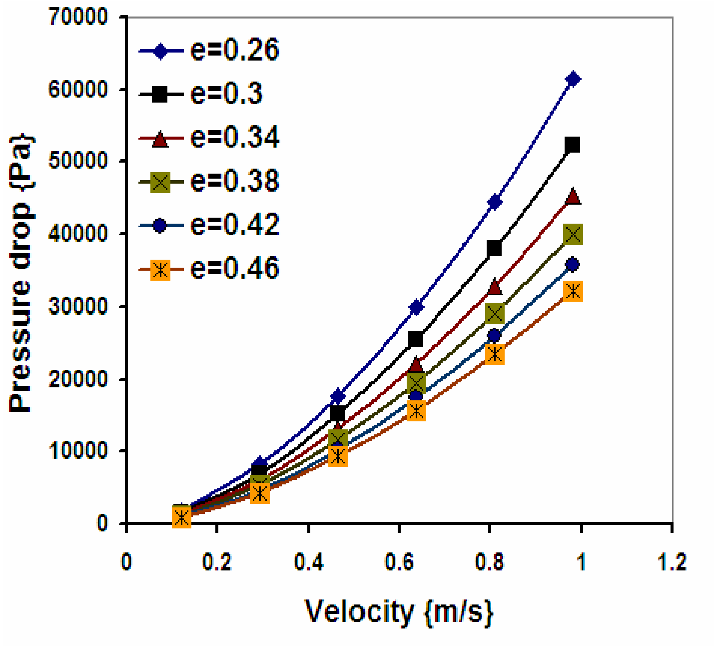


Figure 4.5 Pressure drop vs. velocity for the conditions bed diameter 0.07m, particle diameter 0.003m, bed length 0.1m, at different porosity.

4.2.2.3 Effect of bed length on pressure drop

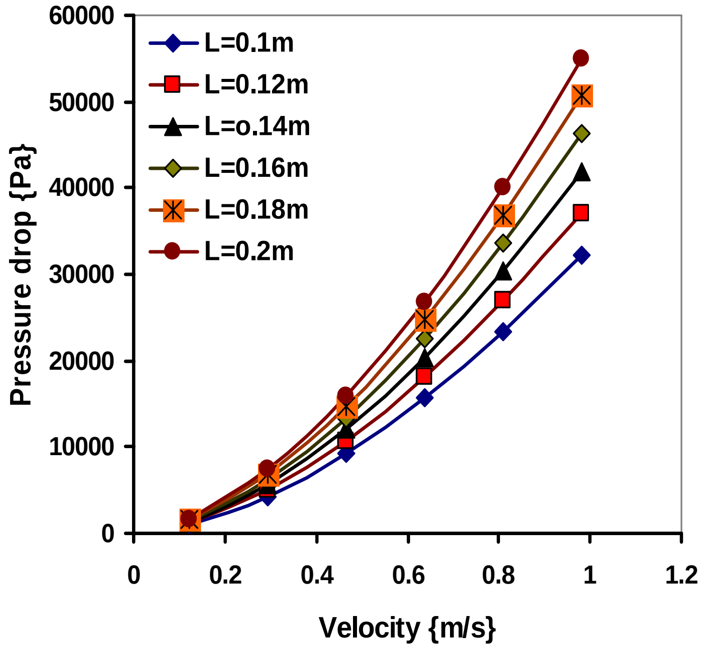


Figure 4.6 Pressure drop vs. velocity for the conditions bed diameter 0.07m, particle diameter 0.003m, porosity 0.46, at different bed length.

4.3 The porosity formula results of the fluid flow of the general equation

The porosity formulas written for the general equation have been tested. This test was with experimental data available and with theoretical calculation (equation 3.11). The results were shown in figures 4.7 and 4.8.

4.3.1 Water flow through packed bed

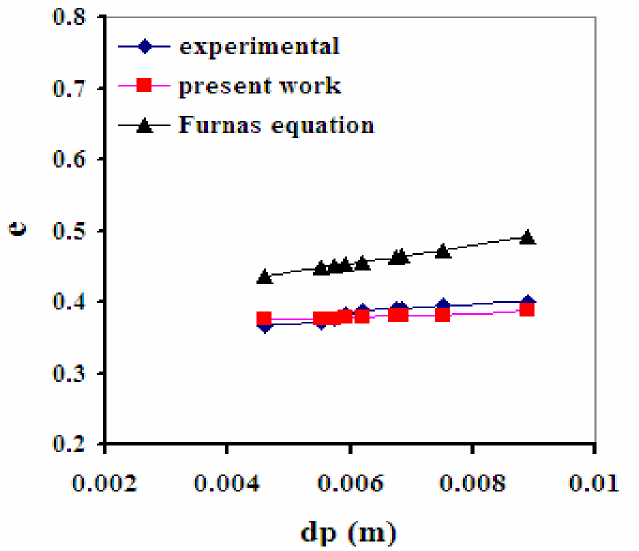


Figure 4.7 Porosity vs. particle diameter at bed diameter of 0.0764m.

4.3.2 Air flow through packed bed

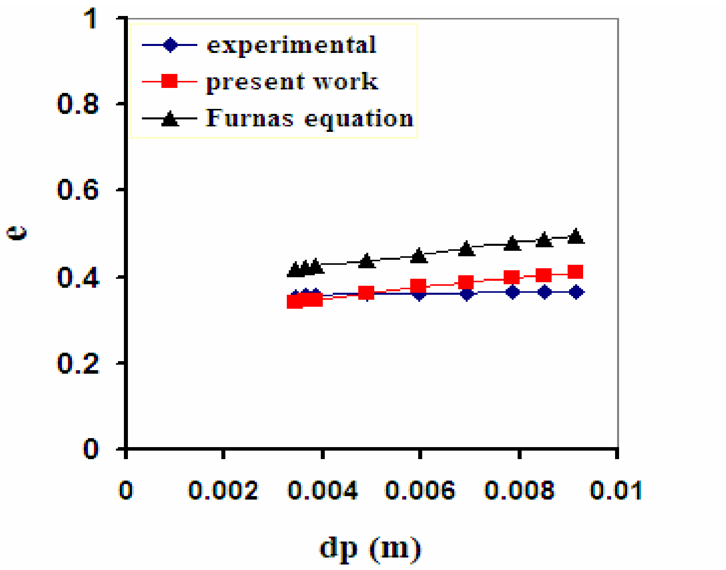


Figure 4.8 Porosity vs. particle diameter at bed diameter of 0.0764m.

4.4 Results of the minimum fluidization velocity equation

The results of the semi-empirical equation 3.13 are shown in the following table. This table show the parameters used in the equation from experiments. It also represents the experimental values for minimum fluidization velocity found in literatures.

Table 4.5 The minimum fluidization velocity results for water flow at room temperature (25°C)

Particle types	u_{mf} (m/s) (experimental)	u_{mf} (m/s) (present work)	dp (m)	Dr (m)	ρ_p (kg/m^3)
Pea gravel[86]	0.0191	0.0229	0.0031	0.08	1600
Glass marbles[86]	0.0175	0.0219	0.0127	0.08	2500
Glass marbles[86]	0.0180	0.0219	0.0127	0.1524	2500
Black marbles[99]	0.0270	0.0218	0.0127	0.1524	2600
Pea gravel[99]	0.0191	0.0229	0.0031	0.1524	1600
Pea gravel[99]	0.0222	0.0229	0.0031	0.1524	1600
Pea gravel[101]	0.0203	0.0217	0.0011	0.089	2500
Pea gravel[60]	0.0192	0.0219	0.0899	0.1524	2800
Pea gravel[60]	0.0169	0.0220	0.0899	0.1524	2600
Pea gravel[60]	0.0167	0.0219	0.0899	0.1524	2800
Pea gravel[60]	0.0175	0.0219	0.0899	0.1524	2800

4.5 Results for water flow through packed bed

4.5.1 Singular equations results for different types of packing

Tables 4.6 to 4.12 contain the calculation results of pressure drop through packed bed using equation (3.9). The constants of equation (3.9) were taken from table 4.1 for each type of packing system. The friction factor results were obtained from equation 2.39, and Reynold's number values were obtained from equation 2.40.

4.5.1.1 Mono size spherical particles system

Table 4.6 For pea gravel spherical particles diameter of 1.27 cm, bed porosity of 0.393, packing height of 41.28 cm, bed diameter of 8.89 cm [87]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0101	350.05	305.501	206.03	9.4221	9.2744
0.011	392.09	336.728	224.77	8.868	8.5896
0.0125	449.87	389.45	255.98	7.8445	7.6592
0.014	520.98	442.941	287.2	7.2169	6.9204
0.0152	580.05	486.237	312.17	6.801	6.4299
0.0168	659.89	540.935	343.39	6.3943	5.9118
0.0183	723.79	596.225	374.61	5.8933	5.4753
0.0192	779.89	629.665	393.34	5.7597	5.2448

4.5.1.2 Mono size non spherical particles system

Table 4.7 For pea gravel of particles diameter 1.27cm, sphericity of 0.7, bed porosity of 0.393, packing height of 41.28cm, bed diameter of 8.89cm [54]

U (m/s)	ΔP (kpa) (experiments)	ΔP (kpa) (present work)	Re_p	f (experiments)	f (present work)
0.016	21.95	22.69	6.021	2.303	2.131
0.018	26.21	27.87	6.773	2.231	2.051
0.02	31.31	33.05	7.526	2.158	1.971
0.022	36.42	38.23	8.279	2.086	1.891
0.024	41.52	43.41	9.031	2.013	1.811
0.028	54.75	57.12	10.54	1.922	1.727
0.032	67.99	70.83	12.04	1.831	1.644
0.036	81.22	84.55	13.55	1.74	1.56
0.04	94.46	98.26	15.05	1.649	1.476
0.042	101.4	106.6	15.8	1.607	1.45
0.044	108.4	114.9	16.56	1.566	1.424
0.046	115.4	123.2	17.31	1.525	1.398
0.048	129.3	131.5	18.06	1.393	1.372

4.5.1.3 Binary sized spherical particles system

In the packing of binary size particles the mixture contains two sizes of sphere particles. The percentage of each size is equal 1/2 from the total packing.

Table 4.8 For Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm, and dp_{eff}=1.016 cm), fractions of (x_1=0.25, x_2=0.75), bed porosity of 0.367, packing height of 49.53cm, and bed diameter of 8cm [88]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.0087	99.544	48.0874	132.056	1.5041	2.772
0.0111	149.316	72.3961	168.738	1.3818	2.501
0.0159	273.746	132.253	242.103	1.2306	2.15
0.0191	373.29	179.798	291.012	1.1614	1.99
0.0239	522.606	261.671	364.376	1.0372	1.811
0.0287	746.58	355.408	437.74	1.0266	1.677
0.0324	920.782	433.067	492.764	0.9992	1.596
0.0352	1094.98	497.651	535.559	1.0059	1.541
0.0384	1294.07	575.802	584.469	0.9982	1.486
0.0439	1617.59	723.334	670.06	0.9493	1.403
0.0488	1965.99	860.307	743.425	0.9373	1.343
0.0540	2289.51	1019.25	822.903	0.8909	1.287
0.0584	2662.8	1162.05	890.153	0.8855	1.245
0.0617	2886.78	1270.56	939.063	0.8626	1.218
0.0681	3409.38	1499.08	1036.88	0.8356	1.168

4.5.1.4 Ternary sized spherical particles system

In the packing of ternary size particles the mixture contains three sizes of sphere particles. The percentage of each size is equal 1/3 from the total packing.

Table 4.9 For glass spherical particles diameter of (0.9987, 0.7955, 0.6015cm, and dp_{eff}=0.765 cm), bed porosity of 0.4111, packing height of 15.15cm, bed diameter of 7.62cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0303	205.674	232.001	405.045	1.54577	1.50087
0.0606	638.298	837.496	810.09	1.19931	1.35449
0.0909	1489.36	1774.58	1215.13	1.24373	1.27558
0.1211	2482.27	3018.65	1618.84	1.16792	1.22254
0.1511	3900.71	4548.03	2019.88	1.17887	1.18313
0.1817	5673.76	6399.51	2428.93	1.18581	1.15127
0.2121	7234.04	8522.6	2835.31	1.10956	1.1252
0.2424	9716.31	10913.7	3240.36	1.14101	1.10317
0.2726	11702.1	13564.6	3644.07	1.08659	1.08416
0.303	14184.4	16498.5	4050.45	1.06605	1.06733

4.5.1.5 Quaternary sized spherical particles system

In the packing of quaternary size particles the mixture contains four sizes of sphere particles. The percentage of each size is equal 1/4 from the total packing.

Table 4.10 For glass spherical particles diameter of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.55cm), bed porosity of 0.3711, packing height of 15.15 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	791	748.6	267.8	1.913	1.81
0.0609	2658	2683	534.8	1.612	1.627
0.0914	5564	5676	802.6	1.498	1.528
0.1218	9644	9644	1070	1.462	1.462
0.1523	14589	14567	1337	1.415	1.413
0.1827	20400	20383	1604	1.375	1.374
0.2132	27941	27104	1872	1.383	1.341
0.2436	35483	34665	2139	1.345	1.314
0.2741	44879	43098	2407	1.344	1.29
0.3046	54152	52364	2675	1.313	1.27

4.5.1.6 Quinary sized spherical particles system

In the packing of quinary size particles the mixture contains five sizes of sphere particles. The percentage of each size is equal 1/5 from the total packing.

Table 4.11 For spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.61 cm), bed porosity of 0.3623, packing height of 15.15 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	767	756.5	290.4	1.868	1.8422
0.0609	2596	2684	579.8	1.586	1.6422
0.0914	5687	5645	870.1	1.542	1.5311
0.1218	9644	9551	1160	1.473	1.4591
0.1523	14465	14381	1450	1.413	1.4051
0.1827	19905	20069	1739	1.351	1.3620
0.2132	26334	26627	2030	1.313	1.3271
0.2436	34000	33989	2319	1.298	1.2980
0.2741	42159	42185	2609	1.271	1.2723
0.3046	52050	51176	2900	1.271	1.2567

4.5.2 General equation results

The following results are for the general equation for all systems considered in the present work. The general equation constants are shown in table 4.1. Tables 4.12 to 4.17 below represents the results of pressure drop through packed bed by using the general equation for (mono size spherical particles system, mono sized non spherical particles system, binary sized spherical particles system, ternary sized spherical particles system, quaternary sized spherical particles system and multi sized spherical particles system, respectively). The friction factor results were obtained from equation 2.39, and Reynolds number values were obtained from equation 2.40.

Table 4.12 For pea gravel spherical particles diameter of 1.27cm, bed porosity of 0.36067, packing height of 41.28cm, bed diameter of 8.89 cm [87]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0101	350.05	183.73	206.03	9.4221	4.1003
0.011	392.09	212.43	224.77	8.868	3.9837
0.0125	449.87	263.91	255.98	7.8445	3.8155
0.014	520.98	319.76	287.2	7.2169	3.6727
0.0152	580.05	367.49	312.17	6.801	3.5725
0.0168	659.89	430.83	343.39	6.3943	3.4614
0.0183	723.79	498.14	374.61	5.8933	3.3629
0.0192	779.89	540.38	393.34	5.7597	3.309

Table 4.13 For pea gravel of particle diameter 1.27 cm, sphericity of 0.7, bed porosity of 0.3064, packing height of 41.28cm, and bed diameter of 8.89cm [54]

U (m/s)	ΔP(kpa) (experiments)	ΔP(kpa) (present work)	Re_p	f (experiments	f (present work)
0.016	21.11	37.87	6.021	2.303	5.713
0.018	26.21	46.1	6.773	2.26	5.494
0.02	31.31	54.95	7.562	2.187	5.305
0.022	36.42	64.43	8.279	2.102	5.14
0.024	41.52	74.49	9.031	2.013	4.994
0.028	54.75	96.34	10.54	1.951	4.745
0.032	67.99	120.4	12.04	1.854	4.54
0.036	81.22	146.5	13.55	1.75	4.366
0.04	94.46	174.7	15.05	1.649	4.216
0.042	101.4	189.5	15.8	1.606	4.148
0.044	108.4	204.8	16.56	1.564	4.085
0.046	115.4	220.6	17.31	1.523	4.025
0.048	122.3	236.8	18.06	1.483	3.968

Table 4.14 For Acrylic balls of particles diameter (dp_1=0.655, dp_2=1.27, and dp_{eff}=1.016 cm), with fraction of (x_1=0.25, x_2=0.75), bed porosity of 0.367, packing height of 49.53 cm, bed diameter of 8 cm [88]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.009	99.54	79.58	132.1	1.504	1.914
0.011	149.3	119.8	168.7	1.382	1.765
0.016	273.7	218.8	242.1	1.231	1.566
0.019	373.3	297.4	291	1.161	1.473
0.024	522.6	432.7	364.4	1.037	1.367
0.029	746.6	587.7	437.7	1.027	1.286
0.032	920.8	716.4	492.8	0.999	1.237
0.035	1095	822.8	535.6	1.006	1.203
0.038	1294	951.9	584.5	0.998	1.169
0.044	1618	1196	670.1	0.949	1.117
0.049	1966	1422	743.4	0.937	1.079
0.054	2290	1685	822.9	0.891	1.043
0.058	2663	1921	890.2	0.885	1.017
0.062	2887	2100	939.1	0.863	0.999
0.068	3409	2477	1037	0.836	0.966

Table 4.15 For glass spheres of particles diameter (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.765 cm), bed porosity of 0.3832, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.03	205.7	257.3	405	1.546	1.28
0.061	638.3	817.8	810.1	1.199	1.018
0.091	1489	1609	1215	1.244	0.889
0.121	2482	2596	1619	1.168	0.809
0.151	3901	3755	2020	1.179	0.752
0.182	5674	5108	2429	1.186	0.707
0.212	7234	6613	2835	1.11	0.672
0.242	9716	8263	3240	1.141	0.643
0.273	11702	10051	3644	1.087	0.618
0.303	14184	11990	4050	1.066	0.597

Table 4.16 For spherical particles diameter of (0.42, 0.51, 0.61 and 0.79 cm, and dp_{eff}=0.55 cm), bed porosity of 0.3771, packing height of 15.15 cm, bed diameter of 7.62 cm [91]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	791	767	267.8	1.913	1.3705
0.0609	2658	2455	534.8	1.612	1.0897
0.0914	5564	4987	802.6	1.498	0.9524
0.1218	9644	8765	1070	1.462	0.8659
0.1523	14589	12998	1337	1.415	0.8041
0.1827	20400	17890	1604	1.375	0.757
0.2132	27941	23458	1872	1.383	0.7192
0.2436	35483	31009	2139	1.345	0.6881
0.2741	44879	39889	2407	1.344	0.6617
0.3046	54152	48987	2675	1.313	0.6389

Table 4.17 For spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, and dp_{eff}=0.61 cm), bed porosity of 0.3623, packing height of 15.15 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	767	750.34	290.4	1.868	1.76
0.0609	2596	2350	579.8	1.586	1.532
0.0914	5687	5331	870.1	1.542	1.4401
0.1218	9644	8505	1160	1.473	1.3705
0.1523	14465	13221	1450	1.413	1.3101
0.1827	19905	185115	1739	1.351	1.2511
0.2132	26334	24353	2030	1.313	1.2103
0.2436	34000	31000	2319	1.298	1.1911
0.2741	42159	39955	2609	1.271	1.173
0.3046	52050	48567	2900	1.271	1.1631

4.6 Results of air flow through packed bed

4.6.1 Singular equations results for different types of packing

Tables 4.18 to 4.22 represent the calculation results of pressure drop through packed bed of spherical particles using equation (3.9). The constants of equation (3.9) were taken from table 4.2 for each type of packing system. The friction factor results were obtained from equation 2.39, and Reynolds number values were obtained from equation 2.40.

4.6.1.1 Mono size spherical particles system

Table 4.18 For spherical particle diameter of 0.9987, bed porosity of 0.4181, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	4.0312	11.31481	1.3567	2.6098	5.5328
0.182	8.0625	22.96189	2.0356	2.3186	4.9875
0.242	13.4376	37.96226	2.7156	2.1713	4.6331
0.303	20.1564	56.02604	3.3945	2.0844	4.3761
0.364	26.8752	76.96539	4.0723	1.9311	4.1771
0.424	36.2816	100.7161	4.7512	1.9152	4.0155
0.485	45.6879	127.138	5.4300	1.8464	3.8807
0.545	57.7818	156.1409	6.1089	1.8449	3.7655
0.606	69.8757	187.6482	6.7878	1.8071	3.6654
0.667	83.3134	221.5929	7.4667	1.7806	3.5772
0.727	98.0948	257.9165	8.1456	1.7616	3.4984
0.788	114.2199	296.5663	8.8245	1.7478	3.4275
0.848	131.6889	337.4258	9.5023	1.7379	3.3633
0.909	158.5642	380.5874	10.1812	1.8228	3.3045
0.97	169.3143	426.0227	10.8612	1.7102	3.2503

4.6.1.2 Binary sized spherical particles system

Table 4.19 For spherical particles diameter of (dp_1=0.24, dp_2=0.42, and dp_{eff}=0.3055 cm), bed porosity of 0.3515, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	19.83	33.0738	0.35773	1.77769	2.63491
0.145	26.345	45.5133	0.42869	1.64462	2.52496
0.181	33.978	67.3089	0.53512	1.36127	2.39645
0.206	37.493	84.5695	0.60903	1.15963	2.32451
0.242	47.823	112.365	0.71546	1.07179	2.23797
0.266	56.607	132.767	0.78642	1.05005	2.18867
0.303	67.343	167.067	0.8958	0.96274	2.12255
0.327	77.103	191.118	0.96676	0.94641	2.08477
0.363	87.839	229.792	1.07319	0.87494	2.03411
0.387	97.599	257.272	1.14415	0.85532	2.00366
0.424	117.119	302.247	1.25353	0.85506	1.96102
0.448	124.927	333.084	1.32449	0.81696	1.93575

4.6.1.3 Ternary sized spherical particles system

Table 4.20 For spherical particles diameters of (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.7104 cm), bed porosity of 0.3796, packing height of 15.15 cm, bed diameter of 7.64 cm [89]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Rep	f (experiments)	f (present work)
0.121	8.6	10.62	0.886	2.577	2.584
0.182	17.47	22.26	1.33	2.325	2.407
0.242	29.56	37.66	1.774	2.211	2.288
0.303	45.69	56.59	2.217	2.187	2.2
0.364	61.81	78.88	2.66	2.056	2.13
0.424	80.63	104.5	3.103	1.97	2.073
0.485	103.5	133.3	3.547	1.935	2.025
0.545	127.7	165.3	3.990	1.887	1.984
0.606	154.5	200.3	4.434	1.85	1.947
0.667	185.4	238.4	4.877	1.834	1.915
0.727	215	279.4	5.32	1.787	1.886
0.788	249.9	323.3	5.764	1.77	1.86
0.848	287.6	370	6.207	1.756	1.836
0.909	327.9	419.7	6.65	1.744	1.814
0.97	370.9	472.2	7.094	1.734	1.793

4.6.1.4 Quaternary sized spherical particles system

Table 4.21 For spherical particles diameters of (0.24, 0.42, 0.82, 1.03 and dp_{eff}=0.4578 cm), bed porosity of 0.3532, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	21.914	20.2134	0.5048	1.96269	2.45363
0.145	28.303	27.5183	0.6049	1.76522	2.32609
0.181	39.039	40.1635	0.7551	1.56258	2.17879
0.206	47.823	50.0764	0.8593	1.47776	2.0972
0.242	58.559	65.9014	1.0095	1.31119	1.99988
0.266	68.319	77.4305	1.1096	1.26613	1.94487
0.303	87.839	96.6827	1.2639	1.2546	1.87156
0.327	97.599	110.101	1.3641	1.19688	1.82995
0.363	115.167	131.562	1.5143	1.14608	1.77443
0.387	125.903	146.736	1.6144	1.10234	1.74123
0.424	145.423	171.454	1.7687	1.06073	1.69495
0.448	156.159	188.329	1.8688	1.02026	1.66764

4.6.1.5 Quinary sized spherical particles system

Table 4.22 For spherical particles diameters of (0.24, 0.42, 0.82, 0.61 and 1.03 cm, with dp_{eff}=0.4818 cm), bed porosity of 0.2977, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	22.447	21.3974	0.50656	1.4632	1.50822
0.145	29.279	28.9305	0.60704	1.3291	1.42003
0.181	43.919	41.8701	0.75775	1.2794	1.31894
0.206	49.775	51.9482	0.86241	1.1194	1.26332
0.242	68.319	67.9474	1.01312	1.1133	1.19734
0.266	78.079	79.5481	1.11361	1.0531	1.16023
0.303	97.599	98.8362	1.26850	1.0145	1.11098
0.327	107.359	112.228	1.36897	0.9582	1.08313
0.363	126.879	133.572	1.51968	0.9189	1.04611

0.387	146.399	148.615	1.62016	0.9329	1.02404
0.424	161.039	173.049	1.77506	0.8549	0.99337
0.448	167.871	189.683	1.87553	0.7982	0.97532

4.6.2 General equation results

The following results are for the general equation for all systems considered in the present work. The general equation constants are shown in table 4.2. Tables 4.23 to 4.27 below represents the results of pressure drop through packed bed by using the general equation for (mono size spherical particles system, binary sized spherical particles system, ternary sized spherical particles system, quaternary sized spherical particles system and multi sized spherical particles system, respectively). The friction factor results were obtained from equation 2.39, and Reynolds number values were obtained from equation 2.40.

Table 4.23 For glass particle diameter of 0.9987 cm, bed porosity of 0.4169, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.1211	4.0312	5.8761	1.3567	2.6098	2.834
0.1817	8.0625	11.515	2.0356	2.3186	2.4669
0.2424	13.438	18.57	2.7156	2.1713	2.2353
0.303	20.156	26.883	3.3945	2.0844	2.0711
0.3635	26.875	36.355	4.0723	1.9311	1.9461
0.4241	36.282	46.946	4.7511	1.9152	1.8461
0.4847	45.688	58.583	5.43	1.8464	1.7637
0.5453	57.782	71.219	6.1089	1.8449	1.6941
0.6059	69.876	84.816	6.7878	1.8071	1.6341
0.6665	83.313	99.339	7.4667	1.7806	1.5817
0.7271	98.095	114.76	8.1456	1.7616	1.5353
0.7877	114.22	131.05	8.8245	1.7478	1.4938
0.8482	131.69	148.15	9.5023	1.7379	1.4565
0.9088	158.56	166.11	10.181	1.8228	1.4225
0.9695	169.31	184.91	10.861	1.7102	1.3914

Table 4.24 For spherical particles diameter of (dp_1=0.24cm, dp_2=0.42cm, and dp_{eff}=0.3055 cm), bed porosity of 0.3343, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	19.83	28.25	0.358	1.778	1.884
0.145	26.35	38.13	0.429	1.645	1.771
0.181	33.98	55.07	0.535	1.361	1.642
0.206	37.49	68.25	0.609	1.16	1.571
0.242	47.82	89.14	0.715	1.072	1.487
0.266	56.61	104.3	0.786	1.05	1.439
0.303	67.34	129.4	0.896	0.963	1.377
0.327	77.1	146.8	0.967	0.946	1.341
0.363	87.84	174.6	1.073	0.875	1.294
0.387	97.6	194.2	1.144	0.855	1.266
0.424	117.1	225.9	1.254	0.855	1.227
0.448	124.9	247.5	1.324	0.817	1.204

Table 4.25 For spherical particles diameters of (0.24, 0.42 and 0.82 cm, with dp_{eff}=0.3862 cm), bed porosity of 0.3495, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	21.46	20.81	0.448	2.304	2.013
0.145	27.32	28.09	0.537	2.042	1.892
0.181	39.04	40.57	0.671	1.873	1.754
0.206	47.82	50.28	0.763	1.771	1.678
0.242	60.54	65.67	0.897	1.624	1.588
0.266	72.19	76.81	0.986	1.603	1.537
0.303	87.84	95.33	1.123	1.503	1.47
0.327	97.6	108.2	1.212	1.434	1.433
0.363	112.2	128.6	1.345	1.339	1.382
0.387	122	143	1.434	1.28	1.352
0.424	144.4	166.4	1.571	1.263	1.311
0.448	148.2	182.3	1.66	1.16	1.286

Table 4.26 For spherical particles diameters of (0.24, 0.42, 0.82 and 1.03 cm, with dp_{eff}=0.4578 cm), bed porosity of 0.3581, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	21.91	16.64	0.505	1.963	2.122
0.145	28.3	22.46	0.605	1.765	1.995
0.181	39.04	32.44	0.755	1.563	1.849
0.206	47.82	40.24	0.859	1.478	1.769
0.242	58.56	52.51	1.009	1.311	1.674
0.266	68.32	61.42	1.11	1.266	1.621
0.303	87.84	76.22	1.264	1.255	1.55
0.327	97.6	86.49	1.364	1.197	1.51
0.363	115.2	102.81	1.514	1.146	1.457
0.387	125.9	114.4	1.614	1.102	1.426
0.424	145.4	133.96	1.769	1.061	1.382
0.448	156.2	145.89	1.869	1.02	1.356

Table 4.27 For spherical particles diameters of (0.24, 0.42, 0.82, 0.61 and 1.03 cm, with dp_{eff}=0.4818 cm), bed porosity of 0.3615, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	22.45	15.55	0.507	1.463	2.157
0.145	29.28	20.99	0.607	1.329	2.028
0.181	43.92	30.32	0.758	1.279	1.88
0.206	49.78	37.58	0.862	1.119	1.798
0.242	68.32	49.08	1.013	1.113	1.702
0.266	78.08	57.41	1.114	1.053	1.648
0.303	97.6	71.24	1.268	1.015	1.576
0.327	107.4	80.84	1.369	0.958	1.536
0.363	126.9	96.13	1.52	0.919	1.482
0.387	146.4	106.9	1.62	0.933	1.45
0.424	161	124.4	1.775	0.855	1.405
0.448	167.9	136.3	1.876	0.798	1.379

Chapter Five

Discussion

This chapter contains the discussions of the achieved equations results, and the comparisons between these results and experimental results taken from literatures, as well as comparisons were made between all these results and similar results taken by using Ergun equation for air and water flow through packed bed. The results are classified into two categories according to the fluid used, i.e. water and air flow through packed bed.

5.1 General equation results

5.1.1 Water flow through packed bed

5.1.1.1 Effect of particle diameter on pressure drop

Figure 4.1 indicates that an increase in the particle diameter causes decrease in the pressure drop, this is due to the fact that when the particle diameter increase's the specific surface area decreases according to equation (2.2), the reason of this relation is that when the surface area decreases the resistance of fluid flow decreases which leads to a decrease in pressure drop. (The pressure drop inversely proportional to particles diameters)[52]. For example at velocity 0.3 m/s when the particle diameter is 0.01m the pressure drop is 5.994 Kpa, while for the same velocity with particle diameter of 0.005m the pressure drop is 14.196 Kpa.

5.1.1.2 Effect of porosity on pressure drop

Figure 4.2 shows that when the porosity increases the pressure drop decreases, where the void fraction between particles become larger this leads to less resistance to fluid flow through the bed [84]. For example at velocity

0.3 m/s when the porosity is 0.5 the pressure drop is 16.797 Kpa, while for the same velocity with porosity of 0.3 the pressure drop is 23.373 Kpa .

5.1.1.3 Effect of bed length on pressure drop

Figure 4.3show that whenever the length of the packing height increases the fluid flow resistance increases this leads to an increase in pressure drop, as shown by the work of Coluson 1949[34]. For example at velocity 0.3 m/s when the packing height is 0.1m the pressure drop is 5.9941Kpa, while for the same velocity with packing height of 0.26 m the pressure drop increased to 19.6516 Kpa, further increase in the packing height to 0.5 m for the same velocity the pressure drop increased to 44.3828 Kpa.

5.1.2 Air flow through packed bed

5.1.2.1 Effect of particle diameter on pressure drop

Figure 4.4 shows that the increase in the particle diameter causes decrease in the pressure drop this is due to the specific surface area as shown in equation (2.2).Where smaller particle diameter used, i.e. more particles numbers, leads to less voids in the packed bed and an increased surface area resisting the fluid flow this will increase the pressure drop (the pressure drop inversely proportional to particles diameters) [55]. For example at velocity 0.98 m/s when the particle diameter is 0.009m the pressure drop is 9452.2688 pa, while for the same velocity with particle diameter of 0.003m the pressure drop is 22112.4602 pa .

5.1.2.2 Effect of porosity on pressure drop

Figure 4.5 show that the pressure drop in the bed is inversely proportional to bed porosity for the same velocity of the fluid entering the bed [92], this is due to the fact that when the void fraction between particles becomes larger this leads to less resistance to fluid flow through the bed. For example at velocity 0.98 m/s when the porosity is 0.46 the pressure drop is 32.195 Kpa, while for the same velocity with porosity of 0.26 the pressure drop increased to 61.399 Kpa .

5.1.2.3 Effect of bed length on pressure drop

Figure 4.6 show that when the length of the packing height increases the fluid flow resistance increase and this is leads to an increase in pressure drop [34]. For example at velocity of 0.98 m/s when the packing height is 0.1m the pressure drop is 32.195 Kpa, while for the same velocity with packing height of 0.2 m the pressure drop increased to 55.039 Kpa.

5.2 The porosity formula of the general equation

The porosity formula used in the achieved semi-empirical equations has been tested as shown in figures 4.7 and 4.8, for water and air flow respectively. This test was by comparing the results of this formula with results of experimental data and theoretical calculations (Furnas equation [31]). The comparisons show a very good agreement between the porosity formula results (equation 3.12) and the experimental data, while Furnas equation of porosity was far away from the experimental data. So the written porosity formula can be used with confidence with any type of packing system.

5.3 The minimum fluidization velocity equation

The results of the semi-empirical equation for the minimum fluidization velocity for water flow through packed bed (equation 3.13) are shown in table 4.5. The table also shows experimental values of minimum fluidization velocity taken from literatures [54, 60, 86, 87, 93, 94, 99, 101]. From this table, it can be seen that the values of the minimum fluidization velocity of the semi-empirical model used are comparable with the experimental values of the minimum fluidization velocity for water flow. So the ranges of calculations for water flow in the achieved equations were taken to be not exceeding this minimum value of velocity.

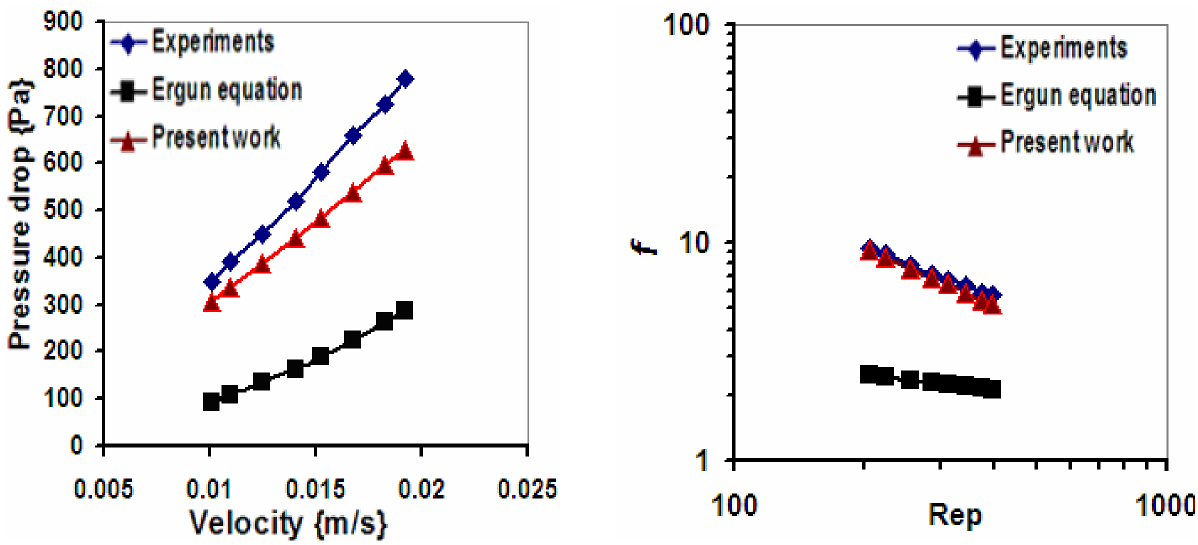
Leva equation [22] for air flow through packed bed was used because there are not available literatures data for minimum fluidization velocity for air flow, besides the data used for air was within the fixed region only.

5.4 Comparisons between achieved equations, Ergun equation and experimental results for water flow through packed bed

5.4.1 Singular equations results for different types of packing

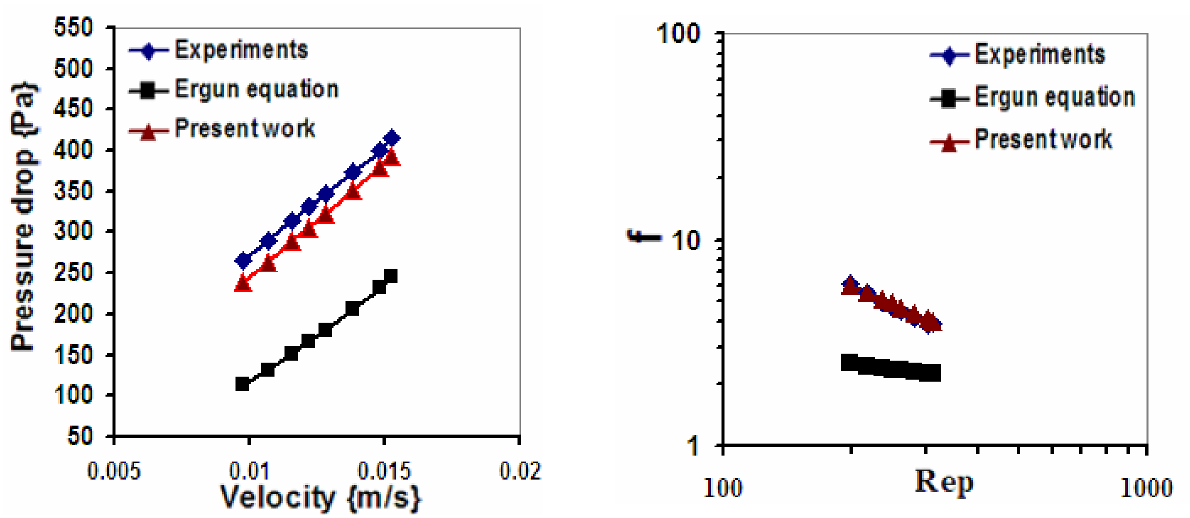
5.4.1.1 Mono size spherical particle system

The values of pressure drop versus velocity for water flow through packed beds of mono size particles were plotted in figures 5.1a, 5.2a, 5.3a and 5.4a.The values of friction factors versus Reynolds numbers were plotted in figures 5.1b, 5.2b, 5.3b and 5.4b.



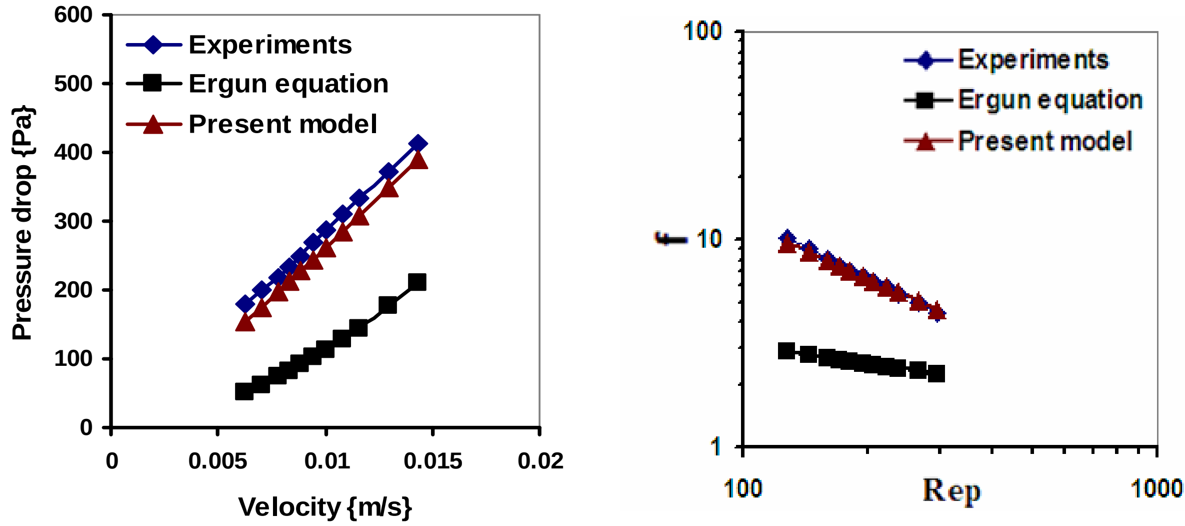
a- ΔP vs. u **b**- f vs. Rep

Figure 5.1 a- Pressure drops vs.velocity, b-Friction factor vs. Reynolds number for pea gravel of particles diameter 1.27cm, bed porosity of 0.393, packing height of 41.28cm, bed diameter of 8.89cm[87] (Table 4.6)



a- ΔP vs. u **b-** f vs. Rep

Figure 5.2 a-Pressure drops vs. velocity, b-Friction factor vs. Reynolds number for pea gravel of particles diameter 1.27 cm, bed porosity of 0.393, packing height of 53.34 cm, bed diameter of 8.89 cm[87] (Appendix A.1)



a- ΔP vs. u **b-** f vs. Rep

Figure 5.3 a-Pressure drop vs. velocity, b- Friction factor vs. Reynolds numbers for pea gravel of particles diameter 1.27 cm, bed porosity of 0.3902, packing height of 50.8 cm, bed diameter of 15.24 cm[94] (Appendix A.2)

Similar comparisons between experimental data and the achieved equations are shown in appendix A (A.3 to A.7).

From these above figures, it can be seen that the achieved equation gave a good fitting to the experiments results rather than Ergun equation, this was expected because of changing the properties of the packing materials leads to large effect on Ergun equation's prediction of pressure drop [85]. In the achieved equation several types of packing were used, which is Pea Gravel, Marbles, Glass Marbles, Black Marbles, Clear Marbles, Acrylic balls and Glass spheres.

Figures 5.1b, 5.2b and 5.3b show that the achieved equation results of friction factor-Reynold's number curve is close to the values of friction factor-Reynolds number curve of the experimental results. The values of friction factor of modified model results (range from 9.27-5.25, 5.96-4.02 and 9.61-4.62 respectively) are close to those of experimental data results (range from 9.42-5.76, 6.13-3.94 and 10.14-4.44 respectively), for Reynolds numbers (range from 206.03-393.34, 238.57-393.12 and 127.99-293.44 respectively). This means that the achieved equation is very close to the experimental results.

The wall affect on bed porosity increases the porosity, this appears clear in figures 5.1 and 5.2 where the bed porosity increased to a value of 0.393, while the bed diameter is (8.89cm), and the particle diameter is (1.27cm). The effect of wall on porosity may be due to the reduction in the ratio of bed diameter to particle diameter than the supposed ratio ($(D_R/d_P) \geq 10$) [31].

For large particles in small column (Fig.5.4), the wall presents an artificial boundary that alters the void fraction, which appears to be smaller than its true value, and the data appears lower than its true value [95].

The achieved equation (3.9) was fitted for water flow through packed beds of mono-sizes spherical particles. In this fitting 40 sets of data from the literature[3,9,20,28,54,84,85, 86,87,93, 94,96, 97,98,100, 101,102,103,104] were used. In these sets 396 values of pressure drop versus velocity were taken. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 230840.9 \left(\frac{L}{d_p}\right)^{0.8302} \left(\frac{1}{\text{Re}}\right)^{0.8815} \varepsilon^{-3.0274}\ \phi^{0.1} \qquad \ldots (5.1)$$

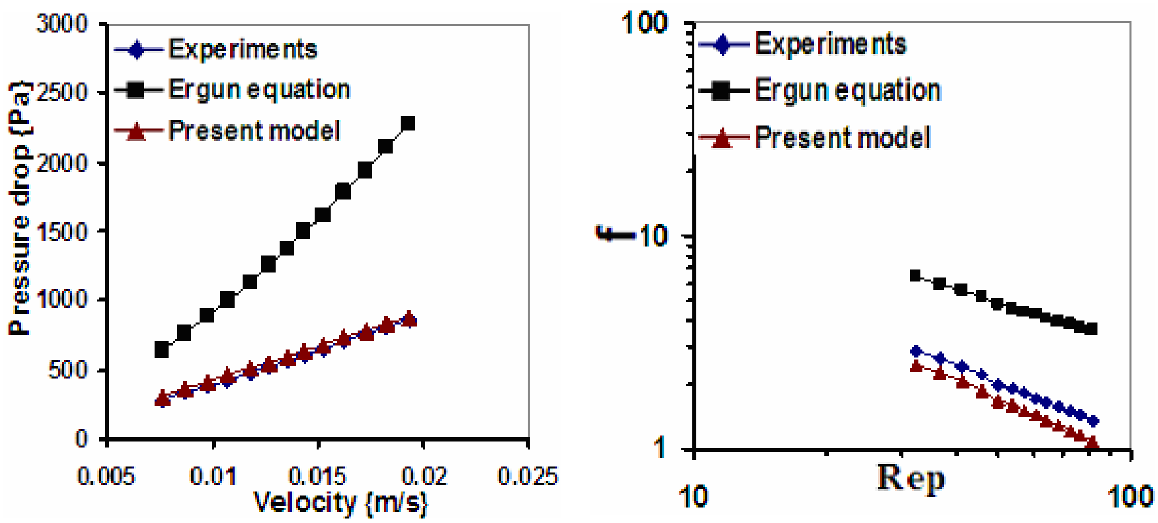
The average percentage error was found to be 4.8397% between experimental work and the achieved equation.

The porosity used in the presented model was evaluated from best fitting of experimental data of water flow through packed beds of mono-sizes spherical particles. This porosity is represented in the following equation.

$$\varepsilon = \frac{0.0624}{\left(0.2125\, D_R^{0.0566} - 0.1803\, d_p^{0.01096}\right)^{0.4625}} \qquad \ldots (5.2)$$

The above correlation deviate's from experimental with a very small average percentage error of $6.11682*10^{-8}$ %. This means that they are identical.

In the following figure a comparisons between present model and experimental data [60, 89, 90, 101, 106], which is not included in the fitting of the present model are presented. These have been done to check that equation 5.1 could be working for different conditions. Figure 5.4 are presented below, while other comparisons are presented in appendix A (A.9 and A.10)



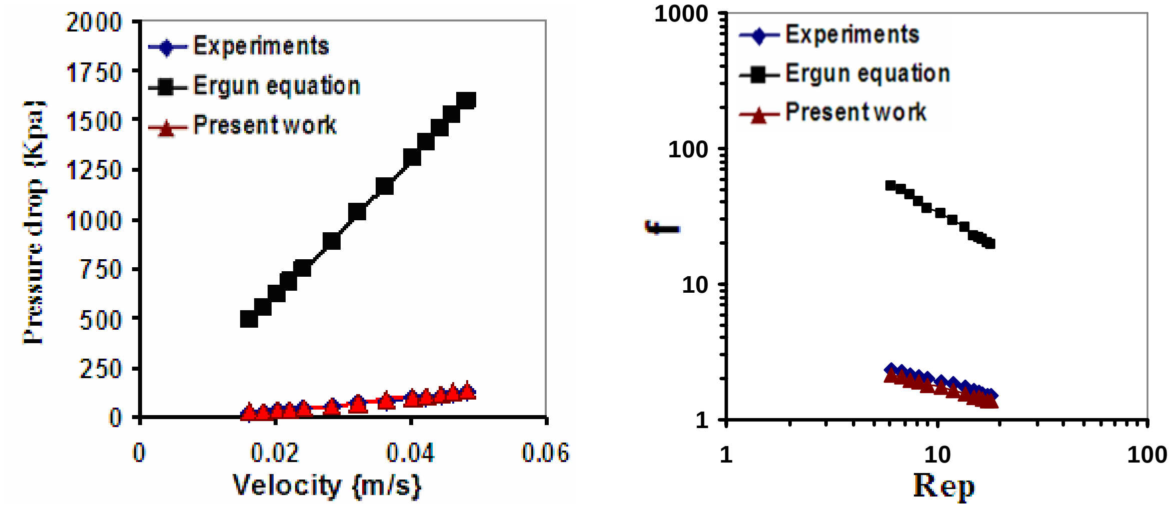
a- ΔP vs. u **b-** f vs. Rep

Figure 5.4 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for pea gravel spherical particles diameter of 0.26, bed porosity of 0.3615, packing height of 41.21cm, bed diameter of 8.26cm [106] (Appendix A.8)

Although these figures are not included in the modeling they show a very good agreement with experimental data [60, 101, 106, 89, 90].

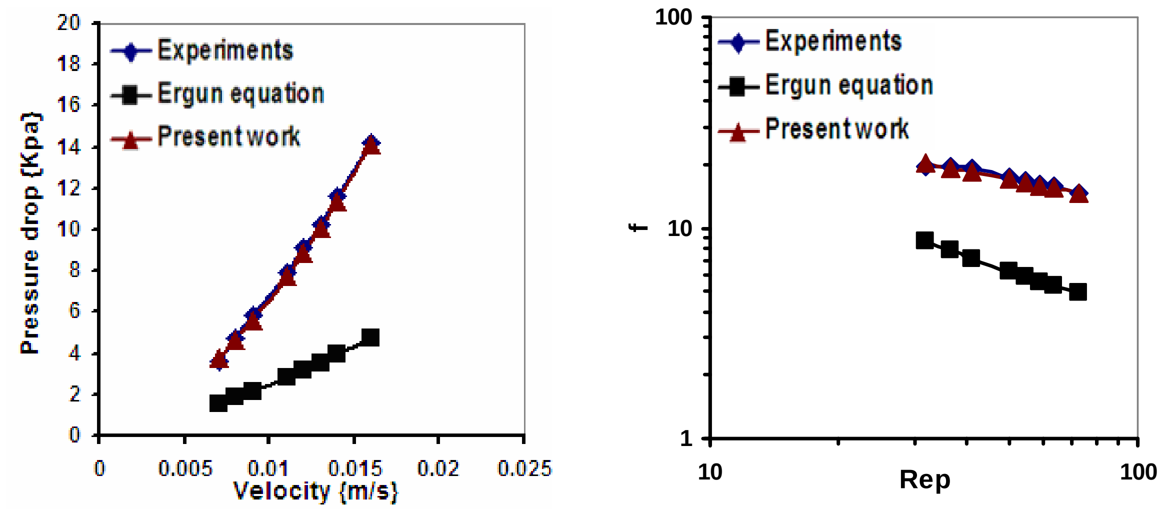
5.4.1.2 Mono size non spherical particles system

The values of pressure drop for mono size non spherical particles versus velocity were plotted in figures 5.5a, and 5.6a. The values of friction factors versus Reynolds numbers were plotted in figures 5.5b and 5.6b.



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.5 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for pea gravel of particle diameter 0.0254cm, sphericity of 0.7, bed porosity of 0.3064, packing height of 43.18cm, bed diameter of 8.89cm [54] (Table4.7)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.6 a-Pressure drop vs. velocity , b-Friction factor vs. Reynolds numbers for rasching rings of particles diameter 0.3cm,sphericity of 0.85, bed porosity of 0.3538,packing height of 67.3cm, bed diameter of 8.89cm [93](Appendix A.11)

Similar comparisons between experimental data and the achieved equation are shown in appendix A (A.12 to A.15).

The above figures show that the achieved equation gave a good fitting to the experimental data rather than Ergun equation. This appears clear in figure 5.6, which shows that the model results of pressure drop-velocity curves coincide with experimental results, while the results from Ergun equation lie above them. The deviation of Ergun curves for rasching rings may be for the reason that Ergun equation was not tested on object with holes (Sabri Ergun did not limit his experiment to spherical packing. He tested his theory with cylinders, tablets and crushed materials, but did not test it with objects with holes [5,46]).

Figure 5.5a show that the achieved results of pressure drop-velocity curve lie on the experimental results curve. The values of pressure drop of the model results in figure 5.5a (range from 22.69 to 131.5 kpa) are close to those of experimental data results (range from 21.95 to 129.39 kpa), for velocity (range from 0.016 to 0.048). The same is seen in figure 5.5b which shows that the model results of friction factor-Reynolds number curve lie on the experimental results. The values of friction factor of figure 5.5b (range from 2.131 to 1.372) are close to those of experimental data results (range from 2.303 to 1.393), for Reynolds numbers (range from 6.021 to 18.06).This means that the achieved equation is very close to the experimental results.

Equation 3.9 was fitted for water flow through packed beds of mono-sizes non spherical particles. In this fitting 18 sets of data from literatures [1, 54, 93, 94, 99, 100, 101, 105] were used, which includes 150 values of pressure drop versus velocity. So, the singular form of the achieved semi-empirical equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 3641.207 \left(\frac{L}{d_p}\right)^{0.7670} \left(\frac{1}{\mathrm{Re}}\right)^{0.4006} \varepsilon^{-3.1935} \phi^{0.9310} \qquad \ldots (5.3)$$

The largest average percentage error was found to be 6.8548% between experimental work and the achieved equation, while the average percentage error between the experimental work and the achieved equation was found to be 5.1117% for the calculations in set one and two [54] which were for packing type of pea gravel with shape factor of 0.7.

The porosity used in the presented model was proposed from best fitting of experimental data of water flow through packed beds of mono-sizes spherical particles. This porosity is represented in the following equation:

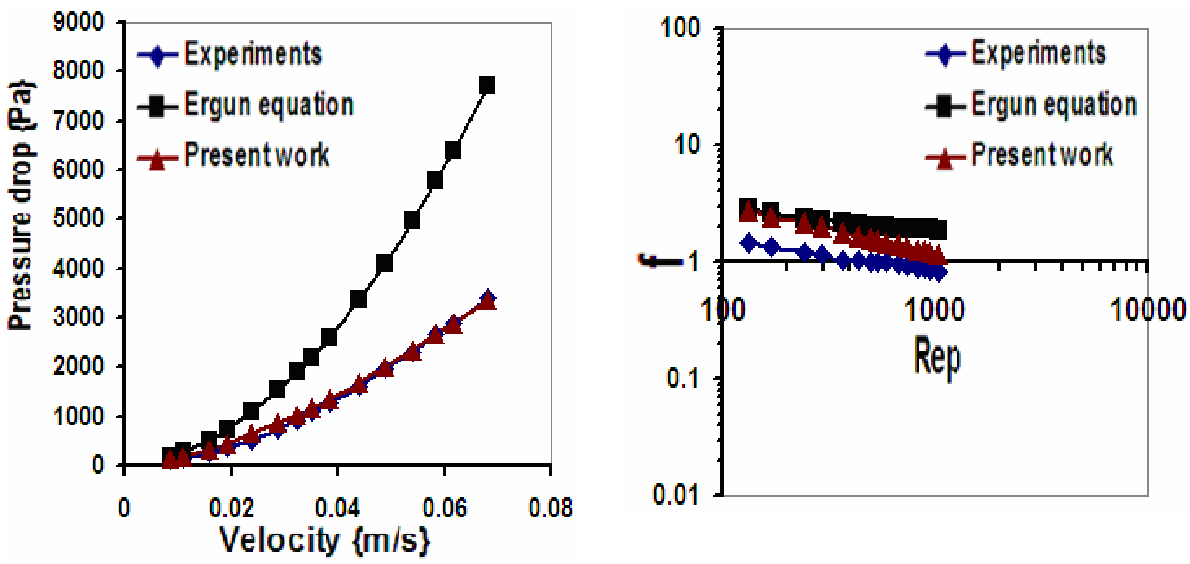
$$\varepsilon = \frac{0.0302}{\left(0.0014\ D_R^{-0.2784} + 0.0125\ d_P^{-0.097}\right)^{0.6634}} \qquad \dots (5.4)$$

The above equation deviates from experimental with a small average percentage error of 0.38895 %.

The average percentage error between the experimental work and the achieved equation in mono non spherical packing system was found to be higher than that of mono spherical packing system (6.8548% for mono non spherical packing system and 4.8397% for mono spherical packing system). This is because in mono size system the shape factor (sphericity) is always one in all spherical particles while in mono non spherical particles system the shape factor (sphericity) is changeable according to the packing type from (0-1) in different experiments .The values (0.3 -0.9) for sphericity were used because data for these values are available in literatures [1, 54, 93, 94, 99, 100, 101, 105].

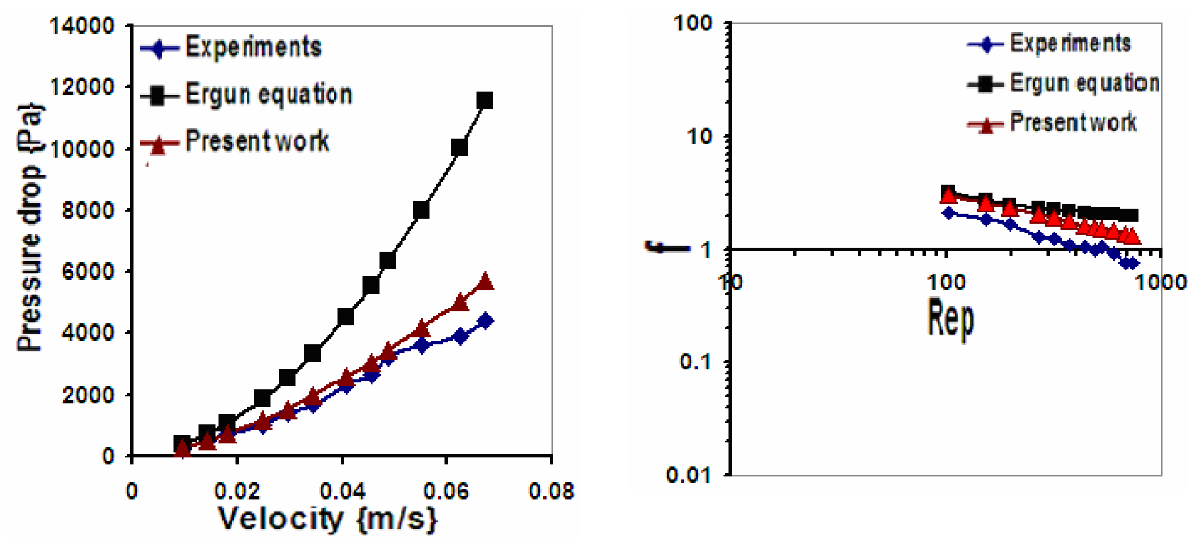
5.4.1.3 Binary sized spherical particles system

The values of pressure drop for water flow through packed beds of binary sized spherical particles were plotted versus velocity in figures 5.7a, 5.8a and 5.9a. The values of friction factors versus Reynolds numbers were plotted in figures 5.7b, 5.8b and5.9b.



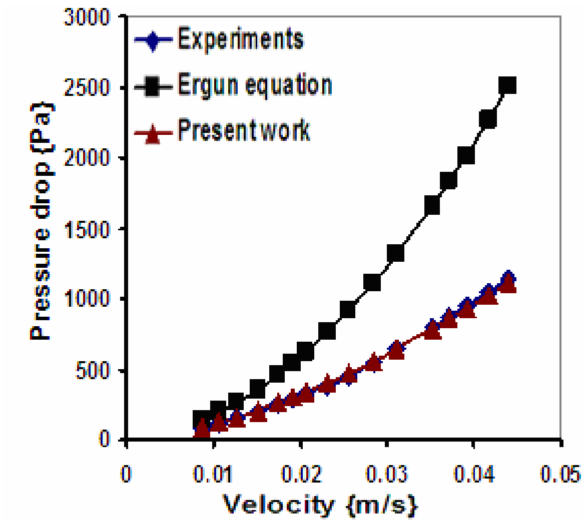
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.7 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for Acrylic balls of particles diameter (dp_1=0.655 cm ,dp_2=1.27 cm, with dp_{eff}=1.016 cm), fractions of (x_1=0.25,x_2=0.75), bed porosity of 0.367, packing height of 49.53 cm, bed diameter of 8 cm[88] (Table 4.8)

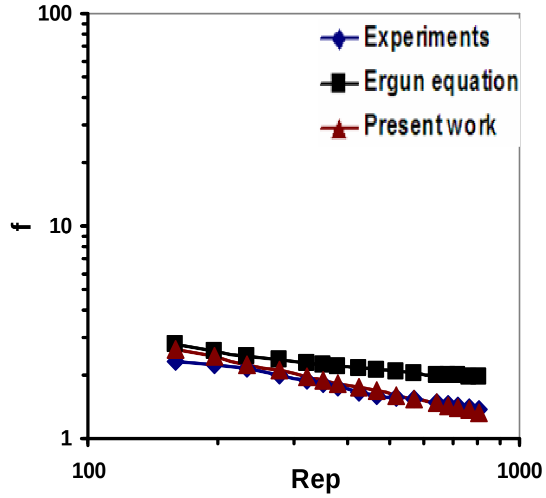


a- ΔP vs. u **b-** f vs. Re_p

Figure 5.8 a-Pressure drop vs.velocity, b-Friction factor vs. Reynolds numbers for Acrylic balls of particles diameter (dp_1=0.655 cm, dp_2=1.27 cm, with dp_{eff}=0.7257 cm), fractions of (x_1=0.75,x_2=0.25),bed porosity of 0.3612, packing height of 50.8 cm, bed diameter of 8 cm[107] (Appendix A.16)



a- ΔP vs. u



b- f vs. Re_p

Figure 5.9 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm, with dp_{eff}=0.7257cm), fractions of (x_1=0.75,x_2=0.25), bed porosity of 0.3612, packing height of 40.64cm, bed diameter of 8cm[107] (Appendix A.17)

Similar comparisons between experimental data and achieved equation are shown in appendix A (A.18 to A.22).

The achieved equation gave a good fitting to the experimental data rather than Ergun equation; this is because Ergun's equation is based on a large ratio of column diameter to particle diameter, neglecting wall effect. (In order to neglect wall effects a ratio of 10 or greater should be used [108]).While figure 5.7 show that the ratio of bed diameter to particle diameter is (8/1.016) which is less than 10. Ergun equation neglects wall effects which cause a great difference from experimental results. The wall effects is included in all experiments, also the present model includes this effect through the equation constants.

The most noticeable effect for mixing two sizes of particles is the decrease in porosity with respect to mono sized particles. This is because in binary system the particles with smaller sizes tend to fill the voids between the larger sizes particles [109].

Figure 5.8 and 5.9 show that as the packing height increases the pressure drop increases, this is because when the packing height increases the fluid flow resistance increases and this leads to an increase in the pressure drop. The packing height increased from 40.64 cm (Fig. 5.9) to 50.8cm (Fig. 5.8), which led to increase the pressure drop values from the range of (85.767-1117.4) Pa (Fig 5.9), to the range of (257.38-5704.8) Pa (Fig 5.8), for the same porosity of 0.3612, bed diameter of 8cm, effective particle diameter of 0.7257cm.

Equation (3.9) was fitted for water flow through packed beds of binary sized spherical particles. In this fitting 26 sets of data were used [88, 89, 90, 91, 107], which include 350 values of pressure drop versus velocity. So, the singular form for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 41.7922 \left(\frac{L}{d_p}\right)^{1.121} \left(\frac{1}{\text{Re}}\right)^{0.4194} \varepsilon^{-0.2435} \phi^{0.1} \qquad \ldots (5.5)$$

The average percentage error was found to be 1.1807% between experimental work and the achieved equation.

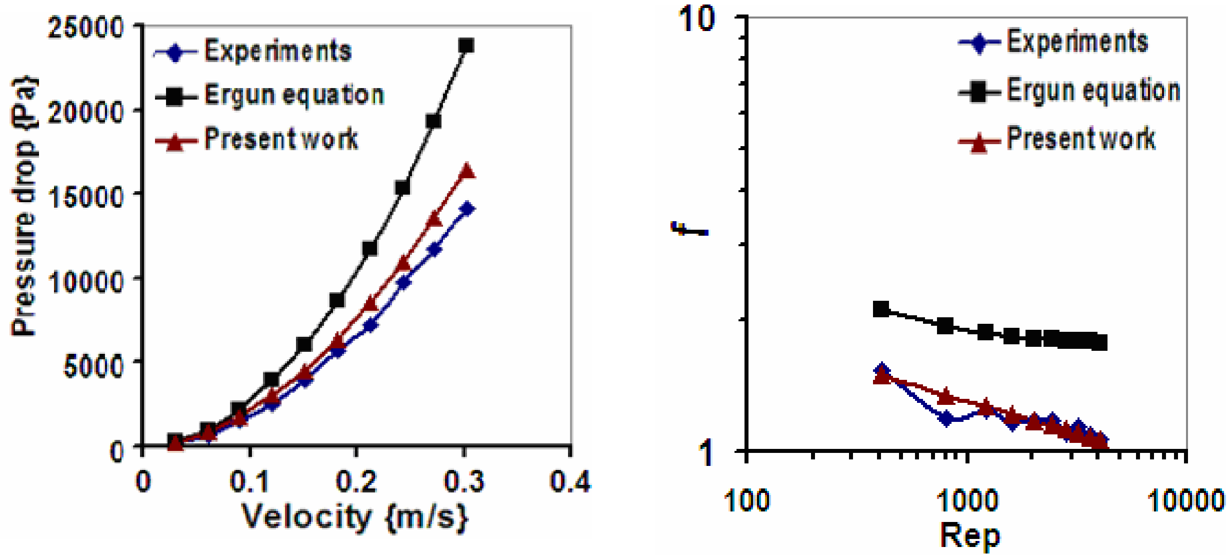
The porosity used in the present model was evaluated from best fitting of experimental data of water flow through packed beds of binary-sizes spherical particles. This porosity is represented in the following equation:

$$\varepsilon = \frac{0.0161}{\left(1.8949\, D_R^{0.8813} - 0.1864\, d_P^{0.0342}\right)^{0.987}} \qquad \ldots (5.6)$$

The above equation deviates from experimental with an average percentage error of 0.00577 %.

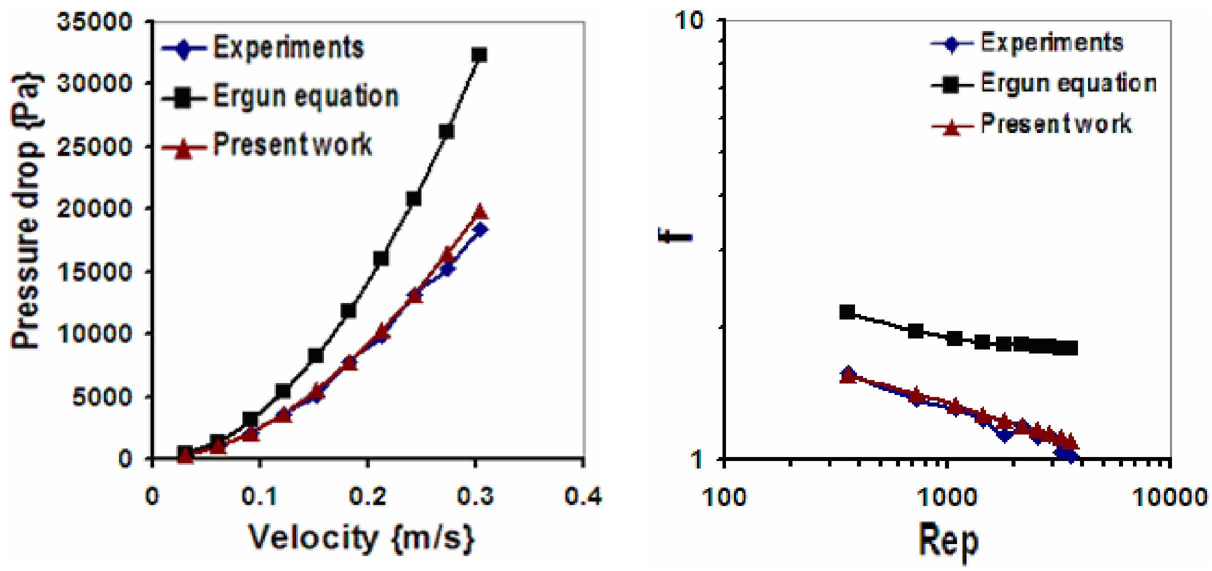
5.4.1.4 Ternary sized spherical particles system

The values of pressure drop for water flow through packed beds of ternary sized spherical particles versus velocity were plotted in figures 5.10a, 5.11a and 5.12a.The values of friction factors versus Reynolds numbers were plotted in figures 5.10b, 5.11b and5.12b.



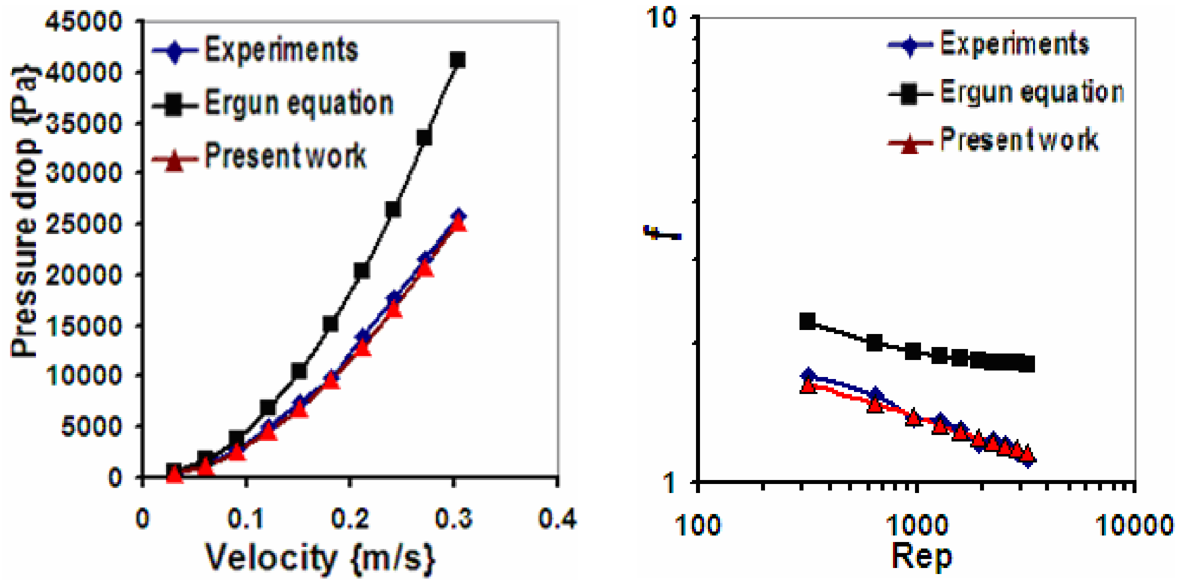
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.10 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass of particles diameter (dp_1=0.9987, dp_2=0.7955 and dp_3=0.6015cm ,with dp_{eff}=0.765 cm), bed porosity of 0.4111, packing height of 15.15cm, bed diameter of 7.62 cm [89] (Table 4.9)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.11 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for spherical particles diameter of (dp_1=0.9987, dp_2=0.7955 and dp_3=0.509 cm, with dp_{eff}=0.71 cm), bed porosity of 0.4023, packing height of 15.15 cm, bed diameter of 7.62 cm [89] (Appendix A.23)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.12 a-pressure drop vs.velocity, b-Friction factor vs. Reynolds numbers for spherical particles diameter of (dp_1=0.9987, dp_2=0.7955 and dp_3=0.421 cm, with dp_{eff}=0.647cm), bed porosity of 0.3921, packing height of 15.15 cm, bed diameter of 7.62 cm [89] (Appendix A.24)

Similar comparisons between experimental data and achieved equation are shown in appendix A (A.25 to A.31)

The above figures show that, the modified model gave a good fit to the experiment rather than Ergun equation, the reason of this difference may lie on the basis of Ergun equation itself [101, 77].

Equation (3.9) was fitted for water flow through packed beds of ternary sized spherical particles. In this fitting 25 sets of data [107, 88, 89, 90] were used. In these sets 250 values of pressure drop versus velocity were taken. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 0.3343 \left(\frac{L}{d_p}\right)^{0.9829} \left(\frac{1}{\mathrm{Re}}\right)^{0.1480} \varepsilon^{-5.0567} \phi^{0.1} \qquad \ldots (5.7)$$

The average percentage errors were found to be 5.5858% between experimental work and the achieved equation.

The porosity used in the presented model was proposed from best fitting of experimental data of water flow through packed beds of ternary-sizes spherical particles. This porosity is represented in the following equation:

$$\varepsilon = \frac{0.0380}{\left(2.0945 D_R^{-0.0569} - 2.4662 d_p^{0.0099}\right)^{0.9163}} \qquad \text{... (5.8)}$$

The above correlation deviates from experimental with an average percentage error of 0.00033 %.

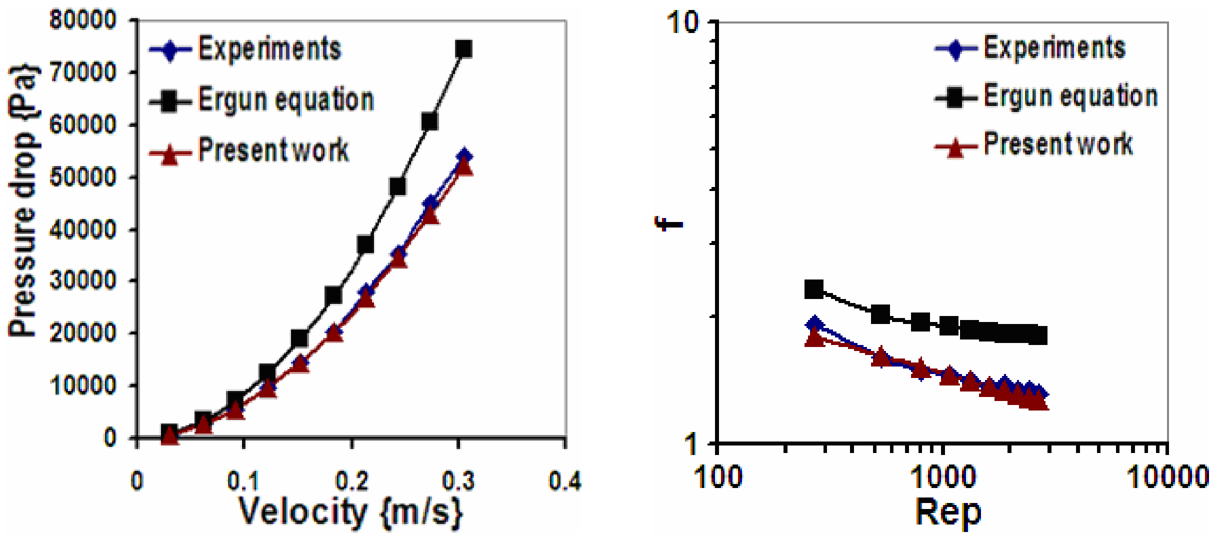
It can be noticed that the porosity of ternary sized packing are generally close to each other, since the voids could be filled with different sizes of particles [12].

The bed porosity highly affects the pressure drop and inversely proportional to it [104], this is appeared in figure 5.10a .This figure shows that for bed porosity of 0.411075 the pressure drop value has a range from 232.001 to 16498.5 Pa, which is less than the range of figure 5.11a for larger value of bed porosity of 0.4023, where the pressure drop values range from 280.459 to 19944.5 Pa.

In all figures, it can be noticed that the divergence values of friction factor is at low values of Reynolds number (laminar and transition region) but its convergence at high values of Reynolds number (turbulent region), because in the turbulent region where the high rate velocity of fluid behaves as a slip velocity and has insignificant effect on the friction values [100,101].

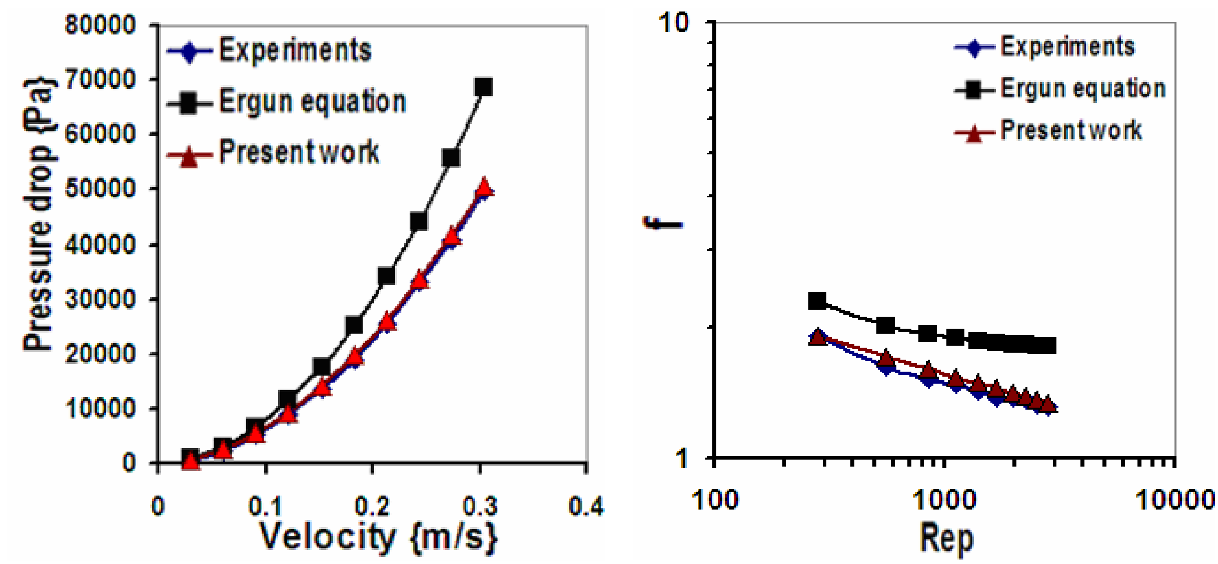
5.4.1.5 Quaternary sized spherical particles system

The values of pressure drop for water flow through packed beds of quaternary sized spherical particles versus velocity were plotted in figures 5.13a, and 5.14a.The values of friction factors versus Reynolds numbers were plotted in figures 5.13b and5.14b.



a- ΔP vs. u　　　　**b-** f vs. Re_p

Figure 5.13 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for spherical particles diameter of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.55 cm), bed porosity of 0.3711, packing height of 15.15 cm, bed diameter of 7.62cm [90] (Table 4.10)



a- ΔP vs u　　　　**b-** f vs Re_p

Figure 5.14 a-pressure drop vs. velocity, b- Friction factor vs. Reynolds numbers for spherical particles diameter of (0.42, 0.51, 0.61 and 1.01cm ,with dp_{eff}=0.5738 cm),bed porosity of 0.3719, packing height of 15.15 cm, bed diameter of 7.62 cm [90] (Appendix A.32)

Similar comparisons between experimental data and the achieved equation are shown in appendix A (A.32 to A.35).

The achieved equation show good results compared with the experimental data [90] as shown in figures 5.13 and 5.14. These results were better than the results from Ergun equation. These figures show clearly that the achieved equation results of pressure drop-velocity curves and the results of friction factor-Reynolds number curves are coincide with experimental results, while the results from Ergun equation lie above them. This is due to the differences in beds dimensions, packing shapes and sizes used by Ergun [7,111]. For example figures 5.13b and 5.14b which show that the values of friction factor of achieved equation results (range from 1.81-1.27 and 1.905-1.336 respectively) are coincide with experimental data results (range from 1.913-1.313 and 1.908-1.311 respectively), for Reynolds numbers (range from 267.8-2675 and 280.3-2799 respectively).

The achieved equation (3.9) was fitted for water flow through packed beds of quaternary sized spherical particles. In this fitting 5 sets of data [90] were used. In these sets 52 values of pressure drop versus velocity were taken. So, the singular form of the modified model for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 10.9685 \left(\frac{L}{d_p}\right)^{0.3339} \left(\frac{1}{\mathrm{Re}}\right)^{0.1542} \varepsilon^{-3.9204} \phi^{0.1} \qquad \dots (5.9)$$

The average percentage error was found to be 2.5303% between experimental work and the achieved equation.

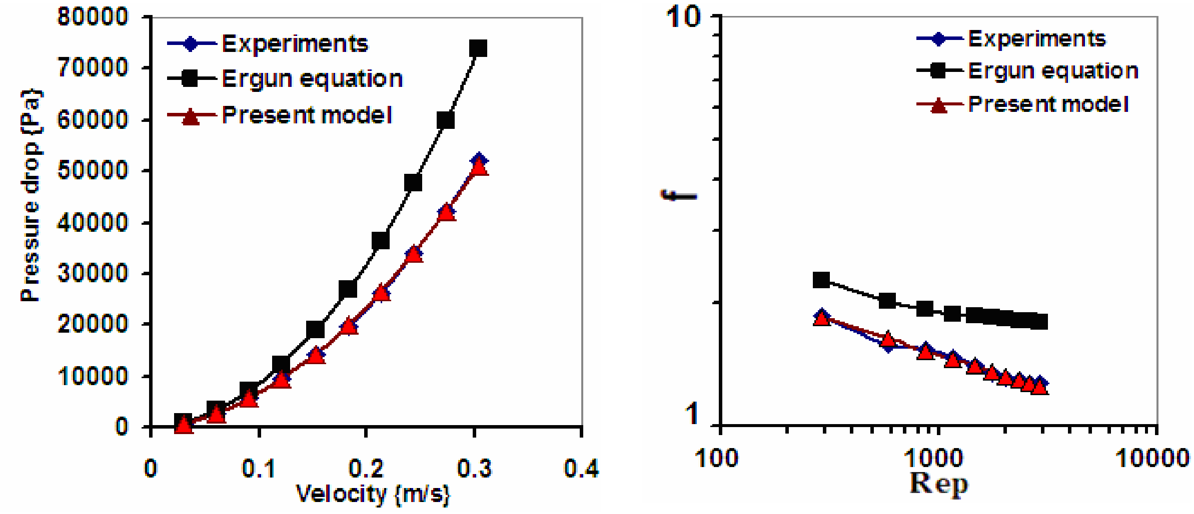
The porosity used in the present model was evaluated from best fitting of experimental data of water flow through packed beds of quaternary-sizes spherical particles. This porosity is represented in the following equation.

$$\varepsilon = \frac{0.1397}{\left(0.4855\, D_R^{3.9653} - 0.0533\, d_P^{1.7936}\right)^{0.0869}} \qquad \dots (5.10)$$

The above equation deviates from experimental with an average percentage error of $2.5*10^{-4}$ %.

5.4.1.6 Quinary sized spherical particles system

The values of pressure drop for water flow through packed beds of quinary sized spherical particles versus velocity were plotted in figure 5.15a. The values of friction factors versus Reynolds numbers were plotted in figure 5.15b.



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.15 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.61 cm), bed porosity of 0.3623, packing height of 15.15cm, bed diameter of 7.62 cm [90] (Table 4.11)

From the above figure, it can be noticed that the achieved equation shows very good fitting to the experiment rather than Ergun equation, this appears in figure 5.15. Figure 5.15a shows that the model results of pressure drop-velocity curve lies on the experimental results curve. The values of pressure drop of figure 5.15a (range from 756.5 to 51176) are close to those of experimental data results (range from 767 to 52050), for velocity (range from 0.0305 to 0.3046). The same is seen in figure 5.15b which shows that the model results of friction factor-Reynolds number curve lies on the experimental results. The values of friction factor of figure 5.15b (range from 1.842 to 1.2567) are close to those of experimental data results (range from 1.868 to 1.271), for Reynolds numbers (range from 290.4 to 2900). This means that the achieved equation is very close to the experimental results.

The achieved equation (3.9) was fitted for water flow through packed beds of quinary sized spherical particles. In this fitting one set of data [90] was used, which included 10 values of pressure drop versus velocity. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 1.6327 \left(\frac{L}{d_p} \right)^{1.8577} \left(\frac{1}{\mathrm{Re}} \right)^{0.1687} \varepsilon^{-0.59} \, \phi^{0.7196} \qquad \ldots (5.11)$$

The average percentage errors were found to be 0.00262% between experimental work and the achieved equation.

The porosity used in the present model was proposed from best fitting of experimental data of water flow through packed beds of quinary sized spherical particles. This porosity is represented in the following equation:

$$\varepsilon = \frac{0.4592}{\left(1.1199\, D_R^{0.5048} - 0.0706\, d_P^{0.0147}\right)^{-0.2369}} \qquad \ldots (5.12)$$

The above correlation deviates from experimental data with an average percentage error of 0.10494 %.

5.4.2 General equation results

The results of the general equation are presented in this section. This presentation takes into account a comparison with Ergun equation and experimental data. The results of the general equation include mono spherical particles, mono non spherical particles, binary spherical particles, ternary spherical particles, quaternary spherical particles and quinary spherical particles systems.

The achieved equation (3.9) was fitted for water flow through packed beds of multi sized particles. In this fitting 115 sets of data from literatures [1, 3, 9, 20, 28, 54, 60, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103,104, 105, 106, 107] were used, which includes 1208 values of pressure drop versus velocity. So, the general form of the achieved equation for water flow through packed beds will be as follows:

$$\frac{\Delta P}{\rho u^2} = 55.3456 \left(\frac{L}{d_p}\right)^{1.2439} \left(\frac{1}{\mathrm{Re}}\right)^{0.3316} \varepsilon^{-0.3947} \phi^{0.1} \qquad \ldots (5.13)$$

The average percentage errors were found to be 8.9981% between experimental work and the achieved equation.

The porosity used in the presented model was evaluated from best fitting of experimental data of water flow through packed beds of multi-sizes spherical particles. Which is as follows:

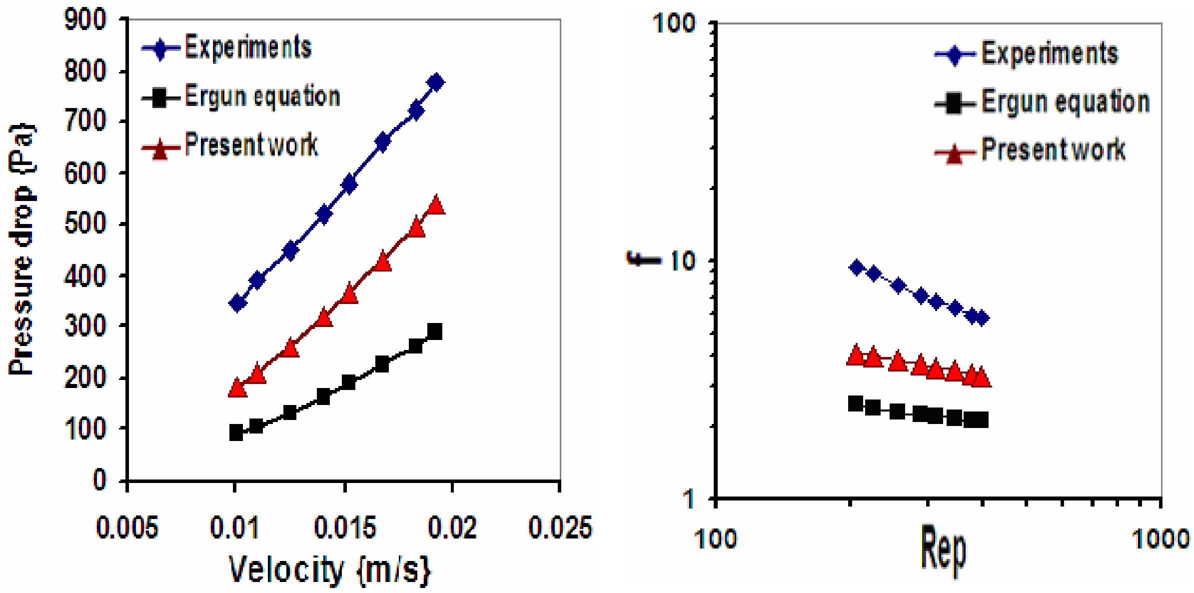
$$\varepsilon = \frac{0.148}{\left(1.6178\, D_R^{6.3578} - 0.00028\, d_p^{1.7666}\right)^{0.0579}} \qquad \ldots (5.14)$$

The above correlation deviates from experimental data with an average percentage error of 0.00082 %.

Equation 5.13 and 5.14 shown above can be used for all types of packing systems.

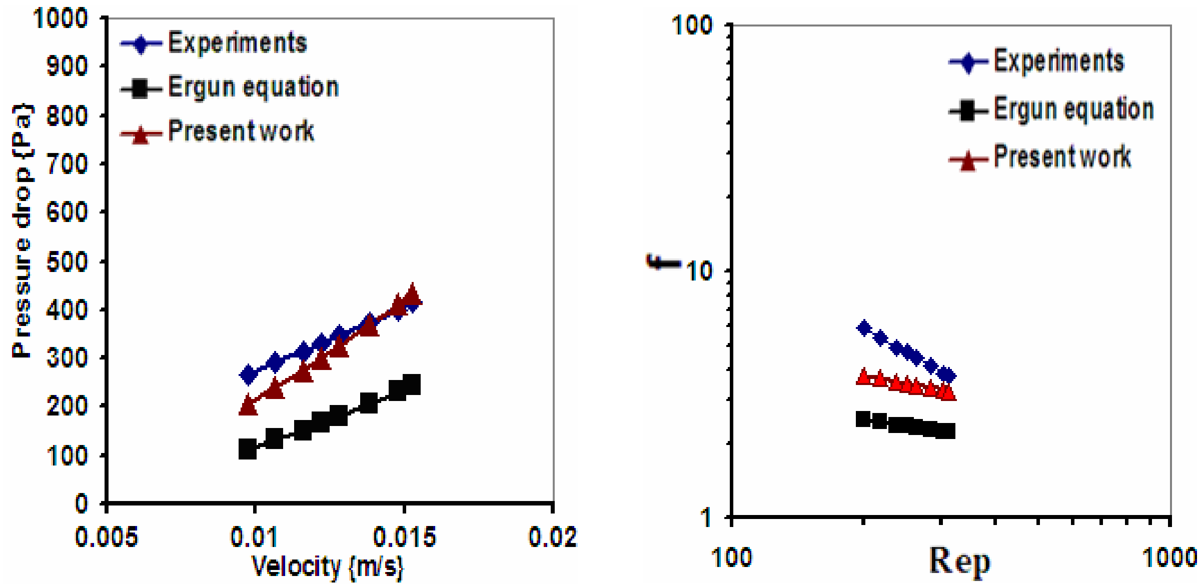
The values of pressure drop versus velocity for water flow through packed beds were plotted in Figs. 5.16a and 5.17a for mono spherical particles, Fig. 5.18a for mono non spherical particles, Figs. 5.19a and 5.20a for binary spherical particles, Figs. 5.21a and 5.22a for ternary spherical particles, Fig. 5.23a for quaternary spherical particles and Fig. 5.24a quinary spherical particles systems.

The values of friction factors versus Reynolds numbers were plotted in Figs. 5.16b and 5.17b for mono spherical particles, Fig. 5.18b for mono non spherical particles, Figs. 5.19b and 5.20b for binary spherical particles, Figs. 5.21b and 5.22b for ternary spherical particles, Fig. 5.23b for quaternary spherical particles and Fig. 5.24b quinary spherical particles systems.



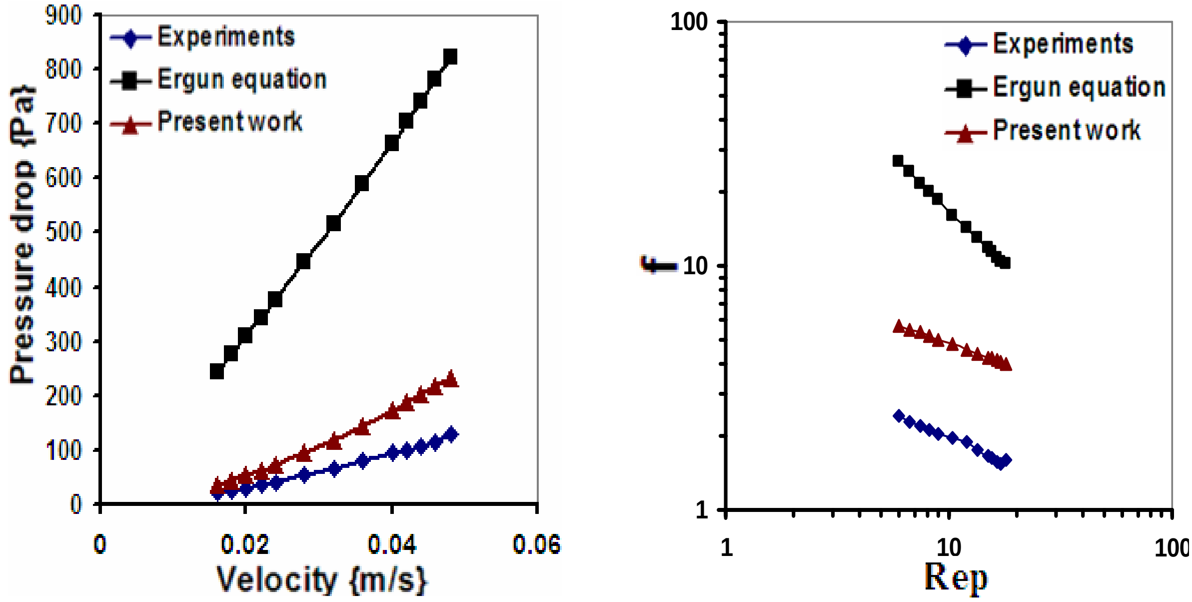
a- ΔP vs. u **b-** f vs. Rep

Figure 5.16 a-Pressure drop vs.velocity, b-Friction factor vs. Reynolds numbers for pea gravel of particles diameter 1.27 cm, bed porosity of 0.36067, packing height of 41.28 cm, bed diameter of 8.89 cm [87] (Table 4.12)



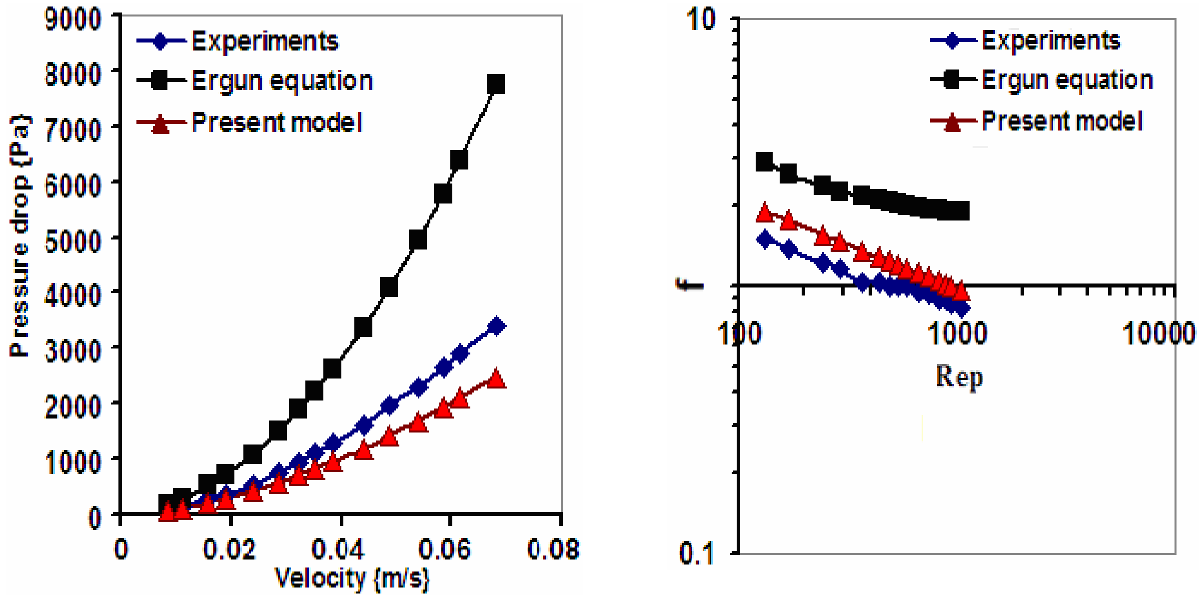
a- ΔP vs. u **b-** f vs. Rep

Figure 5.17 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for pea gravel of particles diameter 1.27 cm, bed porosity of 0.36067, packing height of 53.34 cm ,bed diameter of 8.89 cm[87] (Appendix A.36)



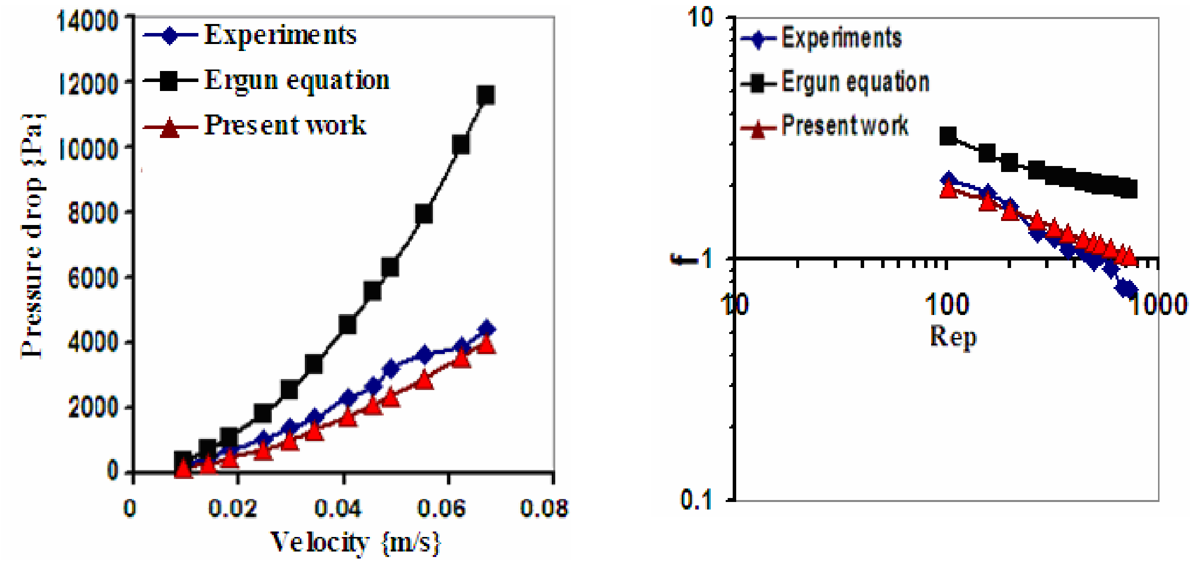
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.18 a-Pressure drop vs.velocity , b-Friction factor vs. Reynolds numbers for pea gravel of particle diameter 0.0254cm, sphericity of 0.7, bed porosity of 0.3064, packing height of 43.18cm, bed diameter of 8.89cm [54].(Table 4.13)



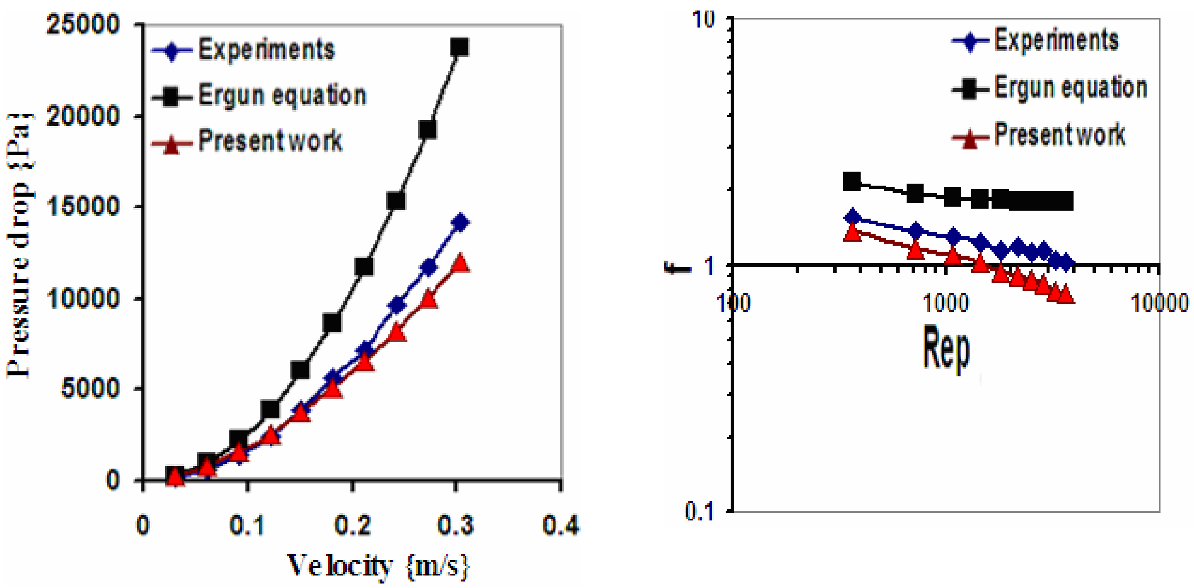
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.19 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm,with dp_{eff}=1.016 cm),fractions of(x_1=0.25,x_2=0.75) ,bed porosity of 0.37435 ,packing height of 49.53cm, bed diameter of 8cm [88].(Table 4.14)



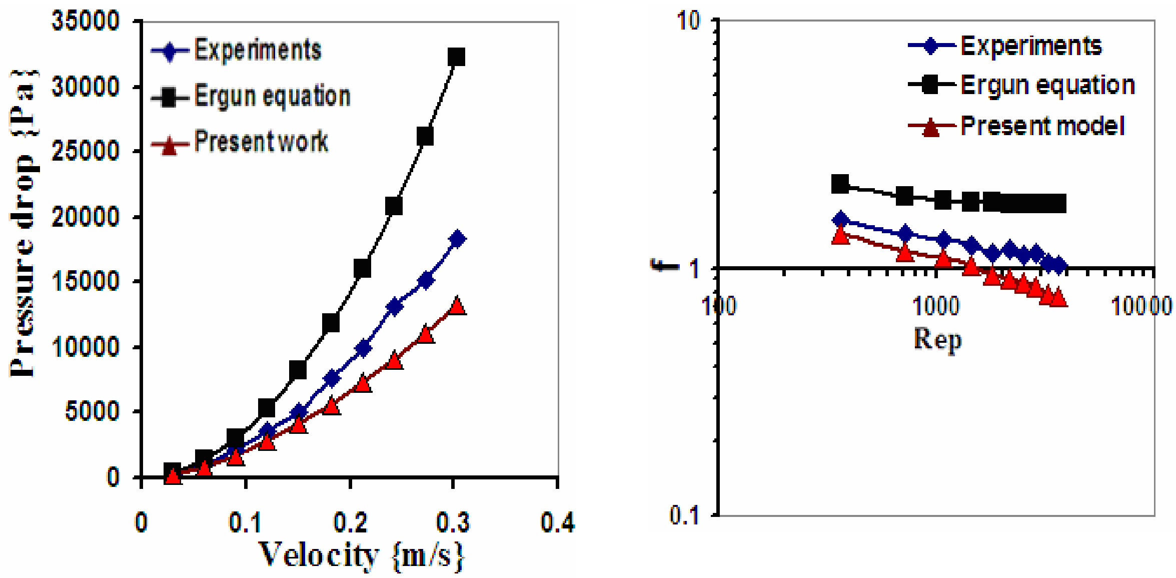
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.20 a-Pressure drops vs. velocity, b-Friction factor vs. Reynold numbers for Acrylic balls of particles diameter (dp_1=0.655, dp_2=1.27, and dp_{eff}=0.7257 cm), fractions of (x_1=0.75, x_2=0.25), bed porosity of 0.37165, packing height of 50.8 cm, bed diameter of 8 cm [88] (Appendix A.37)



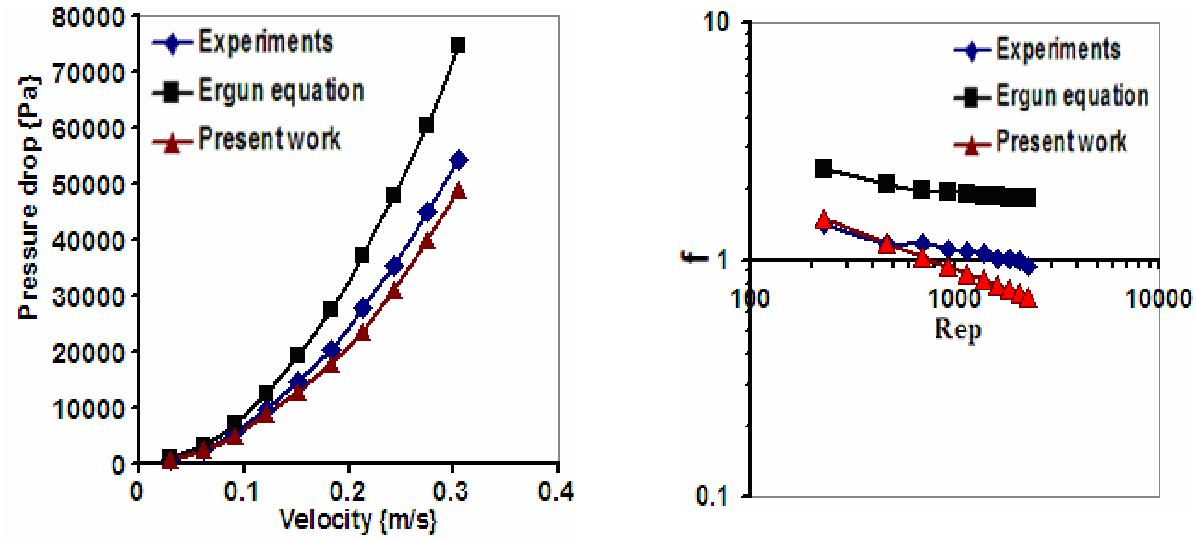
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.21 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass sphere of particles diameter (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.765 cm), bed porosity of 0.3832, packing height of 15.15 cm, bed diameter of 7.62 cm [89] (Table 4.15)



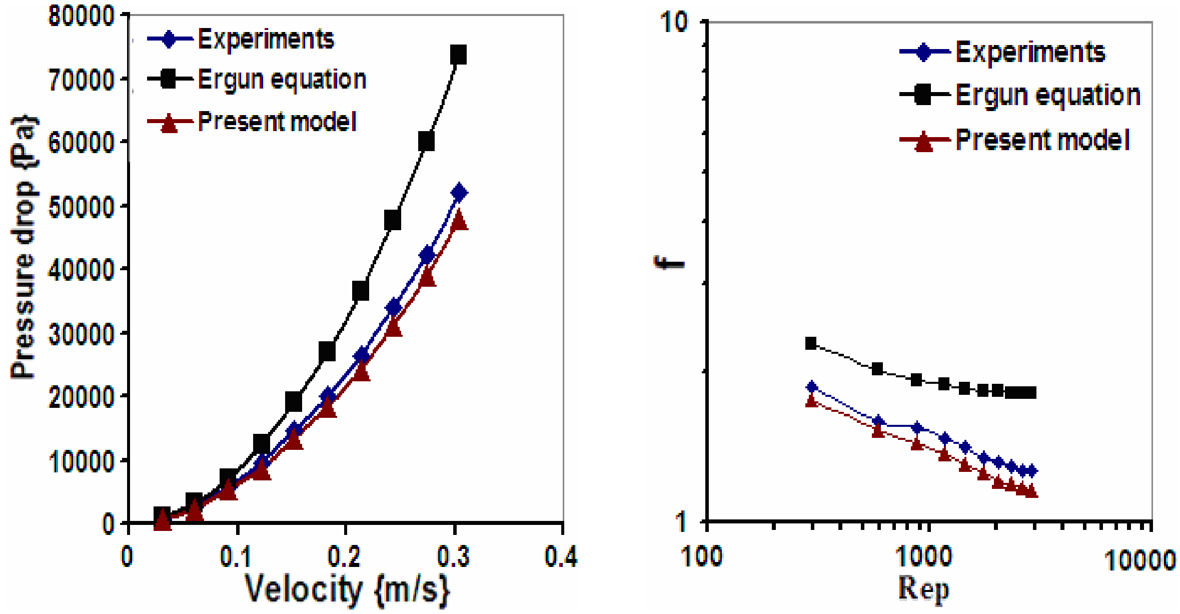
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.22 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass sphere of particles diameter (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.71 cm), bed porosity of 0.3806, packing height of 15.15 cm, bed diameter of 7.64 cm [89] (Appendix A.39)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.23 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spherical particles diameter of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.55 cm), bed porosity of 0.3771, packing height of 15.15 cm, bed diameter of 7.62 cm [91] (Table 4.16)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.24 a-Pressure drops vs. velocity, b- Friction factor vs.Reynolds number for spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.61 cm), bed porosity of 0.3623, packing height of 15.15 cm, bed diameter of 7.62 cm [90](Table 4.17)

It could be noticed from figures (5.16 to 5.24) that the achieved equation gave a good fitting to the experimental results which is better than Ergun equation. This is due to the fact that Ergun equation assumed smooth geometric surface area of the particles, but the irregular of the surface of the particles area would increase the drag force of the fluid moving past the particles (frictions) as well as the pressure drop [107,95]. So there is a cretin deviation between Ergun results and experimental results. This deviation was also found between the achieved equation results and the Ergun equation.

The general equation can be used for any system of packing, while the singular equation for only one types of packing which was written for it ,and can not be used for another type. Figures (5.16 to 5.24) show the results of the general equation for multi sized particles (equation 5.13), which can be compared with the results of singular equation for different types of packing system shown in figures (5.1, 5.2, 5.5, 5.8, 5.9, 5.10, 5.11, 5.13, 5.15 respectively). The comparisons between general and singular equations results show quite good agreement.

Figures 5.24b and 5.25b, shows that the values of friction of figure 5.24b (rang from 1.288 to 0.597) approximately the same values of figure 5.25b (range from 1.295 to 0.605); this is because they have approximately the same values of bed porosities and mean particles diameter.

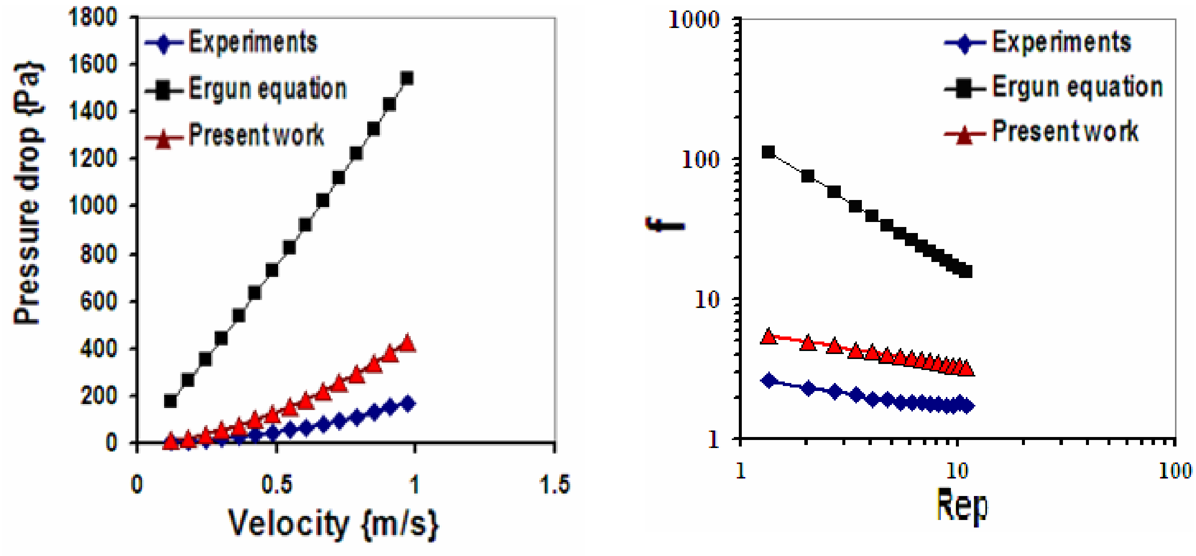
It is clear that as the porosity decreases the friction factor decreases [92], in spite of pressure drop increases, and this is because that the friction factor is proportional to power three with porosity, while it is proportional to power one with pressure drop (equation 2.39).

5.5 Comparisons between achieved equation, Ergun equation and experimental results for air flow through packed bed

5.5.1 Singular equations results for different types of packing

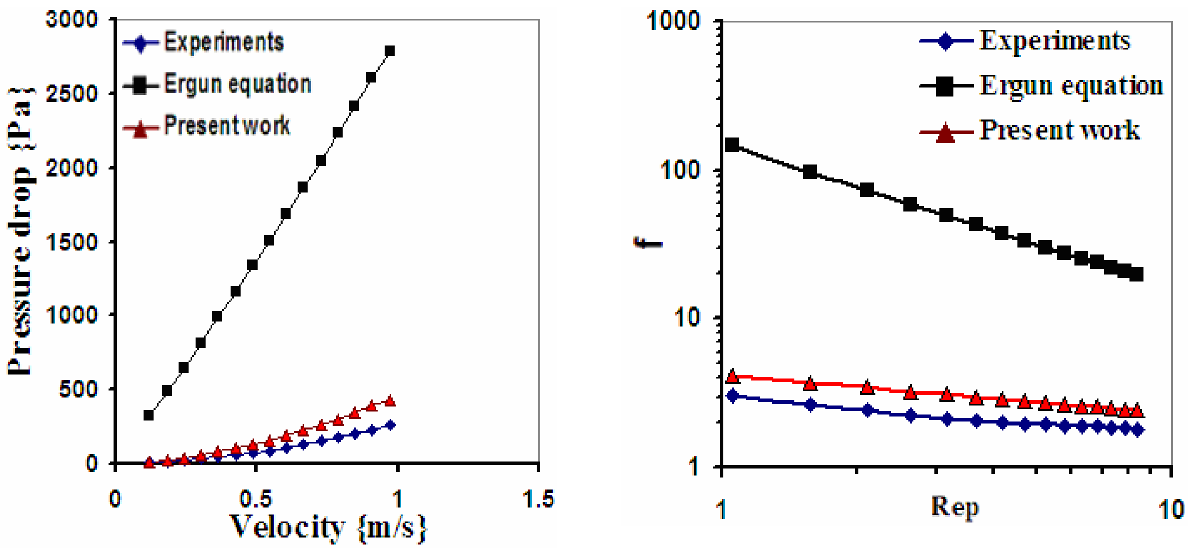
5.5.1.1 Mono size spherical particle system

The values of pressure drop for air flow through packed beds of mono size spherical particles versus velocity were plotted in figures 5.25a, 5.26a and 5.27a. The values of friction factors versus Reynolds numbers were plotted in figures 5.25b, 5.26b and 5.27b.



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.25 a- Pressure drop vs.velocity,b- Friction factor vs. Reynolds numbers for glass spheres of particle diameter 0.9987cm, bed porosity of 0.4181, packing height of 15.15cm, bed diameter of 7.64cm [89](Table 4.18)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.26 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particle diameter 0.7955 cm, bed porosity of 0.4088, packing height of 15.15 cm, bed diameter of 7.64 cm [89] (Appendix B.1)

Similar comparisons between experimental data and the achieved equation are shown in appendix B (B.2, B.3 and B.4).

Figures 5.25 and 5.26 shows that the achieved equation results show good agreement to the experimental results rather than Ergun equation results. Figure 5.26b shows that the achieved equation results of friction factor-Reynolds number curve is close to the values of friction factor-Reynolds number curve of the experimental results. The values of friction factor of achieved equation results of figure 5.26b (range from 4.1295 to 2.4259) are close to those of experimental data results (range from 3.0234 to 1.8114), for Reynolds numbers (range from 1.0502 to 8.4079). This means that the achieved equation results are very close to the experimental results.

The achieved equation (3.9) was fitted for air flow through packed beds of mono-sizes spherical particles. In this fitting 20 sets of data from literatures [89, 91, 96]were used, which include 336 values of pressure drop versus velocity. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 783.6491\left(\frac{L}{d_p}\right)^{0.1937}\left(\frac{1}{\text{Re}}\right)^{0.2557}\varepsilon^{-0.3318}\,\phi \qquad \dots (5.15)$$

The average percentage error was found to be 2.0224% between experimental work and the achieved equation.

The porosity used in the present model was evaluated from best fitting of experimental data of the air flow through packed beds of mono-sizes spherical particles. This porosity can be represented in the following equation.

$$\varepsilon = \frac{0.1202}{\left(0.1106\,D_R^{-0.3381} - 2.4099\,d_p^{0.9253}\right)^{0.8488}} \qquad \dots (5.16)$$

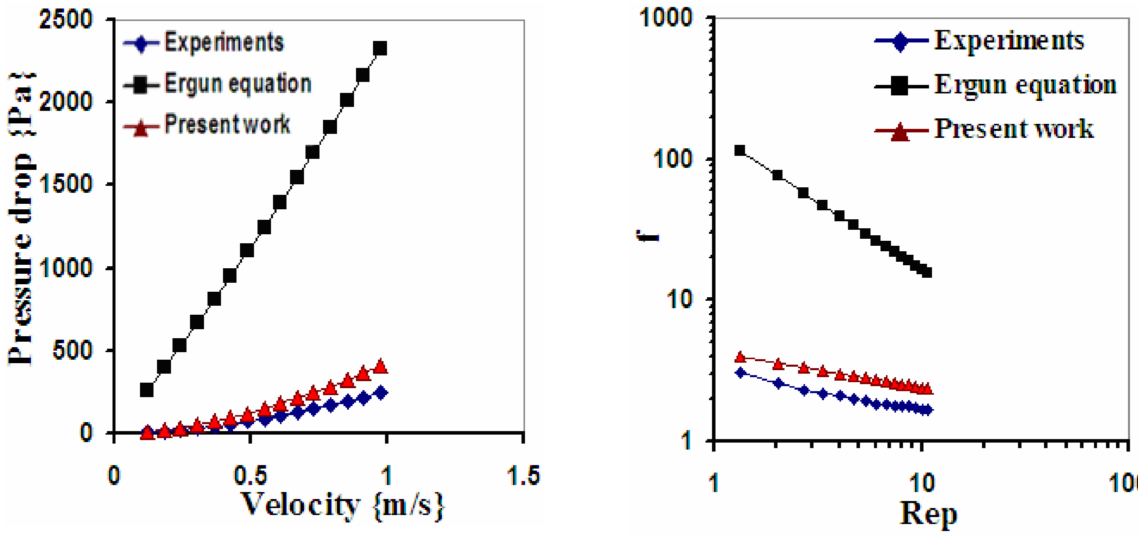
The above correlation deviate's from experimental with an average percentage error of 0.00013%.

The wall affects bed porosity and increases its value. This appears in figure 5.25 where the bed porosity increases to a value of 0.4181, this wall effect may be due to the ratio of bed diameter (7.64cm) to the particles diameter

(0.9987cm) which is less than the supposed ratio (column diameter to the particle diameter should be greater than 10:1 [30,105])

Examining figures 5.25 and 5.26 give that the values of friction factor decrease sharply with increasing Reynolds numbers, because the fluid flow is at the laminar region (where the friction factor-Reynolds number curve is of slope of -1 and the friction factor-Reynolds number curve become straighter)[7].

In the following figures a comparisons between the present model and the experimental data are presented. The experimental data includes 10 sets of data [90], these sets include 217value of pressure drop versus velocity. This comparison is not included in the fitting of the present model and it has been made to check that equation 5.15 could be used for different conditions. Figure 5.27 is represented below for these calculations, while other tables of comparisons are presented in appendix B (B.6 and B.7)



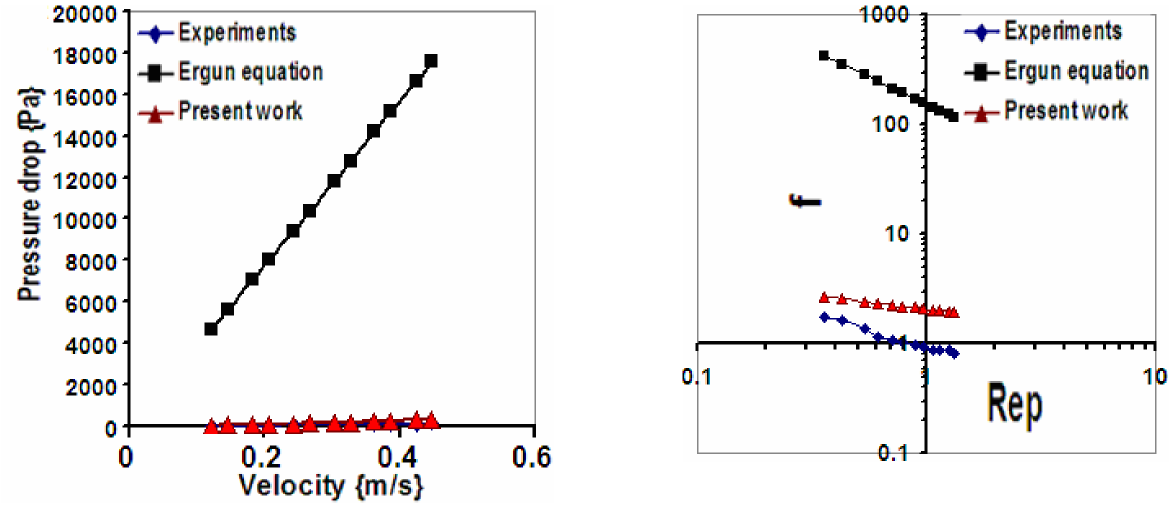
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.27 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particle diameter 1.01cm ,bed porosity of 0.4186 packing height of 20cm ,bed diameter of 7.62cm [90]. (Appendix B.5)

Although these figures are not included in the modeling they show very good agreement with experimental data.

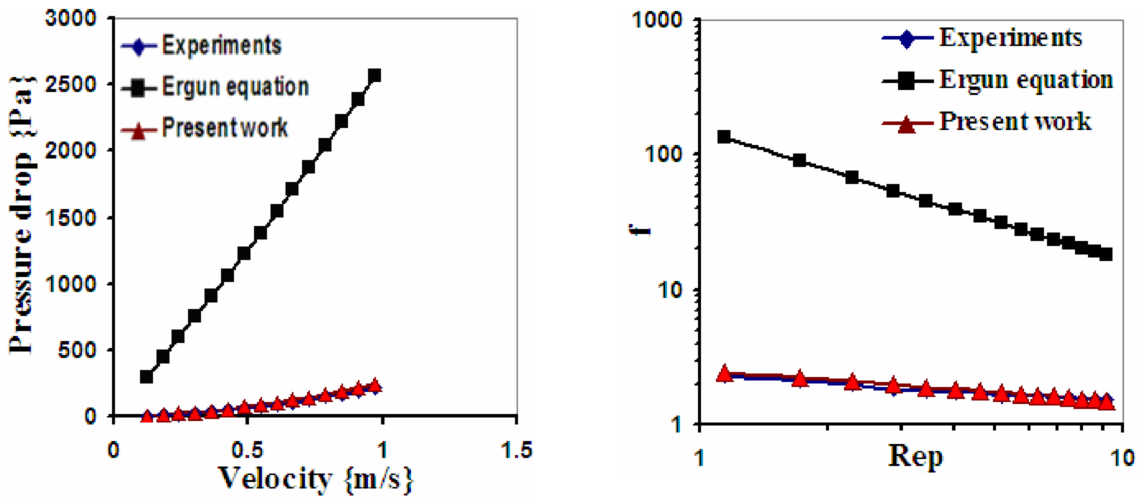
5.5.1.2 Binary sized spherical particles system

The values of pressure drop for air flow through packed beds of binary sized spherical particles were plotted versus velocity in figures 5.28a, 5.29a and 5.30a. The values of friction factors versus Reynolds numbers were plotted in figures 5.28b, 5.29b and 5.30b.



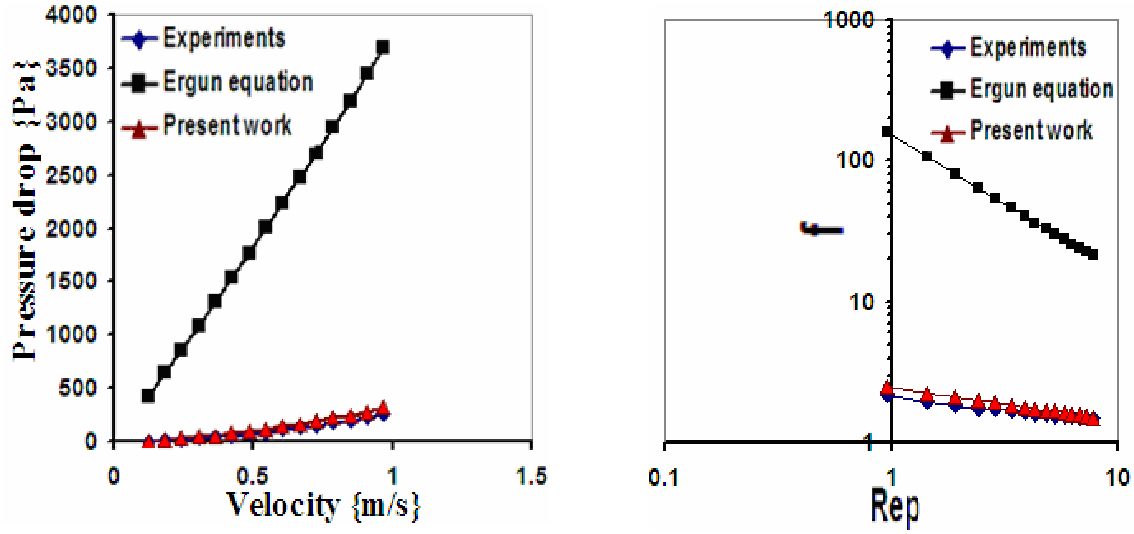
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.28 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameter (dp_1=0.24 and dp_2=0.42, with dp_{eff}=0.3055 cm), bed porosity of 0.3515, packing height of 15.15 cm, bed diameter of 7.64cm [91] (Table 4.19)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.29 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particle diameters (dp_1=0.9987 and dp_2=0.7955cm, with dp_{eff}=0.886 cm), bed porosity is 0.4079, bed diameter is 7.64cm, packing height is 15.15 cm [89] (Appendix B.8)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.30 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particle diameters(dp_1=0.9987 and dp_2=0.6015cm, with dp_{eff}=0.7508 cm), bed porosity is 0.3986, bed diameter is 7.64 cm, packing height is 15.15 cm[89] (Appendix B.9)

Similar comparisons between experimental data and the achieved equation are shown in appendix B (B.10 to B.16).

The above figures show that, the achieved equation results satisfied the experiment results rather than Ergun equation, and this can be noticed from figure 5.28, 5.29 and 5.30, in these figures the pressure drop curves of the achieved equation results lies on the curves of the experimental results from literature [89, 90, 91, 96, 102].

The achieved equation (3.9) was fitted for air flow through packed beds of binary sized spherical particles. In this fitting 30 sets of data [89, 90, 91, 96,102] were used, which includes 391 values of pressure drop versus velocity were taken. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 3.2151\left(\frac{L}{d_p}\right)^{1.105}\left(\frac{1}{\text{Re}}\right)^{0.2356}\varepsilon^{-1.6698}\ \phi \qquad \ldots (5.17)$$

The average percentage error was found to be 3.13327% between experimental work and the achieved equation.

The porosity used in the present model was correlated using the best fitting of experimental data of air flow through packed beds of binary-sized spherical particles. This porosity is represented in the following equation:

$$\varepsilon = \frac{0.1611}{\left(1.6782\ D_R^{-0.0469} - 1.9749\ d_P^{0.0191}\right)^{0.3762}} \qquad \ldots (5.18)$$

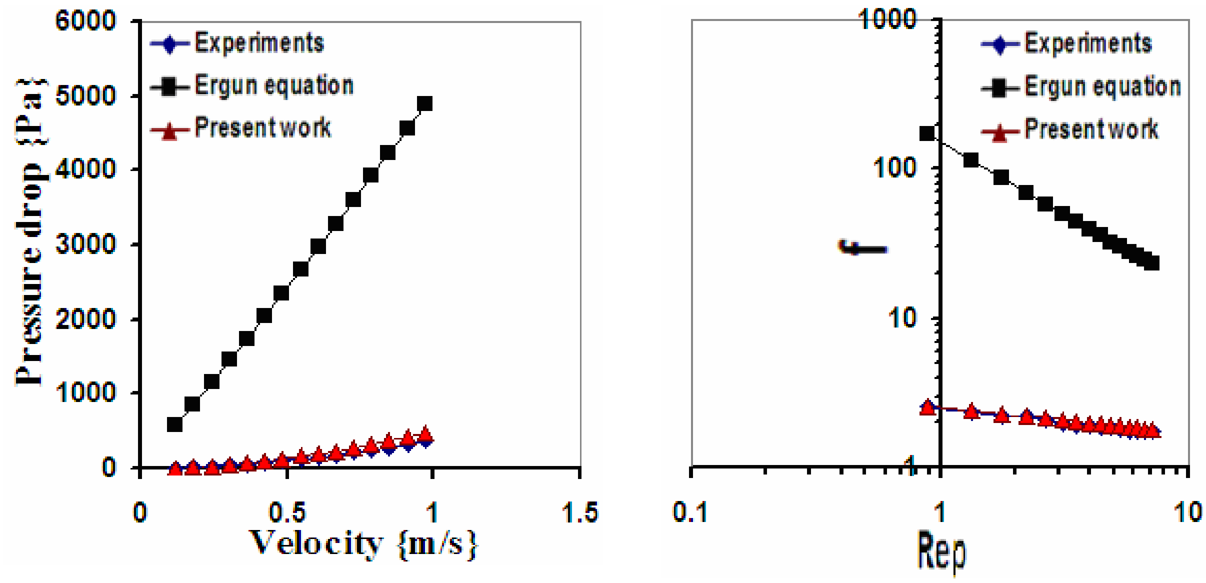
The above correlation deviates from experimental with an average percentage error of $0.991*10^{-3}$ %. This means that they are identical.

It can be noticed that the values of friction factor of binary sizes system are less than those of mono sizes system for approximately near values of bed porosity. For example, Fig. 5.29 (binary system) shows that the values of friction factor range from 2.4479 to 1.4997 are less than those of Fig. 5.25 (mono system) which range from 5.5328 to 3.2503, This is because the contacting surface area of binary size system (677.2009 m^{-1}) is greater than that of mono size system (600.7810 m^{-1}).

As the velocity of fluid increases the pressure drop across the bed increases, e.g., table 4.19 shows that the pressure drop values increase from 33.074 to 333.084 Pa with increasing the fluid velocity from 0.121 to 0.448 m/s.

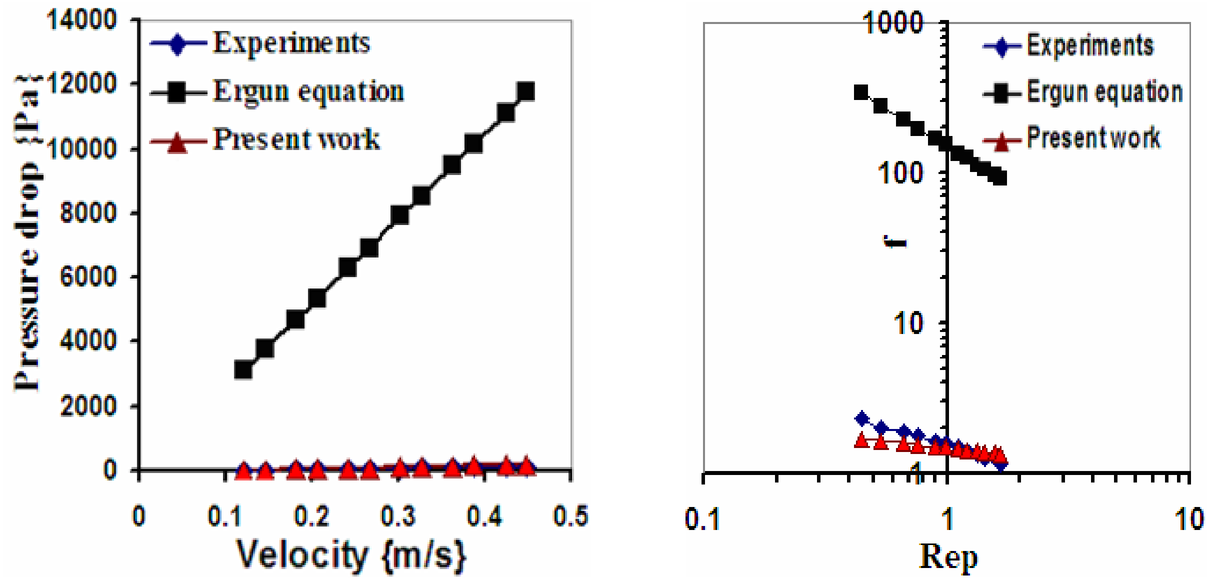
5.5.1.3 Ternary sized spherical particles system

The values of pressure drop for air flow through packed beds of ternary sized spherical particles versus velocity were plotted in figures 5.31a, 5.32a and 5.33a. The values of friction factors are plotted versus Reynolds numbers in figures 5.31b, 5.32b and 5.33b.



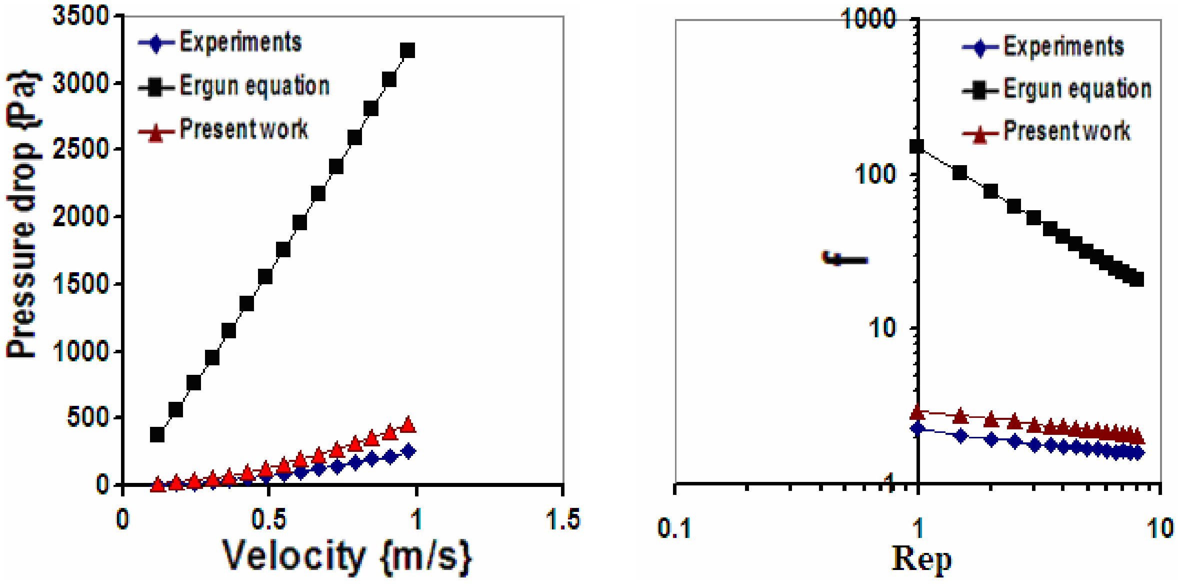
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.31 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particle diameters of (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.7104 cm), bed porosity of 0.3796, packing height of 15.15 cm, bed diameter of 7.64 cm [89] (Table 4.20)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.32 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particle diameters (0.24, 0.42 and 0.82 cm, with dp_{eff}=0.3862 cm), bed porosity of 0.3428, packing height of 15.15 cm, bed diameter of 7.64 cm [91] (Appendix B.17)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.33 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass of particles diameter (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651 cm), bed porosity of 0.3899, packing height of 15.15cm, bed diameter of 7.64 cm [89] (Appendix B.18)

Similar comparisons between experimental data and the modified model are shown in appendix B (B.19 to B.25))

It can be noticed from figures 5.31, 5.32 and 5.33 that the achieved equation results gave a good fitting to the experiment rather than Ergun equation. This is due to the fact that Ergun derived the values for the constants through experiments where the packing was small, non-spherical, and rough [2].

The bed porosity highly affects the pressure drop and inversely proportional to it, this is because that when the porosity increases the resistance to fluid flow through the bed decreases. For example in figure 5.31a for bed porosity of 0.3796 the pressure drop values which range from 10.62 to 472.2 Pa are greater than those of figure 5.33a, while for larger values of bed porosity of 0.3899 the pressure drop values range from 10.36 to 460.6 Pa, for the same value of (bed diameter 7.64cm, packing height 15.15cm, and near values of effective particle diameters (dp_{eff}) of figure 5.31a (0.7104cm) close to the value of figure 5.33a (0.7615cm).

The achieved equation (3.9) was fitted for ternary sized spherical particles system using 30 sets of data from literature [98, 90, 91], which includes 340 values of pressure drop versus velocity. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 4.9298 \left(\frac{L}{d_p}\right)^{1.2343} \left(\frac{1}{\text{Re}}\right)^{0.1757} \varepsilon^{-0.9865} \phi \qquad \ldots (5.19)$$

The average percentage error was found to be 2.18193% between experimental work and the achieved equation.

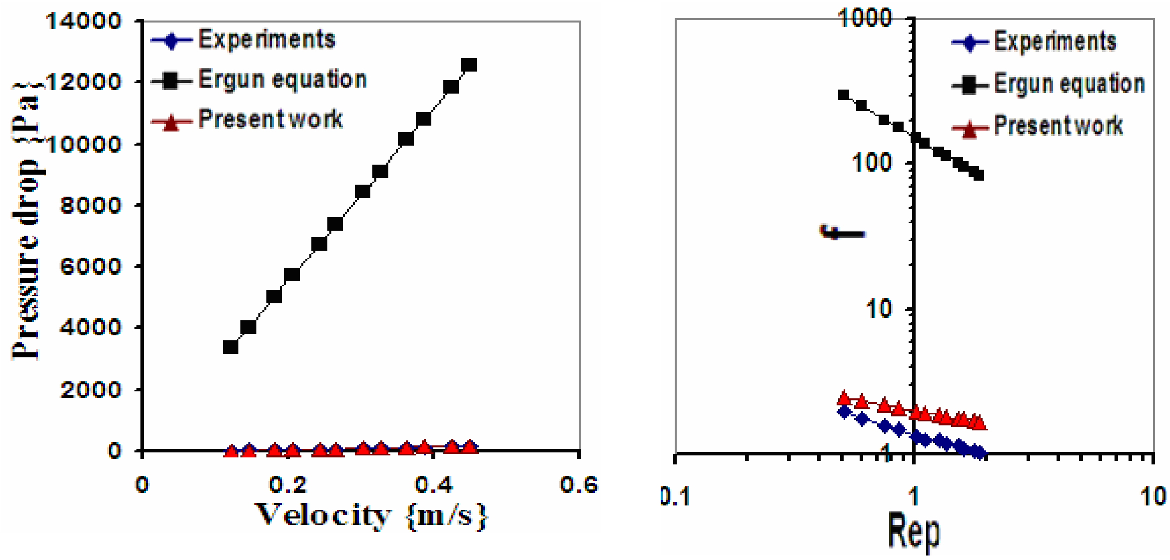
The porosity used in the presented model was proposed from best fitting of experimental data of air flow through packed beds of ternary-sizes spherical particles. This porosity is represented in the following equation:

$$\varepsilon = \frac{0.1834}{\left(4.3115\, D_R^{-0.0771} - 5.6073\, d_P^{0.0167}\right)^{0.3228}} \qquad \ldots (5.20)$$

The above correlation deviates from experimental data with an average percentage error of 0.00054 %.

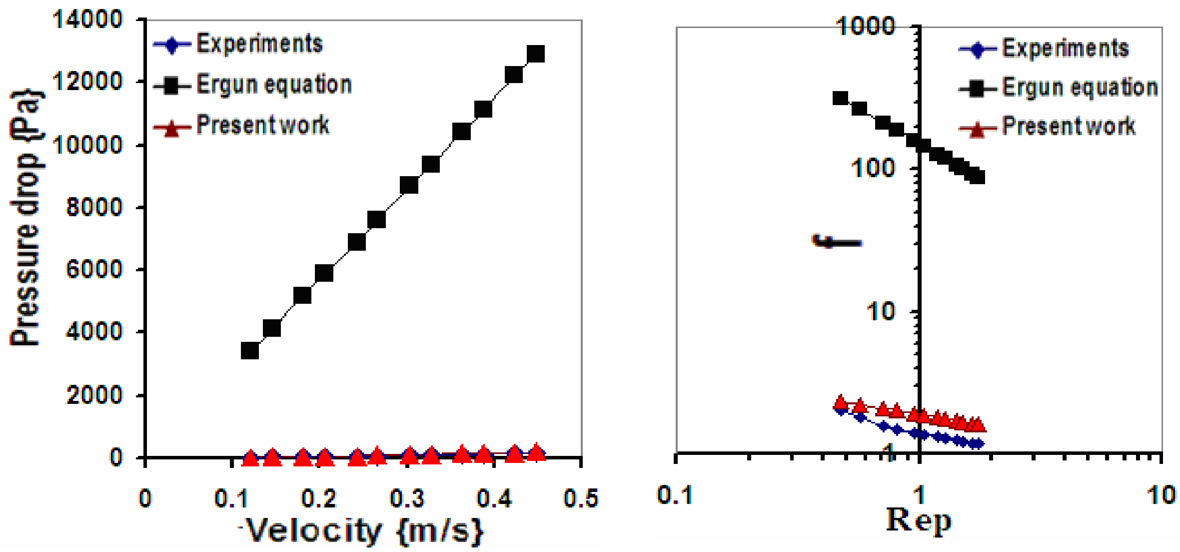
5.5.1.4 Quaternary sized spherical particles system

The values of pressure drop for air flow through packed beds of quaternary sized spherical particles versus velocity were plotted in figures 5.34a, 5.35a and 5.36a. The values of friction factors versus Reynolds numbers were plotted in figures 5.34b, 5.35b and 5.36b.



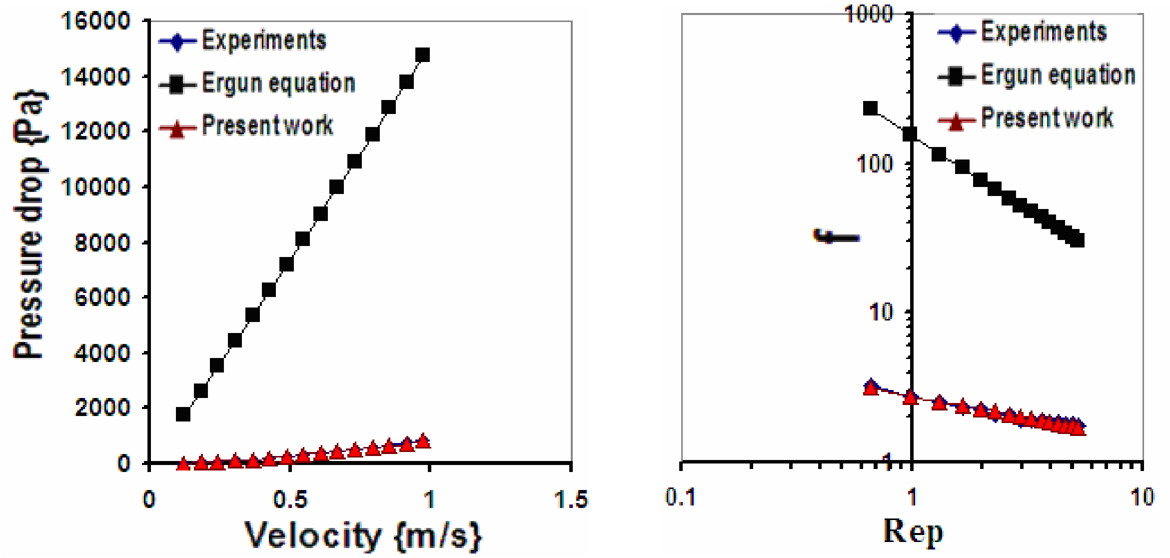
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.34 a-Pressure drop vs.velocity, b-Friction factor vs. Reynolds numbers. for glass spheres of particle diameters of (0.24, 0.42, 0.82 and 1.03 cm, with dp_{eff}=0.4578 cm), bed porosity of 0.3532, packing height of 15.15 cm, bed diameter of 7.64 cm [91] (Table 4.21)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.35 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass of particles diameter (0.24, 0.42, 0.82 and 0.61 cm, with dp_{eff}=0.4252 cm), bed porosity of 0.3474, packing height of 15.15 cm, bed diameter of 7.64cm[91] **(**Appendix B.26)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.36 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameter (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.552 cm), bed porosity of 0.371, packing height of 20 cm, bed diameter of 7.62 cm [90] (Appendix B.27)

Similar comparisons between experimental data and the achieved equation are shown in appendix B (B.28 to B.32).

From the above figures, one can recognize that the achieved equation gave a good fitting to the experiment rather than Ergun equation; this difference may lie on the basic concepts of Ergun equation itself [6, 77]. For example in figure 5.36a the values of pressure drop of experiment results were approximately close to the values of pressure drop of model results range from (14.144 - 537.62 and 14.3906 - 498.884)Pa, since they have the same values of velocity (range from 0.1218 - 0.9746) m/s . The same thing for figure 5.36b the values of friction factor of experiment results were approximately close the values of friction factor of the model results range from (3.4419 - 2.1802 and 4.3011 - 2.3289) since they have the same values of Reynolds numbers (range from 1.0457 - 8.3673).

The values of friction factor of figures 5.34b and 5.35b were approximately close to each other (range from 2.4536-1.6676 and 2.3595-1.6037) since they have close values of Reynolds numbers (range from 0.508-1.8688 and 0.4756-1.76101) and porosities (0.3532 and 0.3474) in spite of the different diameters of particles.

Equation (3.9) was fitted for air flow through packed beds of quaternary sized spherical particles, In this fitting 10 sets of data were used [90,91], which includes 136 values of pressure drop versus velocity. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 5.2649 \left(\frac{L}{d_p}\right)^{1.3899} \left(\frac{1}{\text{Re}}\right)^{0.295} \varepsilon^{-0.2323} \phi \qquad \ldots (5.21)$$

The average percentage error was found to be 2.49298% between experimental work and the achieved equation.

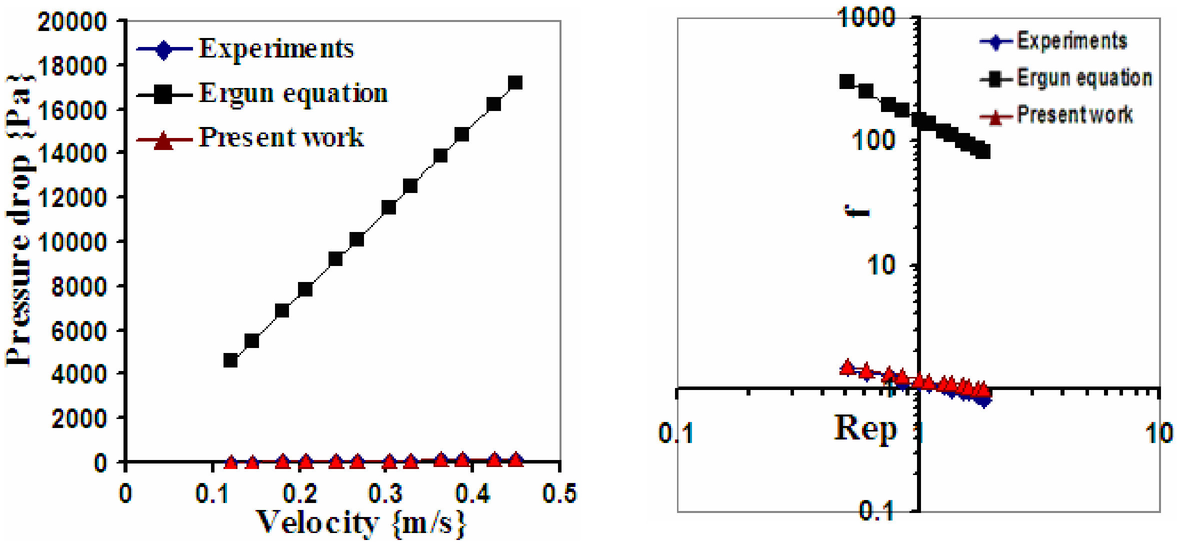
The porosity used in the present model was evaluated from best fitting of experimental data of air flow through packed beds of quaternary sized spherical particles. This porosity is represented in the following equation:

$$\varepsilon = \frac{1.4931}{\left(0.0946 D_R^{-0.6967} + 0.8098 d_P^{0.3707}\right)^{-3.7901}} \qquad \ldots (5.22)$$

The above correlation deviates from experimental data with an average percentage error of 2.11835%.

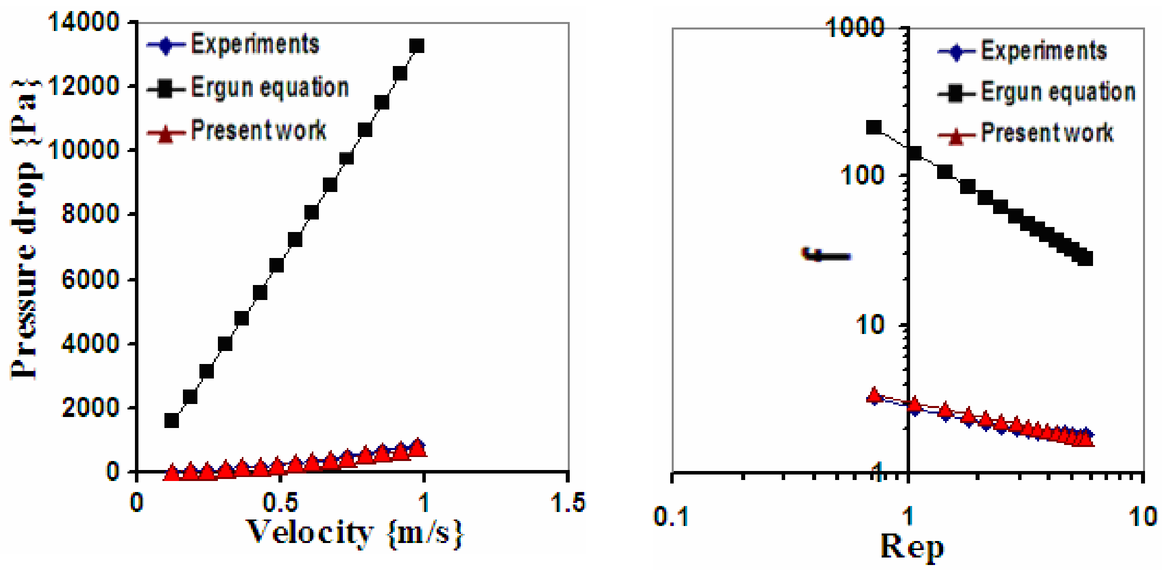
5.5.1.5 Quinary sized spherical particles system

The values of pressure drop for air flow through packed beds of quinary sized spherical particles versus velocity were plotted in figures 5.37a and 5.38a. The values of friction factors versus Reynolds numbers were plotted in figures 5.37b and 5.38b.



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.37 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass sphere of particles diameter (0.24, 0.42, 0.82, 0.61 and 1.03 cm, with dp_{eff}=0.4818 cm), bed porosity of 0.2977, packing height of 15.15 cm, bed diameter of 7.64 cm [91] (Table 4.22)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.38 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass sphere of particles diameter (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.607 cm), bed porosity of 0.3694, packing height of 20 cm, bed diameter of 7.62 cm[90] **(**Appendix B.33)

It can be noticed from the above figures that the achieved equation gave very good fitting to the experiment rather than Ergun equation. The achieved equation results of friction factor-Reynolds number curves and pressure drop-velocity curves lie on the experimental results curves, while the results of Ergun lies above them; this is due to the differences in beds dimensions, packing shapes and sizes [105].

Figure 5.37b shows that the values of friction factor are less than the values of friction factor in all mixtures because minimum porosity value was achieved in this system. In this figure the value of friction factor range from 1.50822 to 0.97532 for a minimum porosity of 0.2977.

The achieved equation (3.9) was fitted for air flow through packed beds of quinary-sized spherical particles, in this fitting two sets of data were used [90,91], which includes 30 values of pressure drop versus velocity. So, the singular form of the achieved equation for this case will be as follows:

$$\frac{\Delta P}{\rho u^2} = 0.5597 \left(\frac{L}{d_p}\right)^{2.1127} \left(\frac{1}{\text{Re}}\right)^{0.3301} \varepsilon^{-0.1012} \phi \qquad \dots (5.23)$$

The average percentage error found to be 0.6547% between experimental work and the achieved equation.

The porosity used in the present model was proposed from best fitting of experimental data of air flow through packed beds of quinary-sized spherical particles. This porosity is represented in the following equation:

$$\varepsilon = \frac{0.2279}{\left(0.5961 D_R^{0.3926} - 0.7136 d_P^{0.2336}\right)^{0.0603}} \qquad \dots (5.24)$$

The above correlation deviates from experimental with an average percentage error of 0.00683 %.

5.5.2 General equation results

The following presentation of results and comparisons are based on general equation fittings for all systems considered in the present work. This system considered includes mono spherical particles, mono non spherical particles, binary spherical particles, ternary spherical particles, quaternary spherical particles and multi-sizes of spherical particles systems.

The achieved equation (3.9) was fitted for air flow through packed beds of multi-sized spherical particles. In this fitting102 sets of data were used, which includes 1450 values of pressure drop versus velocity. So, the general form of the achieved equation for air flow through packed bed will be as follows:

$$\frac{\Delta P}{\rho u^2} = 14.1817 \left(\frac{L}{d_p} \right)^{0.7736} \left(\frac{1}{\text{Re}} \right)^{0.3419} \varepsilon^{-1.1315} \, \phi \qquad \dots (5.25)$$

The average percentage error was found to be 7.05719% between the experimental work and the achieved equation.

The porosity used in the present model was evaluated from best fitting of experimental data of air flow through packed beds of multi-sized spherical particles. This porosity is represented in the following equation:

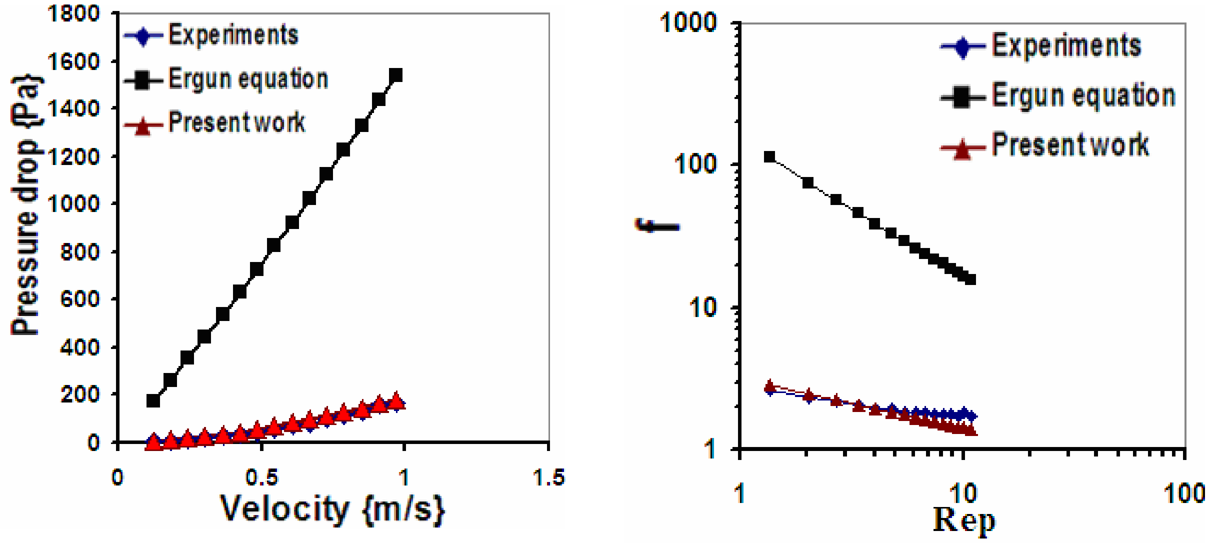
$$\varepsilon = \frac{1.0337}{\left(0.0014 \; D_R^{0.1418} + 1.8929 \; d_p^{1.161}\right)^{-0.197}} \qquad \dots (5.26)$$

The above correlation deviates from experimental with an average percentage error of 0.0309 %.

Equation 5.25 and 5.26 can be used in all types of packing systems.

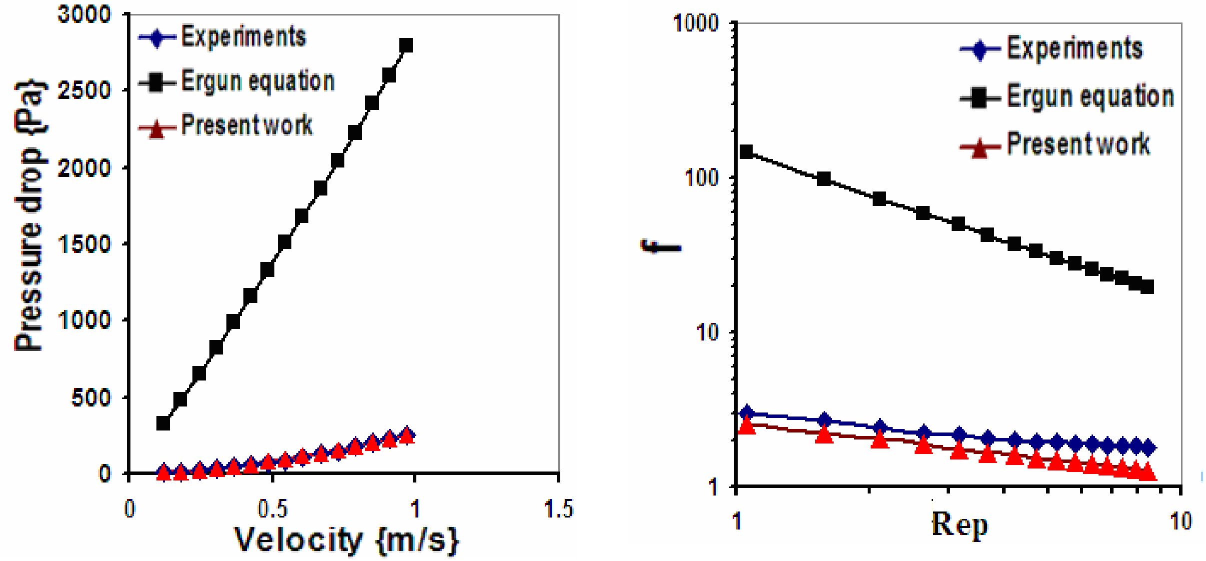
The values of pressure drop versus velocity for air flow through packed beds were plotted in Figs. 5.39a and 5.40a for mono spherical particles, Figs. 5.41a and 5.42a for binary spherical particles, Figs. 5.43a and 5.44a for ternary spherical particles, Fig. 5.45a and 5.46a for quaternary spherical particles, Fig. 5.47a and 5.48a for quinary spherical particles systems. The values of friction factors versus Reynolds numbers were plotted in Figs. 5.39b and 5.40b for mono spherical particles, Figs. 5.41b and 5.42b for binary spherical particles, Figs.

5.43b and 5.44b for ternary spherical particles, Fig. 5.45b and 5.46b for quaternary spherical particles, Fig. 5.47b and 5.48b for quinary spherical particles systems.



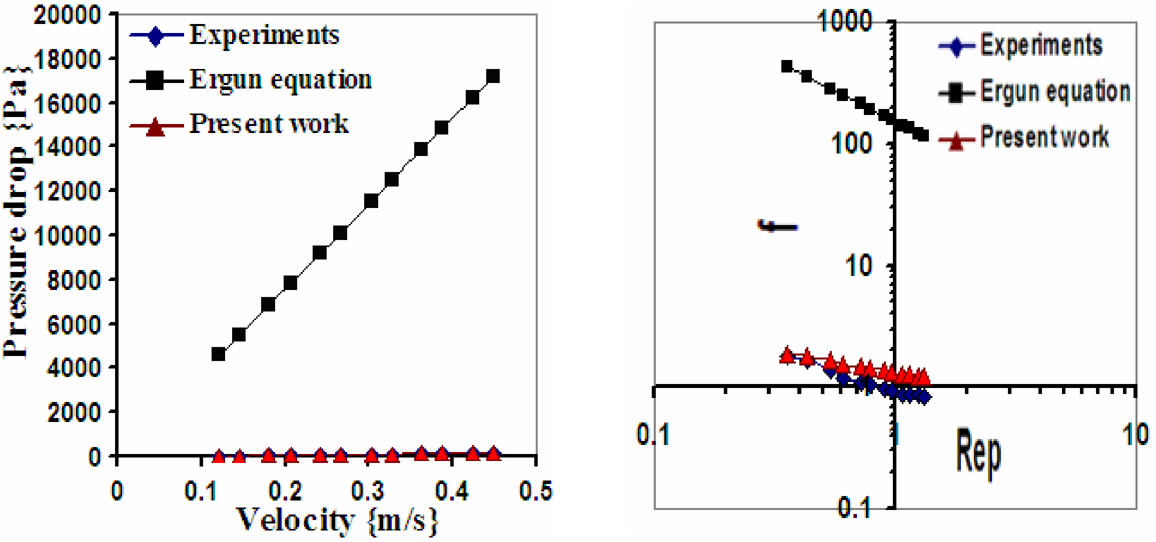
a- ΔP vs. u　　　　**b-** f vs. Re_p

Figure 5.39 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass of particles diameter 0.9987 cm, bed porosity of 0.4169, packing height of 15.15 cm, bed diameter of 7.62 cm [89] (Table 4.23)



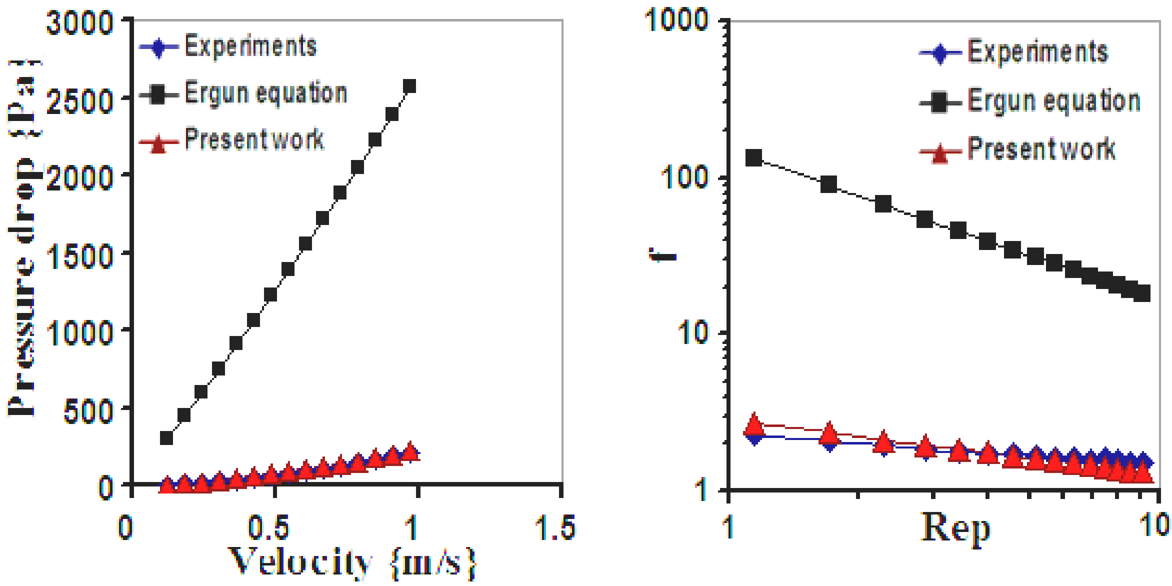
a- ΔP vs. u　　　　**b-** f vs. Re_p

Figure 5.40 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass of particles diameter 0.7955 cm, bed porosity of 0.39804, packing height of 15.15 cm, bed diameter of 7.62 cm [89] (Appendix B.34)



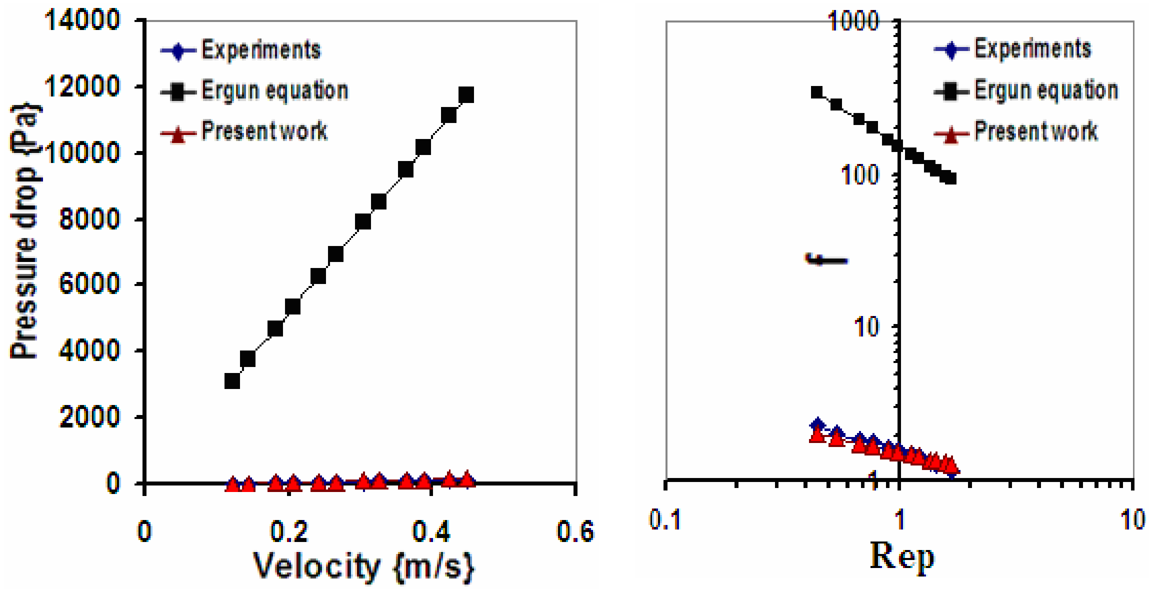
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.41 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameters (dp_1=0.24 and dp_2=0.42 cm, with dp_{eff}=0.3055 cm), bed porosity of 0.3343, packing height of 15.15 cm, bed diameter of 7.64 cm [91] (Table 4.24)



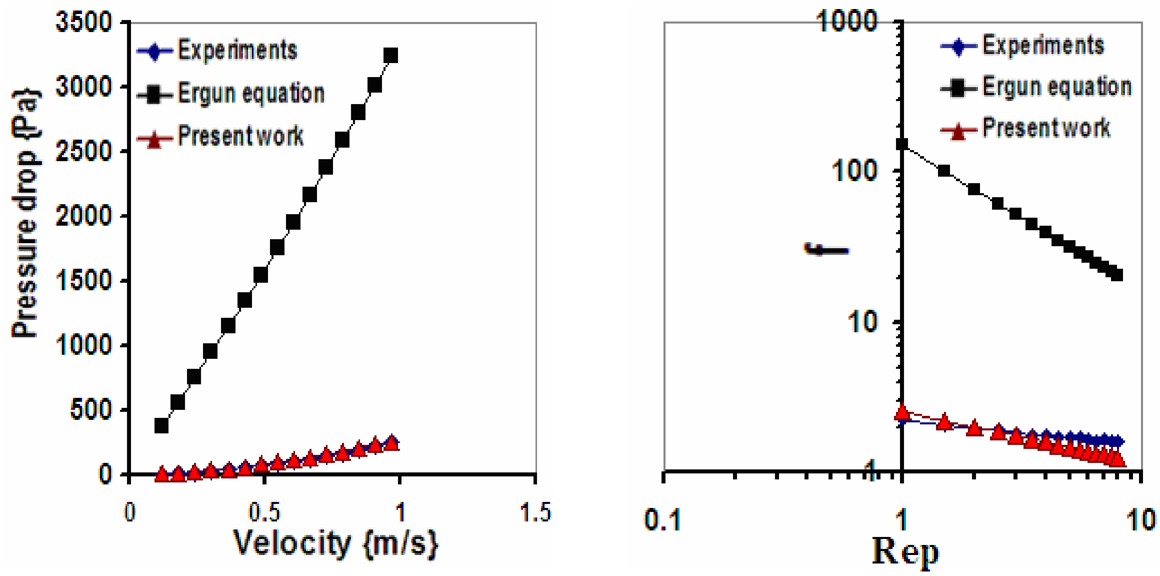
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.42 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameter (dp_1=0.9987 and dp_2=0.7955 cm, with dp_{eff}=0.886 cm), bed porosity is 0.4068, bed diameter is 7.64 cm, packing height is 15.15 cm [89] (Appendix B.35)



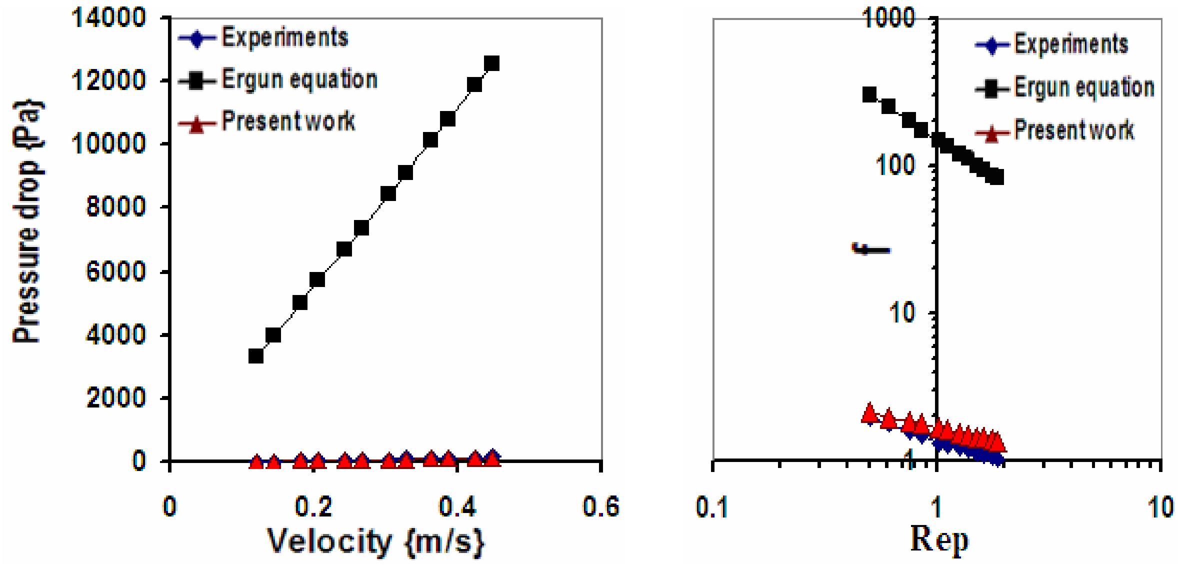
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.43 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameters (0.24, 0.42 and 0.82 cm, with dp_{eff}=0.3862 cm), bed porosity of 0.3495, packing height of 15.15 cm, bed diameter of 7.64 cm [91] (Table 4.25)



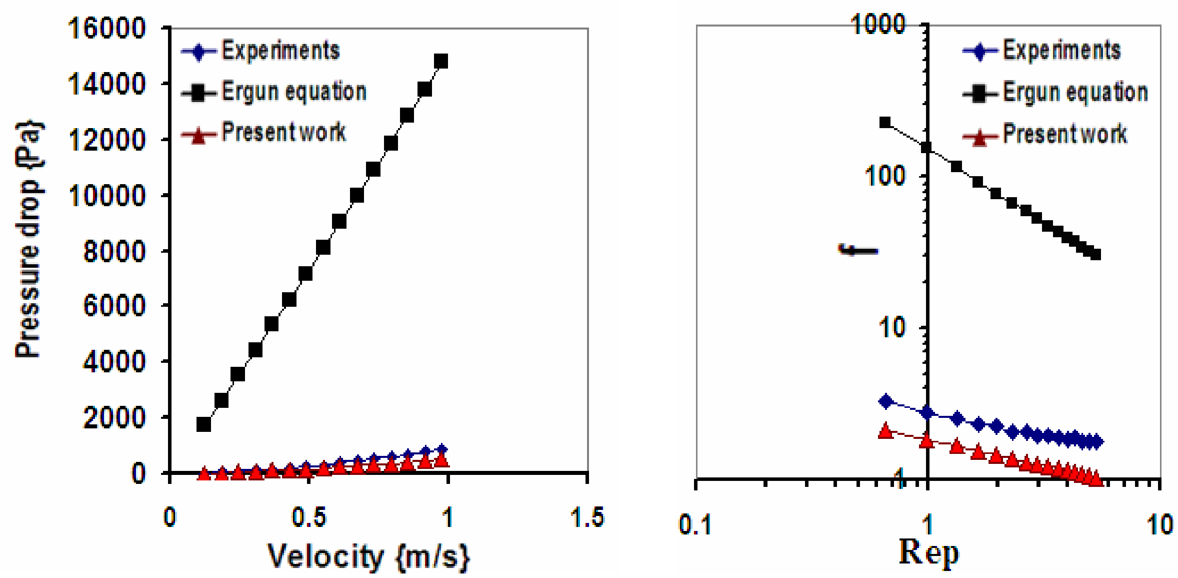
a- ΔP vs. u **b-** f vs. Re_p

Figure 5.44 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameter (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651 cm), bed porosity of 0.3949, packing height of 15.15 cm, bed diameter of 7.64 cm [89] (Appendix B.36)



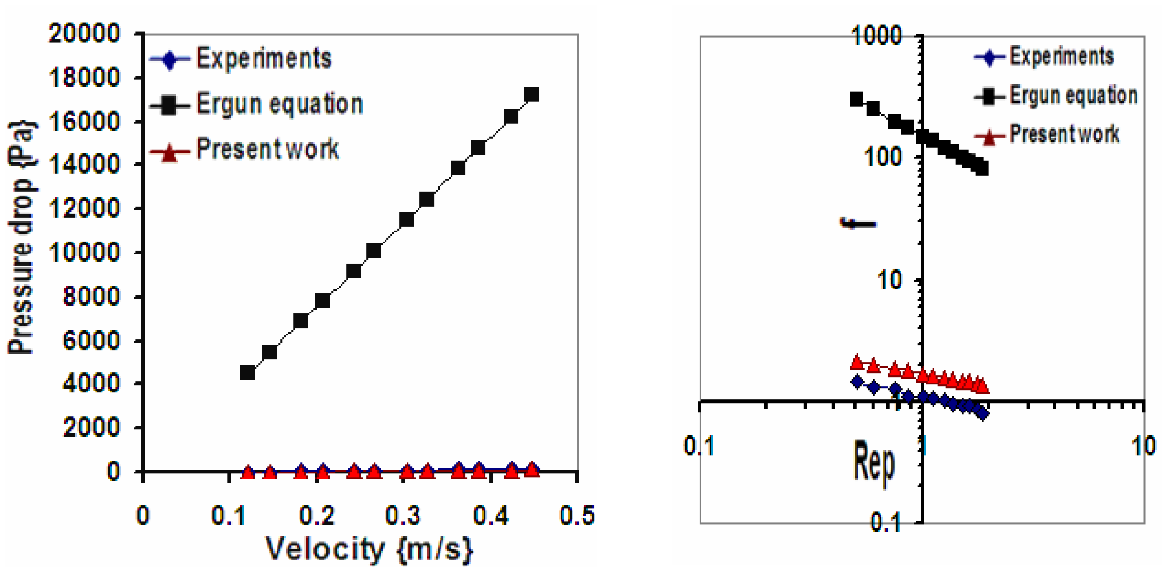
a- ΔP vs. u   b- f vs. Re_p

Figure 5.45 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameter (0.24, 0.42, 0.82 and 1.03 cm, with dp_{eff}=0.4578 cm), bed porosity of 0.3581, packing height of 15.15 cm, bed diameter of 7.64 cm [91] (Table 4.26)



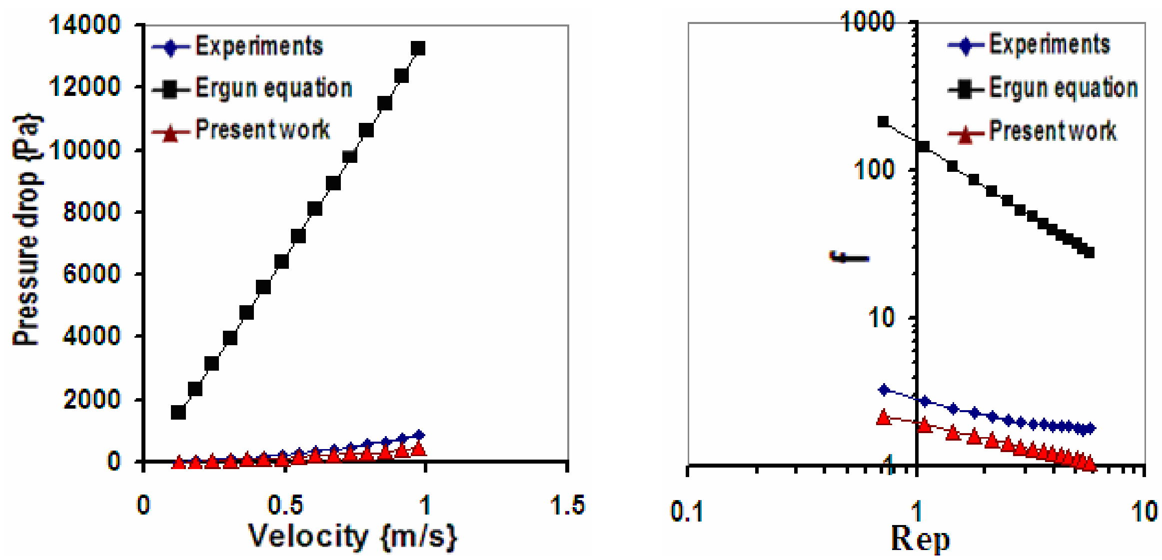
a- ΔP vs. u   b- f vs. Re_p

Figure 5.46 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameter (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.552 cm), bed porosity of 0.3707, packing height of 20 cm, bed diameter of 7.62 cm [90] (Appendix B.37)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.47 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass of particles diameter (0.24, 0.42, 0.82, 0.61and 1.03 cm, with dp_{eff}=0.4818 cm), bed porosity of 0.3615, packing height of 15.15 cm, bed diameter of 7.64 cm [91] (Table 4.27)



a- ΔP vs. u **b-** f vs. Re_p

Figure 5.48 a-Pressure drop vs. velocity, b-Friction factor vs. Reynolds numbers for glass spheres of particles diameter (0.42, 0.51, 0.61, 0.79 and 1.01cm, with dp_{eff}=0.607 cm), bed porosity of 0.3694, packing height of 20 cm, bed diameter of 7.62 cm [91] (Appendix B.38)

The general equation could be used for any packing system, while the singular equation for only one types of packing (which is written for it), and can not be used for another type. For example equation (5.15) can be used for mono spherical system only. This is true for all other equation (5.17, 5.19, 5.21and 5.23). The same thing can be said for the porosity equations. Figures 5.39 to 5.48 show the results of the general equation for multi sized particles (equation 5.25), which can be compared with the results of singular equations for different types of packing system shown in figures 5.25, 5.26, 5.28, 5.29, 5.32, 5.43, 5.34, 5.36, 5.37 and 5.38 respectively.

It can be noticed from figures (5.39 to 5.48) that the achieved equation gave a good fitting to the experiment better than Ergun equation. The achieved equation results of pressure drop-velocity curves and friction factor-Reynolds number curves are close to the experimental results curves, while the results from Ergun equation were far from the experimental results; this may be due to:

1. The difference of bed dimensions (diameter and height of bed) [96].
2. The difference of void fraction (difference of packing shape and size) [101].
3. Ergun designed his equation using completely different procedures than experimental data work, so it was no surprise when its failure was confirmed. Ergun used pea gravel for the packed bed and air for the fluid; unlike experiments were glass is used for the packed bed and air for the fluid [84].
4. Other reasons of this large deviation from Ergun equation, that Ergun's equation does not take in to consideration wall effects, because Ergun considered that in to packed beds it is generally assumed that the diameter of the packing is close to that of the column; therefore, there are no wall effects [93].

Chapter Six

Conclusions and Recommendations for Future Work

6.1 Conclusions

1. The achieved equations had successfully described the effects of different parameters on pressure drop of fluid flow through packed beds, like fluid velocity, height of packing, type of packing particles, particles size, bed porosity and bed diameter, compared with the experimental results.
2. It was found that an increase in particle diameter causes a decrease in pressure drop, this is due to the fact that when the particle diameter increase's the specific surface area of it decreases, and this leads to a decrease in the resistance to fluid flow.
3. The particle size and size distribution highly affect the bed porosity. For mono size packing, the lower the particle size, the lower is the bed porosity. The porosity of multi- size systems are generally less than those of mono size systems, because the particles of smaller sizes tend to fill the void spaces between the larger sizes particles.
4. The bed porosity highly affects the pressure drop and inversely proportional to it, this is because that when the porosity increases the resistance to fluid flow through the bed decreases.
5. It is clear that as the porosity decreases the friction factor decreases, in spite of any increase in pressure drop. This is because the friction factor is proportional to power three with porosity, while it is proportional to power one with pressure drop.

6. It was found that the pressure drop through a packed bed is highly sensitive to the packing height and that as the packing height increases the pressure drop increases.
7. Comparing the results of the achieved equations of pressure drop-velocity and friction factor - Reynolds number curves with those of experimental data from literature and Ergun equation results; it indicates that the achieved equations results coincide with experimental results, while the results from Ergun equation was far away from them.
8. The general equation achieved for water and air flow through packed bed can be used for any system of packing, while the singular equation can be used for only one type of packing which is written for it. The comparison between general and singular equations results show quite good agreement between them; therefore, the general equation could be used for all types of packing instead of singular equations.
9. The porosity formulas written for the achieved equations deviate's from experimental results with a very small average percentage error, which means that they are almost identical. The porosity formulas written for the achieved equations have been compared with the results of experimental data and Furnas equation of porosity. The comparisons show a very good agreement between the porosity formula results and the experimental data, while Furnas equation of porosity was far away from the experimental data. So the written porosity formula can be used with confidence with any type of packing system.
10. The semi-empirical equation of minimum fluidization velocity that has been obtained in the present work is comparable with the experimental values of the minimum fluidization velocity for water flow.

6.2 Recommendations for Future Work

The following suggestions could be considered for future work:

1. The achieved equation can be extended to include fluidization conditions.
2. Studying two phase flow through packed bed using the achieved equation.
3. Studying the effect of temperature and heat transfer on pressure drop using achieved equation of fluid flow.
4. Studying the effect of the surface roughness of the material on sphericity and its effects on the pressure drop of fluid flow through packed bed.

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Appendix A

Water Flow Through Packed Bed

A.1 Singular equations results for different types of packing

A.1.1 Mono sizes spherical particles system

Table A.1 For pea gravel spherical particles diameter of 1.27cm, bed porosity of 0.393, packing height of 53.34cm, bed diameter of 8.89cm [87]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0098	266.56	238.57	199.79	5.9044	5.9601
0.0107	290.85	263.85	218.52	5.3854	5.5411
0.0116	315.15	289.13	237.25	4.9502	5.1222
0.0122	330.97	306.25	249.74	4.6919	4.9059
0.0128	346.79	323.37	262.23	4.4591	4.6896
0.0138	373.33	351.61	282.52	4.1355	4.4103
0.0148	399.87	379.84	302.81	3.8558	4.1309
0.0152	414.54	393.12	312.17	3.761	4.0215

Table A.2 For pea gravel of particles diameter 1.27cm, bed porosity of 0.3902, packing height of 50.8cm, bed diameter of 15.24 cm [94]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0062	178.97	154.08	127.99	10.142	9.6097
0.007	198.93	175.38	143.6	9.0796	8.7688
0.0078	218.88	196.68	159.21	8.0167	7.9278
0.0083	234.32	211.89	170.13	7.5448	7.5029
0.0088	249.76	227.11	181.06	7.0729	7.078
0.0094	269.05	244.76	193.55	6.6894	6.697
0.0101	288.34	262.42	206.03	6.3059	6.316
0.0108	310.24	284.85	221.64	5.892	5.9467

0.0116	332.14	307.27	237.25	5.478	5.5774
0.013	372.19	348.51	265.35	4.9612	5.1009
0.0143	412.23	389.74	293.44	4.4444	4.6244

Table A.3 For pea gravel of particles diameter 1.27cm, bed porosity of 0.3902, packing height of 39.37cm, bed diameter of 15.24cm [54]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0076	255.678	237.705	156.09	12.5713	12.8627
0.0082	278.677	259.167	168.57	11.7973	12.074
0.0085	290.176	269.898	174.82	11.4103	11.6796
0.0088	301.675	280.629	181.06	11.0233	11.2852
0.0096	329.783	307.951	196.67	10.2686	10.547
0.01	343.836	321.612	204.47	9.89126	10.1779
0.0104	357.89	335.272	212.28	9.51391	9.80875
0.0106	365.913	343.586	216.96	9.32393	9.63261
0.0108	373.935	351.9	221.64	9.13396	9.45648
0.011	381.958	360.215	226.33	8.94398	9.28034
0.0113	389.98	368.529	231.01	8.754	9.1042

Table A.4 For pea gravel of particles diameter 0.305cm, bed porosity of 0.3636, packing height of 41.28cm, bed diameter of 8.26cm [106]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0076	295.68	304.54	37.95	2.84	2.925
0.0087	344.26	353.2	43.22	2.575	2.689
0.0097	392.83	401.86	48.49	2.311	2.453
0.0108	441.41	450.53	53.76	2.128	2.217
0.0119	489.99	499.19	59.03	1.945	1.981
0.0127	531.99	539.69	63.25	1.845	1.883
0.0135	573.99	580.2	67.47	1.744	1.784
0.0144	615.99	620.7	71.68	1.662	1.686
0.0152	657.99	661.21	75.9	1.58	1.588
0.0163	708.39	711.24	80.96	1.499	1.513

0.0173	758.79	761.26	86.02	1.418	1.438
0.0193	859.6	861.32	96.14	1.286	1.289
0.0214	965.13	968.24	106.7	1.173	1.186
0.0235	1070.7	1075.2	117.2	1.078	1.082
0.0245	1118.3	1123.2	121.9	1.045	1.049
0.0254	1165.9	1171.2	126.5	1.012	1.017
0.0263	1213.5	1219.2	131.1	0.979	0.984
0.0273	1261.1	1267.2	135.8	0.946	0.951
0.0283	1314.9	1320.5	140.8	0.919	0.923
0.0293	1368.8	1373.7	145.9	0.893	0.896
0.0303	1422.7	1427	151	0.866	0.869
0.0313	1476.6	1480.3	156	0.839	0.841

Table A.5 For glass marbles of particles diameter 1.27cm, bed porosity of 0.3902, packing height of 55.88cm, bed diameter of 15.24cm [86]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.006	143.99	137.12	122.98	8.73	8.3136
0.0064	153.79	146.95	130.66	8.2837	8.0013
0.0068	163.6	156.79	138.35	7.8375	7.689
0.0072	173.41	166.62	146.04	7.4735	7.3766
0.0076	183.22	176.46	153.72	7.1095	7.0643
0.0079	193.03	186.29	161.41	6.8071	6.752
0.0083	202.83	196.13	169.1	6.5047	6.4397
0.0087	212.64	205.96	176.78	6.2495	6.1274
0.0091	222.45	215.8	184.47	5.9944	5.8151
0.0094	232.39	226.04	192.15	5.7793	5.6523
0.0098	242.33	236.28	199.84	5.5642	5.4895
0.0102	252.28	246.51	207.53	5.3777	5.3266
0.0106	262.22	256.75	215.21	5.1913	5.1638
0.011	272.16	266.99	222.9	5.0282	5.001
0.0113	282.1	277.23	230.59	4.8651	4.8382
0.0117	292.04	287.47	238.27	4.7213	4.6754
0.0121	301.99	297.71	245.96	4.5774	4.5125
0.0125	312.75	308.26	253.64	4.4606	4.4118

0.0128	323.52	318.81	261.33	4.3438	4.3111
0.0132	334.28	329.36	269.02	4.2381	4.2104
0.0136	345.05	339.91	276.7	4.1324	4.1096
0.014	355.81	350.46	284.39	4.0364	4.0089
0.0144	366.58	361	292.07	3.9403	3.9082
0.0147	377.34	371.55	299.76	3.8527	3.8075
0.0151	388.11	382.1	307.45	3.765	3.7067

Table A.6 For black marbles of particles diameter 1.9cm, bed porosity of 0.4047, packing height of 67.3cm, bed diameter of 14.606cm [93]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.042	892.57	901.17	1352.5	1.5905	1.6059
0.045	966.57	973.75	1449.2	1.5004	1.5167
0.048	1040.6	1046.3	1545.8	1.4197	1.4275
0.0528	1157.8	1164.6	1698.7	1.3079	1.3366
0.0575	1275	1282.8	1851.7	1.2122	1.2457
0.0623	1392.3	1401.1	2004.7	1.1294	1.1548
0.067	1509.5	1519.4	2157.6	1.057	1.0639
0.07	1587.8	1596.2	2254.2	1.0186	1.028
0.073	1666	1673.1	2350.8	0.9827	0.992
0.076	1744.3	1749.9	2447.5	0.9493	0.9561
0.079	1822.6	1826.8	2544.1	0.918	0.9201
0.082	1901.3	1904.7	2640.7	0.8888	0.8913
0.0843	1960.4	1963.2	2713.1	0.8682	0.8698
0.0865	2019.7	2022.1	2785.6	0.8485	0.8507
0.0895	2098.9	2101	2882.2	0.8237	0.827
0.0918	2158.4	2160.2	2954.7	0.806	0.8093
0.094	2217.8	2219.3	3027.1	0.789	0.7915
0.097	2297	2298.2	3123.7	0.7674	0.7678
0.0985	2335.3	2338.1	3172	0.7566	0.7579
0.1015	2411.8	2417.9	3268.6	0.7359	0.7381
0.1043	2485.1	2491.3	3357.2	0.7188	0.7208
0.1068	2555	2558.2	3437.7	0.7048	0.7059
0.108	2590	2591.6	3478	0.698	0.6984

Table A.7 For black marbles of particles diameter 1.9cm, bed porosity of 0.4047, packing height of 61.6cm, bed diameter of 14.606 cm [93]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0435	929.57	937.46	1400.8	1.5455	1.5613
0.0465	1003.6	1010	1497.5	1.46	1.4721
0.0504	1099.2	1105.5	1622.2	1.3638	1.3821
0.0551	1216.4	1223.7	1775.2	1.2601	1.2912
0.0599	1333.6	1342	1928.2	1.1708	1.2003
0.0646	1450.9	1460.2	2081.1	1.0932	1.1094
0.0685	1548.6	1557.8	2205.9	1.0378	1.0459
0.0715	1626.9	1634.6	2302.5	1.0006	1.01
0.0745	1705.2	1711.5	2399.2	0.966	0.974
0.0775	1783.4	1788.4	2495.8	0.9336	0.9381
0.0805	1861.9	1865.8	2592.4	0.9034	0.9057
0.0835	1940.7	1943.7	2689	0.8963	0.877
0.085	1980.1	1982.7	2737.3	0.9038	0.8626
0.088	2059.3	2061.5	2833.9	0.8697	0.8389
0.091	2138.6	2140.4	2930.5	0.8356	0.8152
0.0925	2178.2	2179.9	2978.8	0.8123	0.8034
0.0955	2257.4	2258.8	3075.4	0.7782	0.7797
0.0978	2316.2	2318.2	3147.9	0.762	0.7629
0.1	2373.6	2378	3220.3	0.7645	0.748
0.103	2450.1	2457.8	3316.9	0.7616	0.7282
0.1055	2520	2524.7	3397.5	0.7298	0.7133

Table A.8 For pea gravel of particles diameter 0.26cm, bed porosity of 0.3615, packing height of 41.21cm, and bed diameter of 8.26cm [106]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.00762	287.35	312.38	32.143	2.9088	2.4986
0.00868	335.3	362.3	36.607	2.6827	2.2971
0.00974	383.24	412.22	41.072	2.4566	2.0956

0.01079	431.19	462.13	45.536	2.2305	1.8941
0.01185	479.13	512.05	50	2.0044	1.6926
0.0127	521.68	553.59	53.572	1.9141	1.6085
0.01355	564.23	595.14	57.143	1.8238	1.5244
0.01439	606.78	636.69	60.715	1.7335	1.4403
0.01524	649.33	678.24	64.286	1.6432	1.3562
0.01626	703.86	729.56	68.572	1.5745	1.2925
0.01727	758.39	780.87	72.857	1.5057	1.2287
0.01829	812.93	832.19	77.143	1.437	1.1649
0.0193	867.46	883.5	81.429	1.3682	1.1011

Table A.9 For pea gravel of particles diameter 8.89cm, bed porosity of 0.5293, packing height of 36.83cm, bed diameter of 15.24cm [60]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0026	40.988	44.787	497.55	449.29	490.94
0.0032	51.071	55.284	595.24	418.35	456.27
0.0037	61.154	65.78	692.93	387.42	421.6
0.0042	71.238	76.277	790.62	356.48	386.93
0.0047	81.321	86.773	888.31	325.54	352.26
0.0052	91.404	97.27	986	294.6	317.59
0.0057	101.49	107.77	1083.7	263.66	282.92
0.0063	111.57	118.26	1181.4	232.73	248.25
0.0068	121.65	128.76	1279.1	201.79	213.58
0.0071	128.07	135.49	1337.7	195.77	207.15
0.0074	134.49	142.22	1396.3	189.75	200.73
0.0077	140.9	148.95	1454.9	183.73	194.3
0.008	147.32	155.68	1513.5	177.71	187.88
0.0086	160.15	169.14	1630.7	165.68	175.02
0.0093	172.99	182.6	1748	153.64	162.17
0.0097	182.74	191.82	1826.1	149.1	156.7
0.0101	192.49	201.04	1904.3	144.56	151.23
0.0105	202.24	210.27	1982.4	140.02	145.75

0.0109	211.99	219.49	2060.6	135.48	140.28
0.0115	226.99	233.63	2177.8	130.45	134.48
0.0122	241.99	247.77	2295	125.42	128.68
0.0128	256.99	261.92	2412.2	120.39	122.88
0.0134	271.99	276.06	2529.5	115.36	117.08

Table A.10 For pea gravel of particles diameter 8.89cm, bed porosity of 0.5293, packing height of 34.925cm, bed diameter of 15.24cm [60]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0026	58.98	57.643	497.55	681.78	666.33
0.003	69.984	67.943	575.7	611.94	595.02
0.0035	80.987	78.243	653.85	542.09	523.72
0.0043	103.38	99.686	810.16	463.73	447.42
0.0051	125.78	121.13	966.46	385.36	371.11
0.0059	148.38	143.43	1122.8	342.22	330.49
0.0068	170.98	165.72	1279.1	299.08	289.88
0.0072	180.06	177.13	1357.2	280.89	275.87
0.0076	189.13	188.53	1435.4	262.7	261.86
0.008	203.56	200.08	1513.5	254.47	250.46
0.0084	217.98	211.63	1591.7	246.23	239.06
0.0088	228.53	223.32	1669.8	235.07	229.59
0.0093	239.07	235.01	1748	223.91	220.11
0.0097	250.05	246.82	1826.1	214.95	212.11
0.0101	261.02	258.64	1904.3	205.99	204.11
0.0105	276.96	270.56	1982.4	201.7	197.25
0.0109	292.89	282.49	2060.6	197.4	190.39
0.0113	302.94	294.53	2138.7	189.82	184.45
0.0117	312.98	306.57	2216.9	182.25	178.51
0.0126	336.27	330.93	2373.2	171.53	168.71
0.0134	359.56	355.3	2529.5	160.82	158.91
0.0138	373.82	367.63	2607.6	157.39	154.82
0.0142	388.09	379.95	2685.8	153.96	150.73
0.0146	401.21	392.36	2763.9	150.38	147.07
0.015	414.34	404.76	2842.1	146.79	143.4

A.1.2 Mono size non spherical particles system

Table A.11 For rasching rings of particles diameter 0.3 cm, sphericity of 0.85, bed porosity of 0.3538, packing height of 67.3 cm, bed diameter of 8.89cm [93]

U (m/s)	ΔP (Kpa) (experiments)	ΔP (Kpa) (present work)	Re_p	f (experiments)	f (present work)
0.016	14.2	14.11	72.73	14.73	14.63
0.014	11.6	11.39	63.64	15.71	15.43
0.013	10.3	10.12	59.09	16.18	15.9
0.012	9.124	8.905	54.55	16.82	16.42
0.011	7.951	7.748	50	17.44	17.11
0.009	5.861	5.621	40.91	19.21	18.42
0.008	4.731	4.656	36.36	19.62	19.31
0.007	3.601	3.76	31.82	19.51	20.37

Table A.12 For rasching rings of particles diameter 1.27 cm, sphericity of 0.3, bed porosity of 0.3885, packing height of 43.62 cm, bed diameter of 15.24cm[99]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0115	1460.9	1443.9	236.05	30.787	30.429
0.0128	1762.1	1731.8	262.19	29.836	29.39
0.0141	2063.4	2019.7	288.33	28.886	28.351
0.0153	2364.6	2307.6	314.47	27.935	27.312
0.0166	2665.9	2595.5	340.61	26.984	26.272
0.0176	2936.5	2875.2	361.99	26.298	25.708
0.0187	3207.1	3154.9	383.38	25.611	25.144
0.0197	3477.7	3434.5	404.76	24.924	24.58
0.0208	3748.3	3714.2	426.15	24.238	24.017
0.0215	3976.8	3942.9	441.88	23.9	23.694
0.0223	4205.2	4171.7	457.6	23.563	23.372
0.0238	4662	4629.2	489.06	22.889	22.728

0.0253	5115.6	5089.1	518.4	22.343	22.223
0.0267	5569.1	5549.1	547.75	21.797	21.719
0.0282	6101.8	6084	578.72	21.359	21.293
0.0297	6634.5	6618.9	609.69	20.92	20.867
0.0312	7167.1	7153.9	640.67	20.482	20.441
0.0327	7699.8	7688.8	671.64	20.044	20.015

Table A.13 For rashing rings of particles diameter 1 cm, sphericity of 0.62, bed porosity of 0.3832, packing height of 45.97 cm, bed diameter of 15.24cm [1]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0144	4670.8	4659.53	233.4	44.69	44.58
0.0171	6185.85	6175.6	276.3	42.03	41.95
0.0197	7700.9	7691.68	319.3	39.37	39.32
0.0217	9050.8	9014.66	351.7	38.05	37.92
0.0237	10400.7	10337.6	384.1	36.74	36.51
0.0244	10850.6	10798.5	394.7	36.3	36.13
0.025	11300.5	11259.4	405.2	35.87	35.74
0.0263	12250.6	12224.5	426.3	35.13	35.05
0.0276	13200.6	13189.7	447.3	34.38	34.35
0.029	14275.7	14251.4	469.2	33.78	33.72
0.0303	15350.9	15313.1	491.1	33.17	33.09
0.0316	16424.8	16390.6	512.2	32.62	32.55
0.0323	16961.7	16929.4	522.7	32.35	32.29
0.0329	17498.7	17468.1	533.2	32.07	32.02
0.0342	18674.2	18646.5	554.7	31.6	31.55
0.0356	19849.7	19824.9	576.2	31.13	31.09
0.0369	21025.3	21003.3	597.6	30.66	30.62
0.0382	22200.8	22181.6	619.1	30.18	30.16
0.0395	23485.3	23462.4	640.6	29.8	29.78
0.0409	24769.8	24743.2	662.1	29.42	29.39
0.0422	26054.3	26024	683.5	29.04	29.01
0.0435	27338.8	27304.8	705	28.66	28.63
0.0442	28025.3	27963.2	715.6	28.52	28.46

0.0448	28711.8	28621.5	726.1	28.38	28.29

Table A.14 For pea gravel of particles diameter 0.11 cm, sphericity of 0.75, bed porosity of 0.3527, packing height of 54.36 cm, bed diameter of 8.89cm[101]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.007	9290.7	9189.249	11.58	24.95	24.68
0.0077	11048.31	10944.77	12.72	24.36	24.04
0.0084	12805.93	12700.29	13.85	23.77	23.4
0.0091	14563.54	14455.81	14.99	23.18	22.76
0.0098	16321.16	16211.33	16.13	22.59	22.12
0.0105	18078.77	17966.85	17.27	21.85	21.48
0.0112	19836.38	19722.37	18.4	21.12	20.84
0.0119	21594	21477.89	19.54	20.39	20.2
0.0126	23351.61	23233.41	20.68	19.66	19.56
0.0133	25644.16	25533.21	21.82	19.32	19.23
0.0139	27936.71	27833.01	22.96	18.98	18.9
0.0146	30229.26	30132.8	24.09	18.63	18.56
0.0153	32521.82	32432.6	25.23	18.29	18.23
0.016	34814.37	34732.4	26.37	17.95	17.9
0.0167	37106.92	37032.19	27.51	17.61	17.57
0.0174	39399.47	39331.99	28.64	17.27	17.23
0.0181	41692.02	41631.78	29.78	16.93	16.9

Table A.15 For pea gravel having a particles diameter of 0.32 cm, sphericity of 0.75, bed porosity of 0.3585, packing height of 53.34 cm, bed diameter of 15.24 cm [100]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0069	3334.7	3025.9	35.025	30.17	27.38
0.0072	3521.4	3261.7	36.575	29.22	26.988
0.0075	3708	3497.5	38.124	28.32	26.597

0.0076	3801.3	3615.4	38.899	27.89	26.401
0.0078	3894.7	3733.3	39.673	27.47	26.206
0.0079	3988.1	3851.2	40.448	27.06	26.01
0.0081	4081.3	3969.1	41.223	26.66	25.815
0.0082	4174.6	4087	41.997	26.27	25.619
0.0084	4267.9	4205	42.772	25.9	25.423
0.0085	4361.3	4322.9	43.547	25.53	25.228
0.0087	4454.6	4440.8	44.321	25.17	25.032
0.009	4641.2	4676.6	45.871	24.49	24.641
0.0093	4827.9	4912.4	47.42	23.83	24.25

A.1.3 Binary sized spherical particles system

Table A.16 For Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm, and dp_{eff}=0.7257cm), with fractions of (x_1=0.75, x_2=0.25), bed porosity of 0.3612, packing height of 50.8cm, bed diameter of 8 cm [107]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0095	248.86	257.38	102.6	2.9236	3.0237
0.0143	497.72	492.92	154.77	2.5696	2.5448
0.0183	721.69	728.97	198.24	2.2709	2.2938
0.0247	1020.3	1172.6	267.8	1.7594	2.022
0.0295	1393.6	1553.6	319.97	1.6833	1.8766
0.0344	1692.2	1972.5	372.13	1.5111	1.7614
0.0408	2314.4	2586.1	441.69	1.467	1.6392
0.0456	2687.7	3085.2	493.86	1.3627	1.5643
0.0488	3235.2	3435.5	528.64	1.4316	1.5202
0.0552	3633.4	4176.9	598.2	1.2556	1.4434
0.0625	4907.1	5072.7	676.45	1.1956	1.3709
0.0673	5404.8	5704.8	728.62	1.1726	1.3288

Table A.17 For Acrylic balls of particles diameter (dp_1=0.655, dp_2=1.27 and dp_{eff}=0.7257 cm) with fraction of (x_1=0.75, x_2=0.25), bed porosity of 0.3612, packing height of 40.64 cm, bed diameter of 8cm [107]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0087	74.658	85.767	159.1	2.288	2.629
0.0107	111.99	121.13	195.93	2.213	2.435
0.0127	149.32	156.49	232.76	2.138	2.241
0.0151	199.09	208.39	276.95	2.005	2.1
0.0175	248.86	260.29	321.15	1.872	1.958
0.0191	286.19	299.99	350.61	1.805	1.891
0.0207	323.52	339.7	380.07	1.738	1.825
0.0231	385.73	406.22	424.27	1.661	1.748
0.0255	447.95	472.74	468.46	1.584	1.671
0.0283	547.49	560.09	520.02	1.56	1.604
0.0311	647.04	647.43	571.58	1.537	1.538
0.0352	796.35	788.85	645.24	1.478	1.467
0.0372	871.01	859.55	682.06	1.449	1.432
0.0392	945.67	930.26	718.89	1.42	1.397
0.0416	1045.2	1023.8	763.09	1.391	1.363
0.044	1144.8	1117.4	807.28	1.363	1.33

Table A.18 For Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm with dp_{eff}=0.9071 cm), fraction of (x_1=0.4, x_2=0.6), bed porosity of 0.3645, packing height of 48.26 cm, bed diameter of 8 cm [86]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.013	243.98	273.95	181.8	2.17	2.44
0.016	371.08	398.71	227.8	2.06	2.248
0.019	498.18	523.46	273.8	1.96	2.055
0.023	659.94	696.27	325.6	1.83	1.926
0.026	821.69	869.08	377.3	1.7	1.796
0.03	1045.7	1096.3	434.8	1.62	1.702
0.034	1269.6	1323.5	492.4	1.54	1.607

0.038	1518.5	1584	549.9	1.48	1.539
0.042	1767.4	1844.4	607.4	1.41	1.471
0.046	2018.6	2104.9	659.2	1.37	1.424
0.05	2269.7	2365.4	710.9	1.32	1.377
0.053	2565.8	2648.9	762.7	1.30	1.339
0.057	2861.9	2932.4	814.5	1.27	1.301
0.06	3073.4	3168.1	854.7	1.24	1.276
0.062	3285	3403.7	895	1.21	1.25
0.064	3521.4	3579.8	923.7	1.21	1.234
0.066	3757.8	3755.8	952.5	1.22	1.218

Table A.19 For Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm and dp_{eff}=0.8447 cm), with fraction of (x_1=0.5, x_2=0.5), bed porosity of 0.3633, packing height of 49.53 cm, bed diameter of 8 cm [107]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0103	239.98	224.69	133.8	2.93	2.74
0.0111	272.71	253.66	144.3	2.86	2.659
0.0119	305.44	282.64	154.7	2.79	2.578
0.0127	333.39	313.99	165.2	2.67	2.512
0.0135	361.35	345.34	175.7	2.56	2.445
0.0143	400.51	378.93	186.1	2.52	2.389
0.0151	439.67	412.53	196.6	2.49	2.332
0.0159	470.51	448.27	207	2.4	2.284
0.0167	501.34	484	217.5	2.32	2.236
0.0173	529.25	512.72	225.3	2.28	2.205
0.0179	557.16	541.44	233.2	2.24	2.174
0.0185	585.07	570.16	241	2.2	2.143
0.0191	612.98	598.87	248.9	2.16	2.113
0.0195	635.02	619.24	254.1	2.15	2.095
0.0199	657.05	639.61	259.3	2.13	2.078
0.0203	679.09	659.97	264.5	2.12	2.06
0.0207	701.12	680.34	269.8	2.1	2.042
0.0215	732.95	723.85	280.2	2.04	2.012
0.0223	764.79	767.37	290.7	1.98	1.982

0.0231	796.62	810.89	301.1	1.92	1.953
0.0239	828.45	854.41	311.6	1.86	1.923
0.0243	855.17	877.28	316.8	1.86	1.909
0.0247	881.89	900.16	322	1.86	1.896
0.0255	939.44	947.22	332.5	1.85	1.871
0.0263	996.98	994.28	343	1.85	1.847

Table A.20 For Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm with dp_{eff}=1.1545 cm), fraction of (x_1=0.1, x_2=0.9), bed porosity of 0.3709, packing height of 40.64cm, bed diameter of 8.001cm [87]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0087	84.658	85.767	159.1	2.595	2.629
0.0107	116.99	121.13	195.93	2.367	2.435
0.0127	149.32	156.49	232.76	2.138	2.241
0.0151	199.09	208.39	276.95	2.005	2.1
0.0175	248.86	260.29	321.15	1.872	1.958
0.0191	286.19	299.99	350.61	1.805	1.891
0.0207	323.52	339.7	380.07	1.738	1.825
0.0231	385.73	406.22	424.27	1.661	1.748
0.0255	447.95	472.74	468.46	1.584	1.671
0.0283	547.49	560.09	520.02	1.56	1.604
0.0311	647.04	647.43	571.58	1.537	1.538
0.0352	796.35	788.85	645.24	1.478	1.467
0.0372	871.01	859.55	682.06	1.449	1.432
0.0392	945.67	930.26	718.89	1.42	1.397
0.0416	1045.2	1023.8	763.09	1.391	1.363
0.044	1144.8	1117.4	807.28	1.363	1.33

Table A.21 For Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm, and dp_{eff}=0.7065 cm), with fraction of (x_1=0.8, x_2=0.2), bed porosity of 0.3609, packing height of 48.26 cm, bed diameter of 8 cm [86]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)

0.011	348.4	324.96	113.3	3.05	2.85
0.014	447.95	473.25	142	2.58	2.62
0.017	547.49	621.54	170.7	2.11	2.4
0.019	647.04	747.61	191.3	1.99	2.29
0.021	746.58	873.67	211.8	1.87	2.19
0.022	871.27	985.44	228.2	1.87	2.13
0.024	995.97	1097.2	244.6	1.87	2.06
0.025	1095.5	1187	256.9	1.86	2.02
0.026	1195	1276.8	269.2	1.85	1.98
0.027	1219.9	1307.9	273.3	1.84	1.97
0.027	1244.8	1338.9	277.4	1.82	1.96
0.028	1344.3	1435.2	289.7	1.8	1.92
0.03	1443.8	1531.5	302.1	1.78	1.89
0.032	1605.6	1702.3	322.6	1.73	1.84
0.033	1686.5	1787.7	332.8	1.71	1.81
0.034	1767.4	1873.1	343.1	1.69	1.79
0.035	1885.6	2003	357.5	1.66	1.76
0.036	2003.8	2132.8	371.8	1.63	1.73
0.038	2122	2262.7	386.2	1.6	1.7
0.039	2240.2	2392.5	400.6	1.57	1.68
0.04	2358.4	2512.8	412.9	1.55	1.66
0.042	2476.6	2633.2	425.2	1.54	1.64
0.043	2594.8	2753.5	437.5	1.52	1.62
0.044	2713	2873.8	449.8	1.51	1.6
0.045	2843.8	3002	462.1	1.5	1.58
0.046	2974.6	3130.3	474.4	1.48	1.56
0.048	3105.3	3258.5	486.7	1.47	1.55
0.049	3236.1	3386.7	499.1	1.46	1.53

Table A.22 For glass of particles diameter (dp_1=0.7955cm, dp_2=0.509 cm, and dp_{eff}=0.6208 cm), with fractions of(x_1=0.5, x_2=0.5), bed porosity of 0.38, packing height of 15.15 cm, bed diameter of 8 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0303	219.858	217.606	322.419	1.21613	2.121

0.0606	815.603	692.023	644.837	1.12786	1.586
0.0909	1631.21	1361.55	967.256	1.00254	1.338
0.1211	2836.88	2197.71	1288.61	0.98237	1.186
0.1511	4255.32	3179.84	1607.84	0.94651	1.081
0.1817	5957.45	4325.98	1933.45	0.91637	1.001
0.2121	7943.26	5600.46	2256.93	0.89669	0.938
0.2424	10212.8	6998.71	2579.35	0.88267	0.887
0.2726	12624.1	8513.95	2900.7	0.86272	0.844
0.303	15319.1	10157.1	3224.19	0.84737	0.807

A.1.4 Ternary sized of spherical particles system

Table A.23 For glass spherical particles diameter of (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.71 cm), bed porosity of 0.4023, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0303	283.688	280.459	359.889	1.57261	1.55963
0.0606	992.908	1012.43	719.777	1.37604	1.40753
0.0909	2127.66	2145.24	1079.67	1.31051	1.32552
0.1211	3546.1	3649.16	1438.37	1.23064	1.27041
0.1511	5106.38	5497.99	1794.69	1.13829	1.22946
0.1817	7730.5	7736.2	2158.14	1.19169	1.19634
0.2121	9929.08	10302.7	2519.22	1.1233	1.16925
0.2424	13120.6	13193.2	2879.11	1.13646	1.14637
0.2726	15248.2	16397.9	3237.81	1.04432	1.12661
0.303	18439.7	19944.5	3598.89	1.0222	1.10911

Table A.24 For spherical particles diameter of (0.9987, 0.7955 and 0.421 cm, with dp_{eff}=0.647 cm), bed porosity of 0.3921, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.0303	390.071	354.306	319.531	1.7017	1.63591
0.0606	1418.44	1279	639.062	1.5469	1.47637
0.0909	2836.88	2710.1	958.593	1.3751	1.39035
0.1211	4964.54	4610.01	1277.07	1.3559	1.33254
0.1511	7446.81	6945.64	1593.44	1.3064	1.28959
0.1817	9929.08	9773.18	1916.13	1.2045	1.25486
0.2121	13829.8	13015.5	2236.72	1.2313	1.22644
0.2424	17730.5	16667.1	2556.25	1.2086	1.20244
0.2726	21631.2	20715.5	2874.72	1.1659	1.18171
0.303	25886.5	25196.7	3195.31	1.1293	1.16336

Table A.25 For spherical particles diameter of (0.51, 0.79 and 1.01 cm, with dp_{eff}=0.6536 cm), bed porosity of 0.3915, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	F (present work)
0.0305	495	470.154	326.532	1.68316	1.62841
0.0609	1855	1692.06	651.994	1.58208	1.46995
0.0914	3956	3588.95	978.526	1.4979	1.3842
0.1218	6553	6108.13	1303.99	1.39722	1.32659
0.1523	10385	9239.43	1630.52	1.4162	1.28342
0.1827	14218	12942.6	1955.98	1.34735	1.2493
0.2132	19163	17226.3	2282.51	1.33355	1.22107
0.2436	24356	22049.7	2607.98	1.29829	1.19721
0.2741	31156	27433.5	2934.51	1.31172	1.17648
0.3046	38080	33353.3	3261.04	1.29824	1.15825

Table A.26 For glass of particles diameter (0.9987, 0.6015, 0.421 cm, with dp_{eff}=0.595 cm, bed porosity of 0.3835, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	F (present work)
0.0303	390.071	436.297	296.025	1.63316	1.70758
0.0606	1418.44	1574.98	592.051	1.48469	1.54104
0.0909	2695.04	3337.25	888.076	1.25374	1.45126
0.1211	4680.85	5676.83	1183.12	1.22689	1.39092
0.1511	7092.2	8552.96	1476.22	1.19405	1.34608
0.1817	9574.47	12034.8	1775.18	1.11474	1.30983
0.2121	13404.3	16027.5	2072.18	1.14533	1.28017
0.2424	16453.9	20524.1	2368.2	1.0764	1.25511
0.2726	20212.8	25509.4	2663.25	1.04555	1.23348
0.303	25177.3	31026.8	2960.25	1.05413	1.21433

Table A.27 For glass of particles diameter (0.9987, 0.509 and 0.421cm, with dp_{eff}=0.562 cm, bed porosity of 0.3777, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0303	453.901	503.294	276.185	1.68165	1.75886
0.0606	1489.36	1816.83	552.37	1.37948	1.58732
0.0909	3191.49	3849.71	828.556	1.31379	1.49484
0.1211	5673.76	6548.55	1103.83	1.31596	1.43269
0.1511	8510.64	9866.33	1377.28	1.26792	1.38651
0.1817	12056.7	13882.9	1656.2	1.24217	1.34916
0.2121	16312.1	18488.6	1933.3	1.23335	1.31861
0.2424	20567.4	23675.7	2209.48	1.19062	1.2928
0.2726	25673.8	29426.6	2484.76	1.17516	1.27052
0.303	30851.1	35791.2	2761.85	1.143	1.25079

Table A.28 For spherical particles diameters of (0.42, 0.51 and 0.61cm, with dp_{eff}=0.5061cm), bed porosity of 0.3655, packing height of 20cm, bed diameter of 7.62cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)

0.0305	1014	897.875	239.044	1.99294	1.86228
0.0609	3338	3231.39	477.304	1.64553	1.68107
0.0914	7047	6853.98	716.348	1.54229	1.583
0.1218	11745	11665	954.608	1.44748	1.51711
0.1523	17680	17645	1193.65	1.3936	1.46774
0.1827	24480	24717	1431.91	1.34088	1.42873
0.2132	32269	32897.9	1670.96	1.29797	1.39644
0.2436	41665	42109.2	1909.22	1.28372	1.36915
0.2741	52050	52390.9	2148.26	1.26665	1.34545
0.3046	62807	63696.3	2387.3	1.23767	1.3246

Table A.29 For spherical particles diameters of (0.42, 0.61 and 0.79cm, with dp_{eff}=0.5627 cm, bed porosity of 0.3762, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	767	680.199	270.913	1.80268	1.75753
0.0609	2596	2447.99	540.938	1.53036	1.58651
0.0914	5564	5192.34	811.851	1.45619	1.49395
0.1218	9520	8836.97	1081.88	1.40302	1.43178
0.1523	14465	13367.2	1352.79	1.36346	1.38518
0.1827	20400	18724.8	1622.81	1.33621	1.34836
0.2132	27323	24922.3	1893.73	1.31424	1.31789
0.2436	34989	31900.5	2163.75	1.28914	1.29214
0.2741	43767	39689.5	2434.67	1.27365	1.26977
0.3046	52792	48254.1	2705.58	1.24403	1.25009

Table A.30 For spherical particles diameter of (0.61, 0.79 and 1.01cm, with dp_{eff}=0.535cm), bed porosity of 0.3715, packing height of 20cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	890	768.868	256.888	1.956	1.80292
0.0609	2967	2767.11	512.933	1.63554	1.62748

0.0914	6429	5869.2	769.821	1.57336	1.53254
0.1218	11003	9988.95	1025.87	1.51633	1.46876
0.1523	16443	15109.7	1282.75	1.4493	1.42096
0.1827	23120	21165.7	1538.8	1.41608	1.38319
0.2132	30909	28171.1	1795.69	1.39024	1.35193
0.2436	39687	36059	2051.73	1.36732	1.32551
0.2741	50319	44863.4	2308.62	1.36928	1.30257
0.3046	61076	54544.4	2565.51	1.34582	1.28238

Table A.31 For spherical particles diameter of (0.51, 0.61 and 1.01 cm, with dp_{eff}=0.6165 cm), bed porosity of 0.3854, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	606	543.619	301.866	1.72873	1.67782
0.0609	2102	1956.45	602.743	1.50402	1.51455
0.0914	4698	4149.75	904.609	1.49236	1.42619
0.1218	8036	7062.57	1205.49	1.43747	1.36684
0.1523	12611	10683.2	1507.35	1.44279	1.32236
0.1827	17309	14965	1808.23	1.3761	1.28721
0.2132	22996	19918	2110.09	1.34256	1.25812
0.2436	29796	25495.1	2410.97	1.33247	1.23353
0.2741	37090	31720.1	2712.84	1.31007	1.21218
0.3046	45127	38565	3014.7	1.29072	1.19339

A.1.5 Quaternary sized spherical particles system

Table A.32 For spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.5738 cm), bed porosity of 0.3719, packing height of 15.15 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	729	728	280.3	1.908	1.905

0.0609	2473	2609	559.6	1.623	1.713
0.0914	5193	5520	839.9	1.513	1.609
0.1218	9025	9378	1119	1.481	1.539
0.1523	13600	14167	1400	1.428	1.487
0.1827	18916	19823	1679	1.38	1.446
0.2132	25592	26359	1959	1.371	1.412
0.2436	33134	33712	2239	1.359	1.383
0.2741	40923	41913	2519	1.326	1.358
0.3046	49948	50925	2799	1.311	1.336

Table A.33 For spherical particles diameter of (0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.848 cm), bed porosity of 0.3889, packing height of 15.15 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.031	519	504.6	423.4	2.281	2.218
0.061	1916	1808	845.4	2.112	1.994
0.091	3833	3826	1269	1.876	1.873
0.122	6553	6501	1691	1.806	1.792
0.152	10014	9820	2114	1.765	1.731
0.183	13847	13740	2536	1.696	1.683
0.213	18669	18271	2959	1.679	1.643
0.244	24232	23368	3381	1.67	1.61
0.274	30291	29052	3805	1.648	1.581
0.305	36720	35299	4228	1.618	1.556

Table A.34 For spherical particles diameter of (0.42, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.6373 cm), bed porosity of 0.3747, packing height of 15.15 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	F (experiments)	f (present work)
0.0305	631	671.7	311.9	1.854	1.973
0.0609	2164	2407	622.7	1.595	1.774

0.0914	4574	5093	934.5	1.496	1.666
0.1218	7913	8653	1245	1.458	1.594
0.1523	12116	13071	1557	1.427	1.54
0.1827	16814	18290	1868	1.377	1.497
0.2132	22625	24321	2180	1.36	1.462
0.2436	29301	31105	2491	1.349	1.432
0.2741	36596	38672	2803	1.331	1.407
0.3046	44632	46987	3114	1.315	1.384

Table A.35 For spherical particles diameter of (0.42, 0.51, 0.79 and 1.01 cm, with dp_{eff}=0.6063 cm), bed porosity of 0.3733, packing height of 15.15 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0305	767	698.6	292.2	1.954	1.784
0.0609	2596	2504	583.4	1.659	1.612
0.0914	5564	5297	875.6	1.578	1.503
0.1218	9520	9000	1167	1.521	1.437
0.1523	14589	13594	1459	1.490	1.389
0.1827	20152	19022	1750	1.431	1.350
0.2132	27447	25294	2042	1.431	1.319
0.2436	35236	32350	2334	1.407	1.292
0.2741	43890	40220	2626	1.384	1.269
0.3046	53410	48867	2918	1.364	1.248

A.2 General equation results

Table A.36 For pea gravel of particles diameter 1.27cm, bed porosity of 0.393, packing height of 53.34cm, bed diameter of 8.89cm [87]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.0098	266.56	206.25	199.79	5.9044	4.5684
0.0107	290.85	239.51	218.52	5.3854	4.4347

0.0116	315.15	274.73	237.25	4.9502	4.3154
0.0122	330.97	299.28	249.74	4.6919	4.2426
0.0128	346.79	324.66	262.23	4.4591	4.1745
0.0138	373.33	367.65	282.52	4.1355	4.0726
0.0148	399.87	412.75	302.81	3.8558	3.9802
0.0152	414.54	434.27	312.17	3.761	3.8411

Table A.37 For Acrylic balls of particles diameter (0.655, 1.27 and dp_{eff}=0.7257 cm), fraction of (x_1=0.75, x_2=0.25), bed porosity of 0.37165, packing height of 50.8 cm, bed diameter of 8 cm [87]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.009	248.9	152.8	102.6	2.125	1.988
0.014	497.7	303.4	154.8	1.868	1.735
0.018	721.7	458.6	198.2	1.651	1.598
0.025	1020	757.5	267.8	1.279	1.446
0.03	1394	1019	320	1.224	1.363
0.034	1692	1312	372.1	1.099	1.297
0.041	2314	1746	441.7	1.066	1.225
0.046	2688	2103	493.9	0.991	1.181
0.049	3235	2356	528.6	1.041	1.154
0.055	3633	2895	598.2	0.913	1.108
0.062	3907	3554	676.5	0.768	1.064
0.067	4405	4024	728.6	0.746	1.038

Table A.38 For glass of particles diameter (0.9987, 0.6015 and dp_{eff}=0.7509 cm), fractions of (x_1=0.5, x_2=0.5), bed porosity of 0.3818, packing height of 15.15 cm, bed diameter of 8 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.03	191.5	264.2	391.3	1.304	1.285
0.061	673.8	839.9	782.6	1.147	1.021
0.091	1418	1652	1174	1.073	0.892

0.121	2553	2666	1564	1.089	0.811
0.151	3546	3857	1951	0.971	0.754
0.182	5319	5247	2346	1.007	0.709
0.212	6738	6792	2739	0.936	0.674
0.242	8865	8486	3130	0.943	0.645
0.273	10638	10323	3520	0.895	0.62
0.303	13546	12314	3913	0.922	0.599

Table A.39 For spherical particles diameter of (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.71cm), bed porosity of 0.3806, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.03	283.7	285.8	359.9	1.573	1.299
0.061	992.9	908.4	719.8	1.376	1.032
0.091	2128	1787	1080	1.311	0.902
0.121	3546	2884	1438	1.231	0.82
0.151	5106	4172	1795	1.138	0.762
0.182	7730	5674	2158	1.192	0.717
0.212	9929	7345	2519	1.123	0.681
0.242	13121	9179	2879	1.136	0.652
0.273	15248	11165	3238	1.044	0.627
0.303	18440	13319	3599	1.022	0.605

Appendix B

Air Flow Through Packed Bed

B.1 Singular equations results for different types of packing

B.1.1 Mono size spherical particles system

Table B.1 For spherical particle diameter of 0.7955, bed porosity of 0.4088, packing height of 15.15 cm and bed diameter of 7.62 cm [89]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	6.7188	11.5653	1.0502	3.0233	4.12949
0.182	13.437	23.4701	1.5758	2.6859	3.722504
0.242	21.500	38.8025	2.1022	2.4147	3.457994
0.303	30.906	57.26609	2.6277	2.2215	3.266194
0.364	43.004	78.6689	3.1524	2.1476	3.117626
0.424	56.4381	102.9453	3.6779	2.0707	2.997089
0.485	71.2195	129.952	4.2035	2.0006	2.896453
0.545	88.6884	159.5969	4.7291	1.9682	2.810496
0.606	107.501	191.8015	5.2546	1.9324	2.735771
0.667	127.657	226.4976	5.7802	1.896415	2.669887
0.727	150.506	263.6251	6.3057	1.878623	2.611128
0.788	176.033	303.1303	6.8313	1.872232	2.558217
0.848	201.564	344.8942	7.3559	1.848863	2.510262
0.909	229.787	389.0111	7.8815	1.835987	2.466352
0.97	258.08	435.4521	8.4079	1.811406	2.425909

Table B.2 For spherical particle diameter of 0.61 cm, bed porosity of 0.4005, packing height of 15.15 cm, bed diameter of 7.62 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)

0.121	12.814	11.82338	0.7147	2.302725	2.998741
0.145	18.002	16.21107	0.8565	2.252749	2.863146
0.181	26.771	23.86733	1.0691	2.149985	2.705294
0.206	33.876	29.90972	1.2168	2.100321	2.617253
0.242	43.077	39.61149	1.4294	1.935275	2.511645
0.266	49.266	46.71454	1.5712	1.831944	2.45164
0.303	60.865	58.62868	1.7898	1.744257	2.371334
0.327	68.653	66.96603	1.9315	1.689243	2.325556
0.363	81.043	80.34761	2.1442	1.618193	2.264266
0.387	89.687	89.84034	2.2859	1.575562	2.227498
0.424	103.569	105.3514	2.5045	1.515745	2.176089
0.448	113.264	115.9711	2.6463	1.484786	2.145664

Table B.3 For spherical particle diameter of 0.6105, bed porosity of 0.3998, packing height of 15.15 cm, and bed diameter of 7.62 cm [89]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	12.6313	11.8545	0.7803	3.9383	2.9493
0.182	24.1877	24.0572	1.1708	3.3499	2.6587
0.242	37.6254	39.7732	1.5619	2.9280	2.4697
0.303	55.0943	58.6986	1.9524	2.7439	2.3328
0.364	76.5945	80.6369	2.3422	2.6506	2.2266
0.424	98.0948	105.5205	2.7327	2.4938	2.1406
0.485	123.6263	133.2028	3.1231	2.4062	2.0687
0.545	150.5016	163.5893	3.5136	2.3144	2.0073
0.606	182.7519	196.5995	3.9041	2.2762	1.9539
0.667	219.0336	232.1635	4.2946	2.2546	1.9069
0.727	252.6277	270.2198	4.6850	2.1849	1.8649
0.788	270.0966	310.7133	5.0755	1.9905	1.8271
0.848	341.1362	353.522	5.4653	2.1682	1.7929
0.909	384.3166	398.7424	5.8558	2.1277	1.7615
0.97	435.3797	446.3452	6.2469	2.1180	1.7326

Table B.4 For spherical particle diameter of 0.81 cm, bed porosity of 0.4099, packing height of 15.15 cm, bed diameter of 7.62 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	10.41	11.5161	1.0105	3.3879	4.2899
0.145	14.65	15.7897	1.2109	3.6937	4.0960
0.181	21.35	23.2478	1.5115	3.5258	3.8701
0.206	27.08	29.1324	1.7203	3.5166	3.7442
0.242	32.87	38.5821	2.0210	3.5302	3.5932
0.266	39.19	45.5005	2.2214	3.4999	3.5073
0.303	51.68	57.1050	2.5304	3.2526	3.3924
0.327	58.75	65.2257	2.7308	3.1729	3.3269
0.363	69.98	78.2596	3.0315	3.1167	3.2392
0.387	81.56	87.5054	3.2319	2.9732	3.1866
0.424	96.87	102.6136	3.5409	2.8789	3.1131
0.448	109.56	112.9574	3.7414	2.8057	3.0696

Table B.5 For spherical particle diameter of 1.01cm, bed porosity of 0.4186, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	7.072	10.82134	1.3435	3.051828	4.013853
0.183	13.358	21.9499	2.0152	2.561986	3.618518
0.244	21.216	36.25436	2.6870	2.288871	3.361869
0.305	31.431	53.53611	3.3598	2.168756	3.175135
0.366	44.004	73.57314	4.0316	2.108772	3.030538
0.426	56.576	96.26388	4.7033	1.992096	2.913427
0.487	71.506	121.5053	5.37511	1.9278	2.815642
0.548	86.436	149.2114	6.0468	1.841319	2.732116
0.609	106.08	179.3089	6.7186	1.830495	2.659499
0.67	125.725	211.7341	7.3903	1.793017	2.595472
0.731	149.298	246.4308	8.0621	1.789165	2.538366
0.792	172.872	283.3491	8.7338	1.765249	2.486944
0.853	196.445	322.5097	9.4067	1.729255	2.440189
0.914	220.019	363.7433	10.078	1.687195	2.397524

0.975	251.45	407.0751	10.750	1.694771	2.358288

Table B.6 For spherical particle diameter of 0.79 cm, bed porosity of 0.4082, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	10.215	11.08039	1.0265	3.0668	2.926187
0.183	18.859	22.47536	1.5397	2.516421	2.637979
0.244	29.074	37.12225	2.0529	2.182187	2.450877
0.305	42.432	54.8177	2.5670	2.036927	2.314743
0.366	58.934	75.3344	3.0803	1.964866	2.209329
0.426	77.007	98.56833	3.5935	1.886416	2.123952
0.487	94.294	124.414	4.1067	1.768613	2.052665
0.548	117.867	152.7833	4.6199	1.746851	1.991772
0.609	145.37	183.6013	5.1332	1.745176	1.938833
0.67	172.872	216.8028	5.6465	1.715208	1.892156
0.731	204.303	252.3301	6.1597	1.703337	1.850525
0.792	235.734	290.1321	6.6729	1.674684	1.813037
0.853	267.166	330.2302	7.1870	1.63617	1.778952
0.914	306.455	372.4509	7.7002	1.634937	1.747848
0.975	345.744	416.82	8.2135	1.621226	1.719244

Table B.7 For spherical particle diameter of 0.61cm, bed porosity of 0.3998, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	15.716	11.3367	0.7773	3.29424	2.143081
0.183	28.288	22.99526	1.1660	2.635317	1.932003
0.244	44.004	37.98097	1.5547	2.305926	1.794973
0.305	62.862	56.08576	1.9440	2.106862	1.695271
0.366	86.436	77.07705	2.3327	2.011999	1.618068
0.426	110.009	100.8484	2.7214	1.881489	1.55554
0.487	141.441	127.2919	3.1101	1.85221	1.503331
0.548	168.943	156.3176	3.4988	1.748115	1.458734

0.609	204.303	187.8485	3.8875	1.7124	1.419962
0.67	243.592	221.8179	4.2762	1.687412	1.385777
0.731	282.881	258.1671	4.6649	1.646629	1.355287
0.792	322.17	296.8435	5.0536	1.597945	1.327832
0.853	377.175	337.8692	5.4429	1.61271	1.302869
0.914	424.322	381.0666	5.8316	1.580506	1.280089
0.975	479.326	426.462	6.2203	1.569228	1.25914

B.1.2 Binary sized spherical particles system

Table B.8 For spherical particle diameters of (dp_1=0.9987 and dp_2=0.7955 cm, with dp_{eff}=0.886cm), bed porosity is 0.4079, bed diameter is 7.64 cm, packing height is 15.15 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1211	5.1063	6.20172	1.1455	2.30925	2.44795
0.1817	10.4813	12.6889	1.7187	2.10552	2.22481
0.2424	17.4689	21.1005	2.2928	1.97176	2.07877
0.303	25.5315	31.2813	2.8660	1.84435	1.97232
0.3635	35.7441	43.1305	3.4383	1.79411	1.88953
0.4241	47.0317	56.6158	4.0115	1.73423	1.82213
0.4847	60.4694	71.6611	4.5847	1.70704	1.7657
0.5453	73.907	88.2178	5.15791	1.64842	1.71737
0.6059	90.0322	106.244	5.73111	1.62648	1.67526
0.6665	107.501	125.705	6.30432	1.60497	1.63806
0.7271	126.314	146.568	6.87753	1.58459	1.60483
0.7877	149.158	168.803	7.45073	1.59433	1.57485
0.8482	167.971	192.347	8.02299	1.54842	1.54763
0.9088	190.815	217.253	8.5962	1.53224	1.52268
0.9695	215.002	243.506	9.17035	1.51705	1.49966

Table B.9 for spherical particle diameters of (dp_1=0.9987, dp_2=0.6015, dp_{eff}=0.7508 cm), bed porosity is 0.3986, bed diameter is 7.64 cm, packing height is 15.15 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1211	5.9125	8.04308	0.96343	2.17637	2.47215
0.1817	11.8251	16.4564	1.44554	1.93351	2.24681
0.2424	19.6189	27.3655	1.92845	1.80244	2.09932
0.303	29.5628	40.569	2.41056	1.73825	1.99182
0.3635	41.6567	55.9363	2.89187	1.70187	1.90821
0.4241	55.0943	73.4256	3.37399	1.65356	1.84014
0.4847	69.8757	92.9379	3.8561	1.60558	1.78315
0.5453	87.3447	114.411	4.33821	1.58568	1.73435
0.6059	106.157	137.79	4.82032	1.56099	1.69182
0.6665	127.658	163.028	5.30243	1.5513	1.65426
0.7271	150.502	190.085	5.78454	1.53675	1.62069
0.7877	174.689	218.923	6.26666	1.51983	1.59042
0.8482	201.565	249.457	6.74797	1.51241	1.56293
0.9088	228.44	281.758	7.23008	1.49309	1.53773
0.9695	258.003	315.806	7.71299	1.48177	1.51449

Table B.10 For spherical particle diameters of (dp_1=0.7955 cm, dp_2=0.6015 cm and dp_{eff}=0.685 cm), bed porosity is 0.3936, bed diameter is 7.64 cm, packing height is 15.15 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1211	8.06258	9.29255	0.87598	2.65994	2.48636
0.1817	16.1251	19.0129	1.31433	2.36308	2.25972
0.2424	26.8752	31.6166	1.75341	2.21295	2.11139
0.303	40.3129	46.8713	2.19176	2.12444	2.00327
0.3635	56.4381	64.6259	2.62939	2.06657	1.91918
0.4241	72.5632	84.8321	3.06774	1.95194	1.85072
0.4847	92.7197	107.376	3.50609	1.90947	1.7934
0.5453	114.22	132.184	3.94444	1.85847	1.74432
0.6059	138.408	159.195	4.38279	1.82408	1.70155
0.6665	165.283	188.354	4.82114	1.80017	1.66377
0.7271	193.502	219.614	5.25949	1.77086	1.63001
0.7877	231.128	252.932	5.69785	1.80225	1.59956

0.8482	258.003	288.21	6.13547	1.73506	1.57192
0.9088	292.941	325.528	6.57383	1.71605	1.54657
0.9695	330.566	364.866	7.0129	1.70156	1.52319

Table B.11 For spherical particle diameters of (dp_1=0.7955, dp_2=0.59 and dp_{eff}=0.551 cm), bed porosity is 0.3817, bed diameter is 7.64 cm, packing height is 15.15 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1211	12.0938	13.1057	0.68066	2.6728	2.52237
0.1817	24.1877	26.8147	1.02127	2.37453	2.29245
0.2424	40.3129	44.5903	1.36244	2.22367	2.14196
0.303	59.1256	66.1047	1.70305	2.08729	2.03228
0.3635	86.0009	91.1447	2.0431	2.10953	1.94697
0.4241	108.845	119.642	2.38371	1.96139	1.87752
0.4847	137.064	151.437	2.72432	1.8909	1.81937
0.5453	172.002	186.425	3.06493	1.8748	1.76958
0.6059	206.94	224.519	3.40554	1.82698	1.72619
0.6665	244.565	265.644	3.74615	1.78438	1.68786
0.7271	287.566	309.731	4.08676	1.76295	1.65361
0.7877	331.91	356.721	4.42737	1.73377	1.62272
0.8482	378.942	406.474	4.76741	1.70714	1.59468
0.9088	421.942	459.106	5.10802	1.6558	1.56897
0.9695	489.13	514.586	5.4492	1.68664	1.54525

Table B.12 For spherical particle diameters of (dp_1=0.61, dp_2=0.79, dp_{eff}=0.688 cm), bed porosity is 0.3628, bed diameter is 7.64 cm, packing height is 20 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	12.573	13.4723	0.85641	2.75758	2.43197
0.1827	25.145	27.5515	1.28461	2.45108	2.21044
0.2436	40.861	45.7711	1.71282	2.24047	2.0656

0.3046	59.719	67.8944	2.14173	2.09429	1.95968
0.3655	83.293	93.6488	2.56993	2.0287	1.87732
0.4264	111.581	122.912	2.99814	1.99682	1.81039
0.4873	141.441	155.559	3.42634	1.93806	1.75434
0.5482	168.943	191.485	3.85455	1.82914	1.70634
0.6091	204.303	230.598	4.28275	1.79177	1.66452
0.67	243.592	272.822	4.71095	1.76562	1.62757
0.7309	290.739	318.087	5.13916	1.77081	1.59456
0.7918	337.886	366.331	5.56736	1.75357	1.56478
0.8528	385.033	417.584	5.99627	1.72261	1.53766
0.9137	432.18	471.629	6.42448	1.68438	1.51288
0.9746	495.042	528.499	6.85268	1.69579	1.49005

Table B.13 For spherical particle diameters of (dp_1=0.79, dp_2=1.01, dp_{eff}=0.89 cm), bed porosity is 0.3832, bed diameter is 7.64 cm, packing height is 20 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	7.858	8.81837	1.15541	2.64608	2.39032
0.1827	15.716	18.0339	1.73311	2.35207	2.17258
0.2436	25.931	29.9596	2.31081	2.18298	2.03023
0.3046	39.289	44.4406	2.88947	2.11542	1.92612
0.3655	53.433	61.2983	3.46717	1.99812	1.84517
0.4264	70.72	80.4529	4.04487	1.94309	1.77938
0.4873	86.436	101.822	4.62258	1.81839	1.7243
0.5482	108.438	125.337	5.20028	1.80256	1.67712
0.6091	133.583	150.939	5.77799	1.79871	1.63602
0.67	157.156	178.577	6.35569	1.74891	1.5997
0.7309	188.587	208.205	6.93339	1.76353	1.56725
0.7918	212.161	239.783	7.5111	1.69052	1.53798
0.8528	251.45	273.331	8.08975	1.7272	1.51132
0.9137	282.881	308.707	8.66745	1.69271	1.48697
0.9746	322.17	345.932	9.24516	1.69441	1.46454

Table B.14 For spherical particle diameters of dp_1=0.61, dp_2=1.01, dp_{eff}=0.7606 cm), bed porosity is 0.3852, bed diameter is 7.64 cm, packing height is 20 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	14.144	12.6994	0.8686	2.82838	2.4259
0.1827	27.502	25.9708	1.3029	2.44426	2.20491
0.2436	44.004	43.1451	1.7372	2.19987	2.06044
0.3046	66.006	63.9992	2.17221	2.11049	1.95478
0.3655	90.365	88.276	2.60651	2.00672	1.87263
0.4264	117.867	115.861	3.04081	1.92317	1.80587
0.4873	149.298	146.635	3.47511	1.86518	1.74996
0.5482	180.73	180.499	3.90941	1.78407	1.70208
0.6091	220.019	217.369	4.34371	1.75931	1.66036
0.67	267.166	257.17	4.778	1.7656	1.62351
0.7309	314.312	299.837	5.2123	1.74544	1.59057
0.7918	361.459	345.313	5.6466	1.71036	1.56087
0.8528	416.464	393.626	6.08162	1.6988	1.53382
0.9137	471.469	444.57	6.51591	1.67535	1.5091
0.9746	526.473	498.178	6.95021	1.64431	1.48633

Table B.15 For spherical particles diameter of (dp_1=0.24, dp_2=0.61 and dp_{eff}=0.3445 cm), bed porosity of 0.3575, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.145	42.771	37.6722	0.47689	2.76203	2.50139
0.181	59.63	55.7128	0.59529	2.47128	2.37407
0.206	75.222	69.9998	0.67752	2.40672	2.30281
0.242	99.146	93.007	0.79592	2.29858	2.21708
0.266	116.975	109.894	0.87485	2.24463	2.16824
0.303	147.149	138.284	0.99654	2.17614	2.10273
0.327	169.291	158.192	1.07548	2.14958	2.06531
0.363	204.507	190.203	1.19388	2.10722	2.01511

0.387	229.728	212.949	1.27281	2.08261	1.98495
0.424	270.239	250.176	1.3945	2.04094	1.94271
0.448	299.493	275.7	1.47344	2.02603	1.91768

Table B.16 For spherical particle diameters of (dp_1=0.42, dp_2=0.61, dp_{eff}=0.7859 cm), bed porosity is 0.4012, bed diameter is 7.64 cm, packing height is 15.15 cm [91]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	8.482	7.47391	0.92113	1.96764	2.46576
0.145	11.704	10.2849	1.10383	1.89067	2.36287
0.181	17.554	15.2102	1.37789	1.81986	2.2426
0.206	21.045	19.1107	1.56821	1.68435	2.17528
0.242	26.351	25.392	1.84226	1.52822	2.0943
0.266	31.231	30.0023	2.02496	1.49914	2.04816
0.303	39.039	37.7532	2.30663	1.44421	1.98628
0.327	44.895	43.1882	2.48934	1.426	1.95093
0.363	53.679	51.9276	2.76339	1.3836	1.90352
0.387	60.511	58.1376	2.9461	1.37224	1.87503
0.424	71.247	68.3008	3.22776	1.34602	1.83513
0.448	78.079	75.2691	3.41047	1.32128	1.81148

B.1.3 Ternary sized spherical particles system

Table B.17 For spherical particles diameters of (0.24, 0.42 and 0.82cm, with dp_{eff}=0.3862 cm), bed porosity of 0.3428, packing height of 15.15 cm, bed diameter of 7.64 cm [91].

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	21.46	18.49	0.448	2.304	1.704
0.145	27.32	25.73	0.537	2.042	1.651
0.181	39.04	38.55	0.671	1.873	1.588
0.206	47.82	48.81	0.763	1.771	1.552

0.242	60.54	65.49	0.897	1.624	1.509
0.266	72.19	77.82	0.986	1.603	1.484
0.303	87.84	98.68	1.123	1.503	1.45
0.327	97.6	113.4	1.212	1.434	1.431
0.363	112.2	137.2	1.345	1.339	1.405
0.387	122	154.2	1.434	1.28	1.389
0.424	144.4	182.2	1.571	1.263	1.367
0.448	148.2	201.4	1.66	1.16	1.354

Table B.18 For spherical particles diameters of (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651cm), bed porosity of 0.3899, packing height of 15.15 cm, bed diameter of 7.64 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	5.644	10.36	0.997	2.293	2.991
0.182	19.56	21.71	1.496	2.085	2.785
0.242	25.35	36.73	1.996	1.962	2.648
0.303	49.56	55.19	2.495	1.918	2.546
0.364	40.31	76.93	2.993	1.818	2.466
0.424	53.75	101.9	3.492	1.78	2.4
0.485	68.53	130	3.992	1.738	2.344
0.545	86.99	161.2	4.491	1.723	2.296
0.606	104.8	195.4	4.99	1.701	2.254
0.667	126.3	232.5	5.489	1.694	2.217
0.727	147.8	272.5	5.988	1.666	2.183
0.788	169.3	315.3	6.487	1.626	2.152
0.848	197.5	360.9	6.985	1.636	2.125
0.909	223.1	409.3	7.484	1.609	2.099
0.97	252.6	460.6	7.984	1.601	2.075

Table B.19 For spherical particles diameters of (0.24, 0.61 and 1.03 cm, with dp_{eff}=0.4427 cm), bed porosity of 0.3469, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	17.45	15.81	0.502	1.836	1.743
0.145	24.45	22.76	0.601	1.787	1.688
0.181	34.16	32.96	0.751	1.606	1.624
0.206	41.69	41.74	0.854	1.513	1.587
0.242	53.68	55.99	1.004	1.412	1.543
0.266	63.44	66.53	1.103	1.381	1.517
0.303	77.1	84.38	1.257	1.293	1.483
0.327	87.84	96.96	1.356	1.265	1.463
0.363	106.6	117.3	1.505	1.246	1.437
0.387	116.6	131.8	1.605	1.199	1.421
0.424	130.2	155.7	1.758	1.115	1.398
0.448	141.4	172.2	1.858	1.085	1.384

Table B.20 For spherical particles diameters of (0.7955, 0.6015 and 0.509 cm, with dp_{eff}=0.3862 cm), bed porosity of 0.6142, packing height of 15.15 cm, bed diameter of 7.64 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	10.75	11.6	0.762	2.7	2.119
0.182	21.5	24.31	1.143	2.398	1.973
0.242	34.94	41.13	1.525	2.19	1.875
0.303	52.41	61.8	1.906	2.102	1.803
0.364	72.56	86.14	2.287	2.022	1.746
0.424	95.41	114.1	2.668	1.953	1.7
0.485	120.9	145.6	3.049	1.896	1.66
0.545	149.2	180.5	3.43	1.847	1.626
0.606	178.7	218.8	3.812	1.793	1.596
0.667	212.3	260.3	4.193	1.76	1.57
0.727	248.6	305.1	4.574	1.732	1.546
0.788	287.6	353.1	4.955	1.707	1.524
0.848	327.9	404.1	5.336	1.678	1.505
0.909	370.9	458.3	5.717	1.654	1.487
0.97	424.6	515.7	6.099	1.664	1.47

Table B.21 For spherical particles diameters of (0.51, 0.79 and 1.01 cm, with dp_{eff}=0.7115 cm), bed porosity of 0.3822, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	15.72	15.39	0.871	3.071	2.879
0.183	29.86	32.25	1.306	2.593	2.681
0.244	48.72	54.51	1.741	2.38	2.549
0.305	72.29	81.95	2.177	2.259	2.451
0.366	100.96	114.3	2.613	2.183	2.373
0.426	125.67	151.4	3.048	2.004	2.31
0.487	157.42	193.1	3.483	1.918	2.256
0.548	209.89	239.4	3.919	1.915	2.21
0.609	253.16	290.1	4.354	2.098	2.17

Table B.22 For spherical particles diameters of (0.51, 0.61 and 1.01 cm, with dp_{eff}=0.6536 cm), bed porosity of 0.3727, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	20.43	16.1	0.784	3.266	2.527
0.183	37.72	33.74	1.176	2.68	2.353
0.244	62.86	57.02	1.568	2.512	2.237
0.305	91.15	85.72	1.961	2.33	2.151
0.366	121.8	119.5	2.353	2.162	2.083
0.426	165	158.3	2.745	2.152	2.027
0.487	196.94	202	3.137	1.962	1.98
0.548	235.87	250.4	3.53	1.936	1.94
0.609	298.96	303.5	3.922	1.909	1.904
0.67	353.96	361.1	4.314	1.868	1.873
0.731	416.75	423.2	4.706	1.849	1.844
0.792	471.25	489.7	5.098	1.783	1.818
0.853	550.56	560.7	5.491	1.794	1.795
0.914	620.98	635.9	5.883	1.763	1.773

0.975	707.2	715.3	6.275	1.766	1.753

Table B.23 For spherical particles diameters of (0.51, 0.61 and 0.79 cm, with dp_{eff}=0.6165 cm), bed porosity of 0.3674, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	19.65	16.75	0.749	2.971	2.356
0.183	36.93	35.18	1.124	2.674	2.194
0.244	58.15	59.33	1.499	2.368	2.085
0.305	86.44	89.19	1.874	2.251	2.005
0.366	117.9	124.4	2.249	2.132	1.942
0.426	157.2	164.7	2.623	2.089	1.89
0.487	196.4	210.2	2.998	1.999	1.846
0.548	235.7	260.5	3.373	1.896	1.808
0.609	290.7	315.7	3.748	1.894	1.775
0.67	353.6	375.7	4.122	1.904	1.746
0.731	408.6	440.3	4.497	1.848	1.719

Table B.24 For spherical particles diameters of (0.24, 0.42 and 1.03 cm, with dp_{eff}=0.3992 cm), bed porosity of 0.3437, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	20.5	17.8	0.441	1.628	1.71
0.145	27.33	24.76	0.528	1.511	1.656
0.181	38.06	37.1	0.66	1.351	1.593
0.206	46.85	46.98	0.751	1.283	1.557
0.242	59.73	63.02	0.882	1.186	1.513
0.266	68.32	74.89	0.969	1.123	1.488
0.303	82.96	94.97	1.104	1.051	1.455
0.327	95.5	109.1	1.192	1.038	1.435
0.363	107.2	132	1.323	0.946	1.409
0.387	121.9	148.4	1.41	0.947	1.394

0.424	136.5	175.3	1.545	0.883	1.371
0.448	146.2	193.8	1.633	0.847	1.358

Table B.25 For spherical particles diameters of (0.9987, 0.7955 and 0.42 cm, with dp_{eff}=0.6474 cm), bed porosity of 0.3696, packing height of 15.15 cm, bed diameter of 7.64 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	12.9	11.18	0.787	3.042	2.251
0.182	25.53	23.43	1.18	2.674	2.096
0.242	43	39.64	1.575	2.531	1.993
0.303	64.5	59.56	1.968	2.429	1.916
0.364	86	83.02	2.361	2.251	1.856
0.424	114.2	110.78	2.755	2.196	1.806
0.485	143.8	140.3	3.149	2.116	1.764
0.545	178.7	174	3.542	2.078	1.728
0.606	216.3	210.9	3.936	2.038	1.696
0.667	260.1	250.9	4.33	2.024	1.668
0.727	299.7	294.1	4.724	1.96	1.643
0.788	346.7	340.3	5.117	1.932	1.62
0.848	396.4	389.5	5.51	1.905	1.599
0.909	448.8	441.8	5.904	1.879	1.58
0.97	510.6	497.1	6.298	1.879	1.562

B.1.4 Quaternary sized spherical particles system

Table 4.26 For spherical particles diameters of (0.24, 0.42, 0.82, 0.61 and dp_{eff}=0.4252 cm), bed porosity of 0.3474, packing height of 15.15cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP(pa) (experiments)	ΔP(pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	21.976	22.1867	0.47563	2.02637	2.3595
0.145	27.998	30.2048	0.56997	1.79776	2.23685

0.181	38.306	44.0845	0.71148	1.57852	2.0952
0.206	46.935	54.9652	0.80975	1.49315	2.01674
0.242	61.28	72.3351	0.95126	1.41263	1.92316
0.266	71.28	84.9899	1.0456	1.36002	1.87025
0.303	89.095	106.122	1.19104	1.31011	1.79976
0.327	101.506	120.85	1.28538	1.28155	1.75974
0.363	121.763	144.406	1.42689	1.2475	1.70635
0.387	135.875	161.061	1.52123	1.22478	1.67442
0.424	158.851	188.193	1.66667	1.19288	1.62992
0.448	175.007	206.715	1.76101	1.17717	1.60366

Table B.27 For spherical particles diameters of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.552 cm), bed porosity of 0.371, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	25.145	24.0913	0.65882	3.28862	3.14262
0.1827	47.147	48.0944	0.98823	2.74053	2.78833
0.2436	78.578	78.5444	1.31765	2.56923	2.56146
0.3046	110.009	114.971	1.6476	2.30052	2.39804
0.3655	157.156	156.875	1.97701	2.28251	2.27251
0.4264	196.445	204.018	2.30642	2.09635	2.17151
0.4873	251.45	256.167	2.63583	2.05455	2.08765
0.5482	298.597	313.127	2.96524	1.92781	2.01637
0.6091	369.317	374.735	3.29465	1.93143	1.95467
0.67	440.037	440.847	3.62407	1.90194	1.90049
0.7309	510.758	511.338	3.95348	1.85505	1.85233
0.7918	605.051	586.097	4.28289	1.87248	1.80911
0.8528	675.772	665.157	4.61284	1.80286	1.76993
0.9137	770.065	748.17	4.94225	1.78969	1.73428
0.9746	872.217	835.178	5.27166	1.78168	1.70158

Table B.28 For spherical particles diameters of (0.24, 0.42, 0.61 and 1.03 cm, with dp_{eff}=0.4368 cm), bed porosity of 0.3495, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	21.109	21.445	0.4882	2.3553	2.3927
0.145	26.895	29.194	0.585	2.0897	2.2684
0.181	38.037	42.61	0.7302	1.8967	2.1247
0.206	43.919	53.127	0.8311	1.6907	2.0451
0.242	53.679	69.915	0.9763	1.4973	1.9502
0.266	63.439	82.147	1.0732	1.4647	1.8966
0.303	78.079	102.57	1.2224	1.3893	1.8251
0.327	87.839	116.81	1.3193	1.342	1.7845
0.363	102.45	139.58	1.4645	1.2701	1.7304
0.387	116.05	155.67	1.5613	1.2658	1.698
0.424	131.59	181.9	1.7106	1.1958	1.6529
0.448	141.52	199.8	1.8074	1.1519	1.6263

Table B.29 For spherical particles diameters of (0.24, 0.82, 0.61 and 1.03 cm, with dp_{eff}=0.5002 cm), bed porosity of 0.3604, packing height of 15.15 cm, bed diameter of 7.64 cm [91]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	19.253	18.095	0.5619	2.7445	2.5794
0.145	24.399	24.634	0.6733	2.422	2.4453
0.181	34.159	35.954	0.8405	2.1761	2.2904
0.206	43.919	44.827	0.9566	2.16	2.2047
0.242	56.607	58.994	1.1238	2.0173	2.1024
0.266	66.367	69.314	1.2352	1.9576	2.0445
0.303	78.079	86.549	1.407	1.7749	1.9675
0.327	87.839	98.561	1.5185	1.7145	1.9237
0.363	107.36	117.77	1.6857	1.7004	1.8654
0.387	117.12	131.36	1.7971	1.6321	1.8305
0.424	136.64	153.48	1.9689	1.5863	1.7818

0.448	146.4	168.59	2.0804	1.5224	1.7531

Table B.30 For spherical particles diameters of (0.51, 0.61, 0.79 and 1.01cm, with dp_{eff}=0.848 cm), bed porosity of 0.414, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	14.144	14.391	1.0457	4.2274	4.3011
0.1827	27.502	28.729	1.5686	3.6533	3.8162
0.2436	45.575	46.918	2.0914	3.4054	3.5057
0.3046	66.791	68.677	2.6151	3.1919	3.2821
0.3655	92.722	93.707	3.138	3.0775	3.1102
0.4264	125.73	121.87	3.6608	3.0661	2.972
0.4873	157.16	153.02	4.1837	2.9345	2.8572
0.5482	192.52	187.04	4.7065	2.8404	2.7597
0.6091	243.59	223.84	5.2294	2.9113	2.6752
0.67	282.88	263.33	5.7522	2.7942	2.6011
0.7309	330.03	305.44	6.2751	2.7392	2.5352
0.7918	385.03	350.1	6.7979	2.7231	2.476
0.8528	440.04	397.32	7.3216	2.6828	2.4224
0.9137	510.76	446.91	7.8445	2.7127	2.3736
0.9746	573.62	498.88	8.3673	2.6777	2.3289

Table B.31 For spherical particles diameters of (0.42, 0.51, 0.61 and 1.01 cm, with dp_{eff}=0.5738 cm), bed porosity of 0.3745, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	22.788	22.972	0.6883	3.1952	3.221
0.1827	43.218	45.86	1.0325	2.6932	2.8579
0.2436	73.863	74.895	1.3767	2.5892	2.6253
0.3046	102.15	109.63	1.7214	2.2902	2.4578
0.3655	145.37	149.59	2.0656	2.2635	2.3292

0.4264	188.59	194.54	2.4098	2.1576	2.2257
0.4873	227.88	244.26	2.7539	1.9962	2.1397
0.5482	275.02	298.58	3.0981	1.9036	2.0666
0.6091	345.74	357.32	3.4423	1.9385	2.0034
0.67	400.75	420.36	3.7865	1.857	1.9479
0.7309	471.47	487.58	4.1306	1.8358	1.8985
0.7918	550.05	558.86	4.4748	1.825	1.8542
0.8528	628.63	634.25	4.8195	1.798	1.8141
0.9137	715.06	713.4	5.1637	1.7817	1.7775
0.9746	817.21	796.37	5.5079	1.7897	1.744

Table B.32 For spherical particles diameters of (0.42, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.6373 cm), bed porosity of 0.3843, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	18.859	20.214	0.7678	3.2238	3.4555
0.1827	35.36	40.355	1.1518	2.6864	3.0659
0.2436	56.576	65.905	1.5357	2.4178	2.8164
0.3046	86.436	96.47	1.9202	2.3625	2.6368
0.3655	113.94	131.63	2.3041	2.1629	2.4987
0.4264	157.16	171.19	2.6881	2.192	2.3877
0.4873	188.59	214.94	3.072	2.014	2.2955
0.5482	235.73	262.74	3.4559	1.9892	2.2171
0.6091	290.74	314.43	3.8398	1.9873	2.1493
0.67	345.74	369.9	4.2237	1.9532	2.0897
0.7309	408.61	429.05	4.6076	1.9397	2.0367
0.7918	463.61	491.78	4.9916	1.8753	1.9892
0.8528	526.47	558.12	5.3761	1.8358	1.9461
0.9137	612.91	627.77	5.76	1.8618	1.9069
0.9746	691.49	700.78	6.1439	1.8462	1.871

B.1.5 Quinary sized spherical particles system

Table B.33 For spherical particles diameters of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.607 cm), bed porosity of 0.3694, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1218	24.359	24.2001	0.71683	3.28203	3.41954
0.1827	45.575	47.5727	1.07524	2.72915	2.98763
0.2436	73.863	76.8474	1.43365	2.488	2.71469
0.3046	106.08	111.536	1.79266	2.28534	2.52
0.3655	145.37	151.136	2.15107	2.17509	2.37159
0.4264	188.587	195.407	2.50948	2.07327	2.25295
0.4873	235.734	244.113	2.8679	1.9843	2.15498
0.5482	290.739	297.06	3.22631	1.93376	2.07211
0.6091	353.602	354.086	3.58472	1.90509	2.00068
0.67	424.322	415.048	3.94314	1.8894	1.93818
0.7309	495.042	479.824	4.30155	1.85227	1.88284
0.7918	581.478	548.306	4.65997	1.85387	1.83332
0.8528	667.914	620.515	5.01897	1.83571	1.78856
0.9137	746.492	696.128	5.37738	1.78729	1.74795
0.9746	856.501	775.181	5.73579	1.80241	1.71079

B.2 General equation results

Table B.34 For glass spherical particle with diameter of 0.7955, bed porosity 0.39804, packing height of 15.15 cm, bed diameter of 7.62 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.1211	6.7188	7.9811	1.0502	3.0234	2.5848
0.1817	13.438	15.64	1.5758	2.686	2.2499
0.2424	21.5	25.222	2.1022	2.4147	2.0387
0.303	30.907	36.514	2.6277	2.2215	1.8889
0.3635	43	49.379	3.1524	2.1476	1.7749

0.4241	56.438	63.763	3.678	2.0707	1.6837
0.4847	71.22	79.569	4.2035	2.0005	1.6086
0.5453	88.688	96.732	4.7291	1.9683	1.5451
0.6059	107.5	115.2	5.2546	1.9324	1.4904
0.6665	127.66	134.92	5.7802	1.8964	1.4426
0.7271	150.5	155.87	6.3057	1.8786	1.4003
0.7877	176.03	177.99	6.8313	1.8722	1.3625
0.8482	201.56	201.23	7.356	1.8489	1.3284
0.9088	229.78	225.62	7.8815	1.836	1.2974
0.9695	258	251.15	8.4079	1.8114	1.2691

Table B.35 For spherical particle diameters of (dp_1=0.9987, dp_2=0.7955, and dp_{eff} =0.886 cm), bed porosity is 0.4068, bed diameter is 7.64 cm, packing height is 15.15 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	5.106	6.909	1.145	2.309	2.697
0.182	10.48	13.54	1.719	2.106	2.348
0.242	17.47	21.83	2.293	1.972	2.128
0.303	25.53	31.61	2.866	1.844	1.971
0.364	35.74	42.75	3.438	1.794	1.852
0.424	47.03	55.2	4.011	1.734	1.757
0.485	60.47	68.88	4.585	1.707	1.679
0.545	73.91	83.74	5.158	1.648	1.612
0.606	90.03	99.73	5.731	1.626	1.555
0.667	107.5	116.8	6.304	1.605	1.505
0.727	126.3	134.9	6.878	1.585	1.461
0.788	149.2	154.1	7.451	1.594	1.422
0.848	168	174.2	8.023	1.548	1.386
0.909	190.8	195.3	8.596	1.532	1.354
0.97	215	217.4	9.17	1.517	1.324

Table B.36 For spherical particles diameters of (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651 cm), bed porosity of 0.3949, packing height of 15.15 cm, bed diameter of 7.64 cm [89]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.121	5.644	8.41	0.997	2.293	2.546
0.182	11.56	16.48	1.496	2.085	2.216
0.242	19.35	26.58	1.996	1.962	2.008
0.303	29.56	38.47	2.495	1.918	1.86
0.364	40.31	52.03	2.993	1.818	1.748
0.424	53.75	67.19	3.492	1.78	1.658
0.485	68.53	83.84	3.992	1.738	1.584
0.545	86	101.9	4.491	1.723	1.522
0.606	104.8	121.4	4.99	1.701	1.468
0.667	126.3	142.2	5.489	1.694	1.421
0.727	147.8	164.2	5.988	1.666	1.379
0.788	169.3	187.5	6.487	1.626	1.342
0.848	197.5	212	6.985	1.636	1.308
0.909	223.1	237.7	7.484	1.609	1.278
0.97	252.6	264.6	7.984	1.601	1.25

Table B.37 For spherical particles diameters of (0.42, 0.51, 0.61, and 0.79 cm, with dp_{eff}=0.552 cm), bed porosity of 0.3707, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	25.15	16.28	0.659	3.289	2.116
0.183	47.15	31.88	0.988	2.741	1.842
0.244	78.58	51.36	1.318	2.569	1.67
0.305	110	74.4	1.648	2.301	1.547
0.366	157.2	100.6	1.977	2.283	1.453
0.426	196.4	130	2.306	2.096	1.379
0.487	251.5	162.1	2.636	2.055	1.317
0.548	298.6	197.1	2.965	1.928	1.265

0.609	369.3	234.7	3.295	1.931	1.221
0.67	440	274.9	3.624	1.902	1.181
0.731	510.8	317.6	3.953	1.855	1.147
0.792	605.1	362.6	4.283	1.872	1.116
0.853	675.8	410.1	4.613	1.803	1.088
0.914	770.1	459.8	4.942	1.79	1.062
0.975	872.2	511.7	5.272	1.782	1.039

Table B.38 For spherical particles diameters of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.607cm), bed porosity of 0.3775, packing height of 20 cm, bed diameter of 7.62 cm [90]

U (m/s)	ΔP (pa) (experiments)	ΔP (pa) (present work)	Re_p	f (experiments)	f (present work)
0.122	24.36	14.34	0.717	3.282	2.189
0.183	45.58	28.09	1.075	2.729	1.905
0.244	73.86	45.27	1.434	2.488	1.727
0.305	106.1	65.57	1.793	2.285	1.6
0.366	145.4	88.7	2.151	2.175	1.503
0.426	188.6	114.5	2.509	2.073	1.426
0.487	235.7	142.9	2.868	1.984	1.362
0.548	290.7	173.7	3.226	1.934	1.308
0.609	353.6	206.9	3.585	1.905	1.262
0.67	424.3	242.3	3.943	1.889	1.222
0.731	495	279.9	4.302	1.852	1.186
0.792	581.5	319.6	4.66	1.854	1.154
0.853	667.9	361.4	5.019	1.836	1.125
0.914	746.5	405.2	5.377	1.787	1.099
0.975	856.5	451	5.736	1.802	1.075

الخلاصة

تم صياغة معادلات شبه عملية لجريان الموائع خلال عمود حشوي بالاعتماد على نظرية باكنكهام. تم استخدام نوعان من الموائع (ماء و هواء) بشكل منفصل في كل مرة (جريان طور واحد). تم استخدم انواع واشكال مختلفه من الحشوات وبحجوم مختلفة، وتم دراسة كل واحد منها بشكل منفصل.

تم دراسة العوامل المختلفة التي تؤثر على هبوط الضغط عند جريان الموائع في عمود حشوي كل على حده، هذه العوامل هي سرعة جريان الموائع، مسامية الحشوة، طول الحشوة في العمود الحشوي، قطر العمود الحشوي، معامل كروية الحشوات، قطر الحشوة ودراسة تاثير جدار العمود الحشوي.

تم صياغة معادلة شبه عملية محددة لجريان الموائع لكل نوع محدد من الحشوات المستخدمة وتم تسميتها بالمعادلة المفردة (حشوة كروية الشكل ذات حجم واحد، حشوة غير كروية الشكل ذات حجم واحد ، حشوة كروية الشكل ثنائية الحجوم، حشوة كروية الشكل ثلاثية الحجوم، حشوة كروية الشكل رباعية الحجوم، حشوة كروية الشكل خماسية الحجوم و حشوة كروية الشكل متعددةالحجوم). تم صياغة احد عشر معادلة احادية من هذه النوعية، ستة منها لجريان الماء وخمسة منها لجريان الهواء خلال العمود الحشوي.

تم صياغة معادلة عامة لجريان الموائع خلال العمود الحشوي تصلح لكافة انواع واشكال الحشوات.

لقد كانت النتائج المستحصلة من المعادلات الجزئية والعامة متقاربة. تم مقارنة النتائج المستحصلة من الحسابات لجريان الموائع خلال عمود حشوي مع عدد كبير من النتائج العملية المستحصلة من المصادر الموثقة،و هذه المقارنة اعطت تطابق جيد جدا، وتم عرض ذلك في جداول و رسومات. تم وضع النتائج المستحصلة باستخدام معادلة ايرجن في هذه الرسومات لنفس الظروف السابقة لغرض المقارنة.

تم كتابة علاقات تجريبية لحساب مسامية المواد لجميع المعادلات المستخدمه في الحسابات، وتم مقارنة النتائج المستحصلة منها مع معادلة فرناس وتجارب عملية ماخوذة من مصادر عملية موثقة وكانت نتائجنا مطابقة بشكل جيد جدا للنتائج العملية.

ان النقطة الدنيا لسرعة الجريان غير المنتظم هي دلالة لبداية عدم الانتظام. وعليه تم كتابة معادلة شبه عملية بالاعتماد على معادلة ليفا لحساب السرعة الدنيا للجريان وذلك لحساب نقطة تغير الجريان الى غير المنتظم.

شكر وتقدير

في البداية اشكر الله عز وجل الذي وفقني لاكمال متطلبات هذا البحث. وانا انهي بحثي لا يسعني واعترافا بالفضل الا ان اتقدم بوافر الشكر والامتنان للاستاذ المشرف الدكتور محمد نصيف لطيف لاقتراحه موضوع الرسالة واشرافه عليها ولمواصلته ومتابعته العلمية للبحث وما ترتب على ذلك من توجيهات قيمه واراء سديدة.

كما اتقدم بجزيل الشكر الى رئيس قسم الهندسة الكيمياوية، و جميع اساتذة قسم الهندسة الكيمياوية لمساعدتهم القيمة لي طيلة فترة الدراسة ولمدهم يد العون لي خلال اعداد هذه الرسالة.

واتقدم بشكري وامتناني الى عمادة جامعة النهرين،لمساعدتهم ودعمهم الدائم لي طيلة فترة الدراسة.

كما اتقدم بالشكر الجزيل الى كافة العاملين في المكتبة المركزية بجامعة النهرين، وذلك لمساعدتهم لي في الحصول على بعض المصادر المستعملة في البحث.

وشكري الجزيل الى جميع زملائي و زميلاتي الذين مدو يد العون عند حاجتي اليها في البحث.

ولا انسى ان اتقدم بجزيل الشكر والتقدير الى من ساندني وساعدني على تخطي الصعوبات خلال فترة البحث الى الذين لا مثيل لهم في الدنيا الى أبي وأمي الأعزاء، أخوتي وأخواتي.

م. زينب طالب عبدزيد

دراسة العوامل المؤثرة على هبوط الضغط خلال العمود الحشوي

رسالة

مقدمة إلى كلية الهندسة في جامعة نهرين

وهي جزء من متطلبات نيل درجة ماجستير علوم

في الهندسة الكيمياوية

من قبل

زينب طالب عبدزيد

(بكالوريوس علوم في الهندسة الكيمياوية 2005)

شعبان 1428

اب 2007