OPTIMUM SINGLE AND LAYERED PIPE INSULATION FOR A GIVEN HEAT LOAD UNDER NATURAL CONVECTION HEAT TRANSFER

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by

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Certification

I certify that this thesis entitled "**Optimum Single and Layered Pipe Insulation for a Given Heat Load under Natural Conduction Heat Transfer**" was prepared by **Samar Saadi Hussein** under my supervision at Nahrain University/College of Engineering in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering.

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Abstract

The present work is an attempt for gaining some standard and technical information to analyze the problem of heat loss from pipelines carrying hot streams. The analysis covers the use of a single cylindrical insulation or composite structure of different insulating materials. Applying the quasi-Newton's method in STATISTICA program, the thermal conductivity equation function of factors affecting insulation efficiency (temperature and density) is found by using three types of insulations (mineral wool, calcium silicate, and cellular glass). The temperature field is obtained for each layer by trial and error in a linear program MATLAB. The temperature of the outermost insulation layer is calculated and observed not to exceed a maximum limit set by safety regulations in oil refinery industry (60°C). The analysis is applied to three cases for which different fluids (hot water, petroleum products in refineries, and steam) at different temperature levels are considered. The temperature fields and heat loss rates for each case is studied with and without insulation, single and double layers.

Economic analysis of the problem is presented where an annual cost function is derived on basis of the insulation layer cost and cost of lost energy. The objective is to minimize the cost function subject to the only imposed constrain (the temperature of the outermost insulation layer is not to exceed a maximum limit set). An interactive MATLAB program is used to find the annual cost for different insulation configuration for three cases. The optimum thickness chosen for single and multilayer insulation is based on economic analysis.

The results show significant cost saving with optimum insulation compared to the dimensions of engineering practice. Detailed results for these cases are presented, analyzed, and discussed. The thermal conductivity generally increases with temperature for the three insulations studied. The results indicate that the lowest thermal conductivity was obtained using the mineral wool insulation at a density of 200 kg/m^3. From the results of composite two layers, it is concluded that using composite two layers of (calcium silicate & mineral wool) and (cellular glass & mineral wool) show a better optimum from using single layer of calcium silicate or cellular glass. On the other hand, the minimum total cost of double-layer insulation is higher than that of using only mineral wool insulation.

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Notations

Symbols	Notations	Units
β	=volume coefficient of expansion	1/K
Cp_w	= Specific capacity of water	J/kg. K
D_i	= Inside diameter	m
D_0	= Outside diameter	m
%EQ	= Percentage error of heat transfer	-
ε	= Emissivity of insulation surface	-
ε_0	= Emissivity of pipe surface	-
Gr_D	= $\frac{g\ \beta\,(T_s - T_0)d^3}{\upsilon^2}$ = Grashof number	-
g	= acceleration of gravity = 9.81	m/s^2
h_r	= Heat transfer coefficient of radiation	W/m^2.K
h_i	= Inside heat transfer coefficient of convection	W/m^2.K
h_0	= Outside heat transfer coefficient of convection	W/m^2.K
k_t	= Thermal conductivity of steel pipe	W/m. K
k_w	= Thermal conductivity of water	W/m. K
k_a	= Thermal conductivity of air	W/m. K
k_{ins}	= Thermal conductivity of insulation	W/m. K
$k_{ins\,1}$	= Thermal conductivity of first insulation	W/m. K
$k_{ins\,2}$	= Thermal conductivity of second insulation	W/m. K
μ_w	= Dynamic viscosity of water	Kg/m. s
Nu	= $\frac{h_0 d}{k}$ = Nusselt number	-
Pr_w	= Prandtl number of water	-
Pr_a	= Prandtl number of air	-
Q	= Heat transfer	W/m
Q_a	= Actual heat transfer	W/m
q_c	= Heat transfer by convection	W/m
q_r	= Heat transfer by radiation	W/m
$\%Q_{sav}$	= Percentage of energy saving of an insulated pipe while neglecting heat radiation	-
$\%Q_{asav}$	= Percentage of energy saving of an insulated pipe while considering heat radiation	-
r_1	= Interior radius of pipe	m
r_2	= Exterior radius of pipe (Interior radius of first insulation)	m

r_3	=	Exterior radius of first insulation (Interior radius of second insulation)	m
r_4	=	Exterior radius of second insulation	m
R_1	=	Thermal resistance of fluid	K/W
R_2	=	Thermal resistance of pipe	K/W
R_3	=	Thermal resistance of first insulation	K/W
R_4	=	Thermal resistance of second insulation	K/W
Re	=	Reynolds number	-
T_1	=	Interior pipe temperature	K
T_2	=	Exterior pipe temperature	K
T_3	=	Exterior first layer insulation temperature	K
T_4	=	Exterior second layer insulation temperature	K
T_a	=	actual temperature	K
T_{b1}	=	Interior bulk temperature	K
T_{b2}	=	Exterior bulk temperature	K
T_f	=	film temperature	K
T_i	=	Bulk mean temperature	K
t_{ins}	=	Insulation thickness	m
T_m	=	Mean temperature	K
T_{mp}	=	Mean temperature of pipe	K
T_0	=	ambient temperature	K
t_{opt}	=	Optimum insulation thickness	m
T_s	=	Surface temperature	K
u	=	Average velocity	m/s
ν	=	kinematic viscosity	m^2/s
ρ_w	=	Density of water	kg/m^3
ρ	=	Density of insulation	kg/m^3
σ	=	Stefan Boltzmann constant= 5.67*10^{-8}	W/m^2.K^4

Abbreviations

ASTM	=	American Society for Testing and Materials
NIST	=	National Institute of Standards and Technology

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Chapter one

Introduction

1-1 Thermal insulation

In both industry and building, when equipment or a pipe operates at temperatures sensibly different from ambient temperature, it is usually necessary to thermally insulate it. Thermal insulation may be necessary for the following reasons [1, 2, and 3]:

1. For energy savings. The energy lost to the ambient needs to be supplied at the expense of a higher fuel or electricity consumption. In these cases, the cost of a higher insulation thickness must be economically balanced again the higher fuel consumption

2. To reduce the external temperature of equipment and piping to avoid burns to the operating personnel. This is called personal-protection thermal insulation, and the insulation thickness is calculated so as to have external surface temperatures lower than 55 or 60°C.

3. For process reasons, in some cases it is necessary to keep process fluids at certain minimum temperatures to avoid undesirable reactions, solidification, precipitation, polymerization, etc. In this case, insulation thicknesses result from process calculations.

4. In low temperature processes to avoid condensation of ambient humidity over the surface of piping and equipment.

1-2 Characteristics of a good thermal insulation system

A good thermal insulation system must meet the following conditions [2]:

1. The materials employed must be compatible with the process and must be inert when the process fluids come in contact with them.

2. The insulation system must combine a low thermal conductivity with adequate mechanical resistance.

3. It must have a barrier to avoid humidity penetration. The presence of water within the insulation increases the thermal conductivity dramatically.

4. In plants where combustible materials are processed, it is necessary to evaluate the behavior of the insulation in case of fire. Insulation material must present low combustibility and smoke production.

1-3 Thermal insulation materials

Materials used in thermal insulation applications generally are grouped according to the temperature range in which they are employed. The temperatures that limit these ranges are somewhat arbitrary. When the temperature reaches an upper limit, materials may be damaged or become uneconomical because their thermal conductivity increases. A lower limit usually means that the material is not competitive because there are cheaper materials that can perform satisfactorily. Within each temperature range, the selection is made taking into consideration other properties and cost [2].The temperature range within which the term “thermal insulation” applies, is from -

73˚C to 982˚C. All applications below -73˚C are termed cryogenic and those above 982˚C are termed refractory.

Thermal insulations are further divided into three general application temperature ranges [4].

1-3-1 Low temperature thermal insulation

Insulation used for low temperature applications is subdivided into the general temperature ranges.

- 16˚C through 0˚C – cold or chilled water.
- -1˚C through -39˚C – refrigeration.
- -40˚C through -73˚C - refrigeration.

The major problems on low temperature installations are moisture penetration and cost effectiveness. Ideally, the insulation material or system should absorb no moisture and readily give up any that enters. It should also resist water deterioration [4]. Four generic insulation materials are [2]:

- Foamed glass
- Urethane foam
- Polystyrene foam
- Fiberglass

1-3-2 Intermediate temperature thermal insulation

This temperature range, from 16˚C to 315˚C includes conditions encountered in most industrial processes and in hot water and steam systems found in commercial installations. Selection of material in this range is based more on thermal values than with low temperature applications. However, other factors such as mechanical and chemical properties, availability of forms, installation time and cost are also significant [4].

The commonly used materials include the following [2]:

- Calcium Silicate

- Mineral Wool
- Fiberglass
- Cellular Glass or Cellular Foam

1-3-3 High temperature thermal insulation [4, 2]

High temperature thermal insulation is used in the temperature range of 315˚C to 870˚C. The materials usually employed are

- Miner fiber or calcium silicate up to 900˚C
- Ceramic fibers (Al_2O_3-SiO_2) up to 1400˚C
- Molded ceramic refractory up to 1650˚C
- Metallic oxides fibers such as Al_2O_3 or ZrO_2 up to 1650˚C
- Carbon fibers up to 2000˚C

1-4 Scope of the present work

The major objective of the present work is to analyze the problem of heat loss from pipelines carrying hot streams. The analysis covers the use of a single cylindrical insulation or composite structure of different insulating materials. The analysis is applied to three cases for which different fluids (hot water, petroleum products in refineries, and steam) at different temperature levels are considered. The temperature fields and heat loss rates for each case are studied for no insulated and insulated conditions. Economic analysis of the problem is presented where an annual cost function is derived on basis of the insulation layer cost and cost of lost energy. The objective is to minimize the cost function subject to the only imposed constrain (the temperature of the outmost insulation layer is not to exceed a maximum limit set). The optimum thickness chosen for single and multilayer insulation is based on economic analysis.

Chapter two

Literature survey

2-1 Insulation materials

2-1-1 General

Insulation materials are extensively used to reduce the heat losses (or gains) from thermal systems like buildings, pipes and ducts, components of HVAC installations, etc. Most mass-type thermal insulation materials are highly porous, and consist of a solid matrix full of small voids that comprise 90% or more of the total volume. These voids contain air or some other harmless gas such as CO_2. The apparent conductivity of the material is the macroscopic result of various basic heat transfer mechanisms: solid and gas conduction, gas convection and long-wave radiation within the voids. From the macroscopic point of view, the apparent conductivity mainly depends on the kind of insulation, bulk density, temperature, water content, thickness and age. From the microscopic point of view, factors such as cell size, diameter and arrangement of fibers or particles, transparency to thermal radiation or type and pressure of the gas come into play [5].

There are many different types of insulation materials available for both commercial and industrial piping applications. The following list, a few of the most common to the industrial and commercial piping industry [1]:

- Calcium silicate insulation
- Cellular glass insulation
- Elastomeric foam insulation

- Fiberglass and mineral wool insulations
- Perlite insulation
- Phenolic foam insulation
- Polystyrene foam insulation
- Polyurethane and polyisocyanurate foam insulations

2-1-2 Factors affecting thermal conductivity:

Insulation materials usually consist of a fibrous, cellular, or porous structure in which a solid matrix encloses spaces where air is present. Within this structure, heat transmission is due to a combination of the three basic mechanisms: conduction, convection, and radiation.

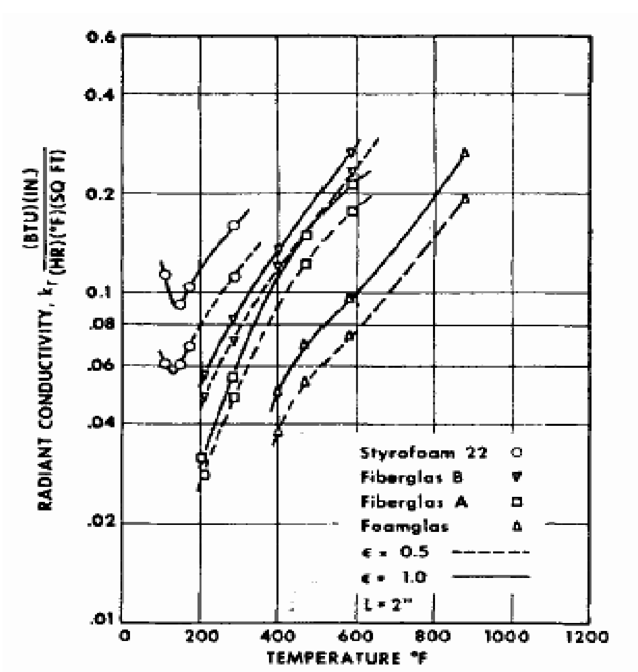
Heat transfer through a fibrous insulation involves combined modes of heat transfer: solid conduction through fibers, gas conduction and natural convection in the space between fibers, and radiation interchange through participating media. The relative contributions of the different heat transfer modes vary during reentry. Radiation becomes more dominant at high temperatures and with large temperature gradients through the insulation, while gas conduction and natural convection contributions are minimal at low pressures and become more significant with increasing pressure. The complex coupling of the heat transfer modes makes the analysis and design of high temperature insulation difficult [2, 6, and 7].

In general, the ‘‘effective’’ conductivities depend on density, temperature, moisture content as well as the constituents and voids present in their structures. Radiation characteristics across these voids add to the complexity of the problem

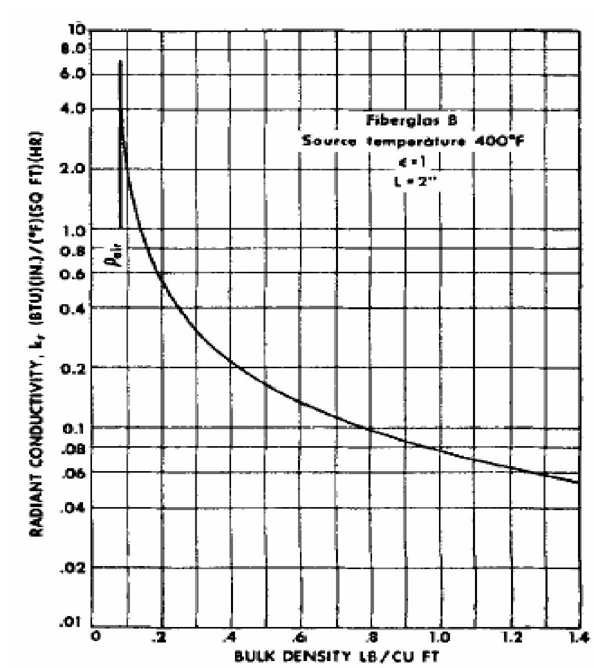
[8]. Radiative heat transfer could be a significant contribution to the total heat transfer within the highly porous materials [9].

Principle of heat transfer through insulation materials has been investigated by various researchers:

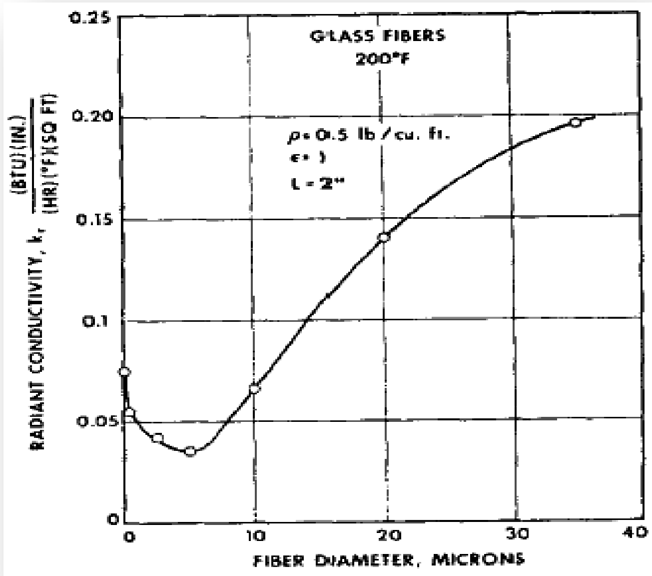
In 1959 Larkin and Churchill [10] introduced two-flux model and defined total heat flux to consist of conduction and radiation. Experimental and theoretical study of heat transfer in fibrous insulation was performed at 760 torr and temperatures up to 400°C. The influence of temperature level, bulk density and fiber diameter on radiant heat transfer were investigated as shown in fig. 2-1a, b and c.



(a)



(b)



(c)

Figure 2-1a, b, and c Effect of temperature, bulk density and fiber size on radiant conductivity respectively [10].

Tong and Tien (1983) [11] modeled heat transfer through insulation at atmospheric pressure with temperatures up to 150°C. Their dependence on the physical characteristics of fibrous insulations has been investigated. It has been found that the radiant heat flux can be minimized by making the mean radius of the fibers close to that which yields the maximum extinction coefficient. It was found that results were useful to those concerned with the design and application of lightweight fibrous insulations. **Tong et al. (1983) [12]** modeled heat transfer through insulation at atmospheric pressure with temperatures up to 150°C. A guarded hot plate apparatus has been used to measure the radiant heat flux in the insulation. Also it proved by experiments that no convection occurs even at the very low material density, as the small size of the pores and tortuous nature of air channels prevent heat convection effectively.

In 1997 Petrov [13] analyzed and compared three different approaches for describing combined radiation and conduction heat transfer in fiber thermal insulation at high temperatures. The considered approaches include the radiation transfer equation or its approximations, approximation of radiation thermal conductivity and the radiation diffusion approximation for radiation transfer. It was shown that the most preferable approach was the radiation diffusion approximation for radiation transfer.

In 1999 Daryabeigi [14] solved numerically the combined radiation and conduction heat transfer through fibrous insulation. The radiation heat transfer was modeled using the optically thick approximation. Various models for gas and solid conduction and their combination were evaluated. The experimental results were used to evaluate the analytical models. It was concluded that the best analytical model resulted in effective thermal conductivity predictions that were within 8% of experimental results.

In 2002 Tseng and Kuo [15] presented measurement results of radiative properties and effective thermal conductivity of phenolic foams. Effective thermal conductivity (k_{eff}) of various samples was measured using heat flow thermal conductivity testers that comply with ASTM C518. Fig. 2-2 shows the effective thermal conductivity of samples *A* and *C* at various temperatures increases with temperature, quadratically, in the temperature range.

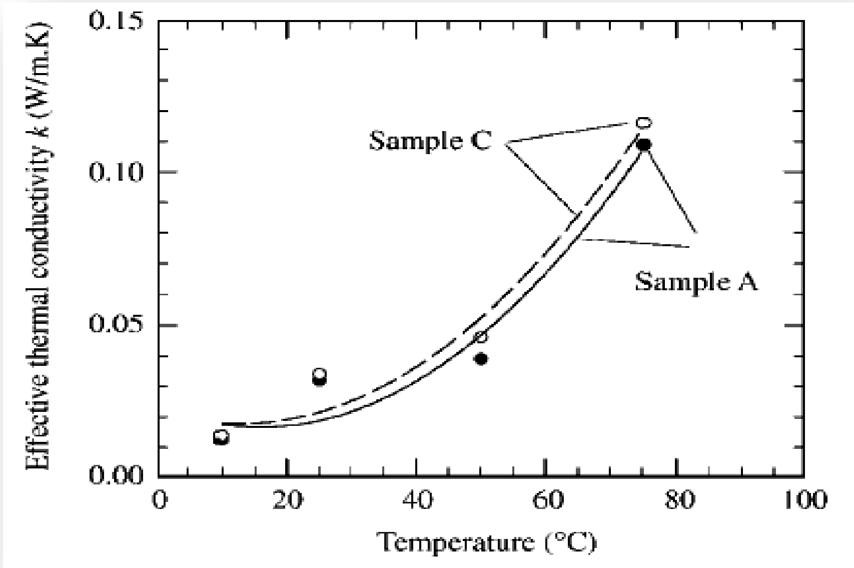


Figure 2-2 The effective thermal conductivity of samples A and C at various temperatures. Density of sample A=46.3 kg/m^3, density of sample C=66.6 kg/m^3 [15].

In 2003Wang et al. [16] tested thermal conductivity of Owens Corning glass fibers from room temperature to 700° C using the Hot Disk system. A more sensitive Heat Flow Meter system was used to measure thermal conductivities at lower temperatures. The results showed a lowering of thermal conductivity with increasing densities. The low temperature data fits and physical laws were used to extrapolate and estimate the high temperature thermal conductivity.

In 2004 Karamanos et al. [17] described the theoretical approach and the resulting data on the assessment of the changes in stone wool's thermal conductivity, under various operational conditions. They concluded that it was hydrophilic and water absorption increases its thermal conductivity factor dramatically. High temperatures cause an increase of the thermal conductivity factor. The results depicted in fig. 2-3 indicate the variations arising for each different heat transfer mode.

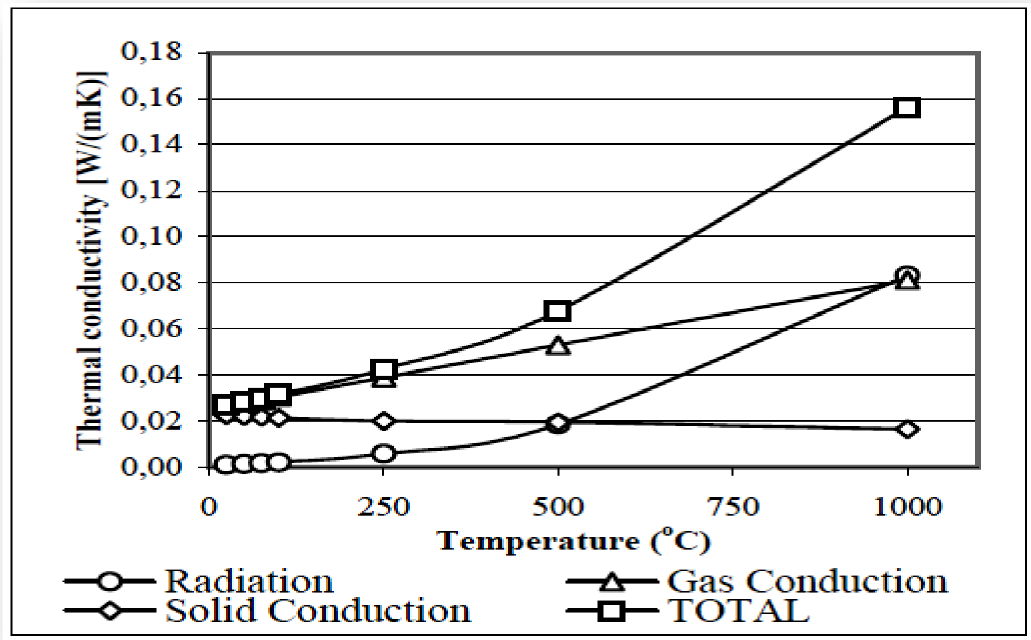


Figure 2-3 Results of the calculations of the model described [17].

In 2005 Placido et al. [18] investigated the dependence of the radiative and conductive properties on geometrical parameters characterizing the internal foam structure, such as the mean wall thickness, the mean strut and cell diameters. This type of study may be used mostly for optimization purposes of foam thermal performances.

In 2006 Abdulagatov et al. [19] measured the effective thermal conductivities (ETC) of five dry rocks (sandstone, limestone, amphibolite, granulite, and pyroxene−granulite) over a temperature range from (273 to 423) K and at pressures up to 350 MPa with a steady-state parallel-plate apparatus. It is an absolute, steady-state measurement device with an operational temperature range of (273 to 1273) K and hydrostatic pressures up to 1500 MPa. The estimated uncertainty of the method was 2 %. A sharp increase of ETC was found for rocks at low pressures between (0.1 and 100) MPa along various isotherms between (273 and 423) K. At high pressures (P> 100 MPa), a weak linear dependence of the ETC with pressure was observed. The measured values of ETC of rocks were used to test and confirm of applicability of the various theoretical and semiempirical models.

Al-Ajlan (2006) [8] used a new and accurate experimental technique for measuring the thermal conductivity of some commonly used insulation materials produced by local Saudi manufacturers. The effects of temperature and density on the thermal conductivity are also examined. Other thermal properties like the density and specific heat (and hence the thermal diffusivity) are measured too. Comparisons with manufacturers' claimed values of properties were made.

Manohar et al. (2006) [20] investigated the potential of naturally occurring biodegradable fibers for use as building thermal insulation. The apparent thermal conductivity (k) for biodegradable coconut and sugarcane fibers were investigated in accordance with ASTM C 518 over the density ranges 40 to 90 kg/$m^3$and 70 to 120 kg/$m^3$for the test temperature ranges 13.2 to 21.8˚C and 18 to 32˚C, respectively. The experimental data were used to determine empirical equations for (k) variation with density and temperature for both coconut and sugarcane fiber. Comparisons of k at 24˚C for coconut . and sugarcane fiber were made with seven different conventional insulation from published data. It was found that (k) variation with density and mean temperature for both coconut and sugarcane fibers were consistent with the behavior of loose-fill thermal insulation. The optimum (k) values at 24°C averaged about 0.0488 W/m. K and 0.0483 W/m. K for coconut and sugarcane fiber, respectively.

In 2007 Wei et al. [21] investigated experimentally the effective thermal conductivity of high-porosity xonotlite insulation material as a function of pressure and temperature. Fig. 2-4 shows that the thermal conductivity of xonotlite decreases significantly with a drop of pressure and reaches a minimum value at about 100 Pa. The thermal conductivity of xonotlite is mainly attributed to solid conduction and radiation as shown in fig. 2-5 where the thermal conductivity of xonotlite increases significantly with an increase in temperature. It was expected that the validated effective thermal conductivity model would be important for the thermal design and thermal analysis of xonotlite insulation material.

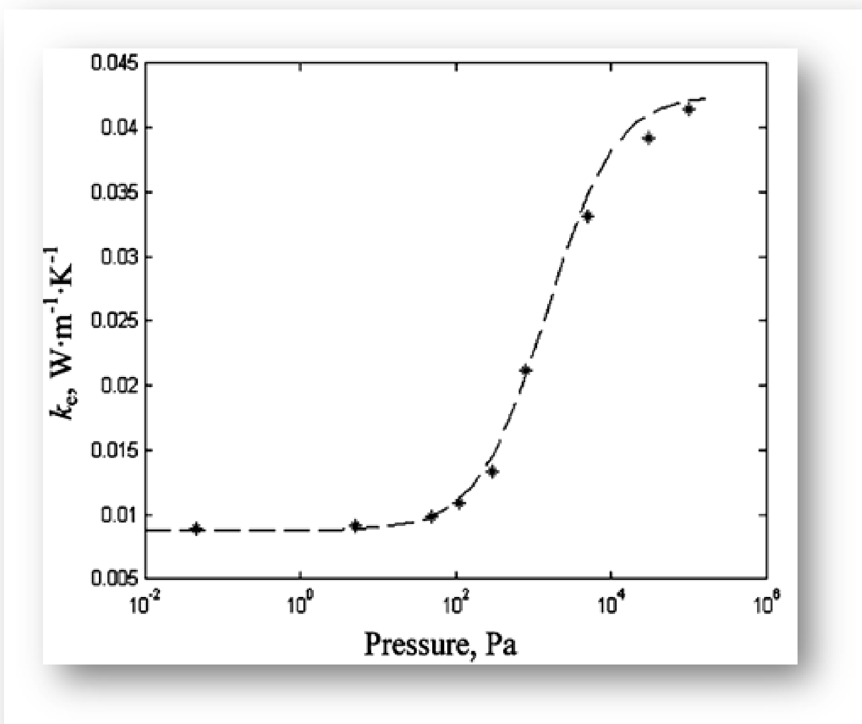


Figure 2-4 Pressure dependence of thermal conductivity of xonotlite insulation material [21].

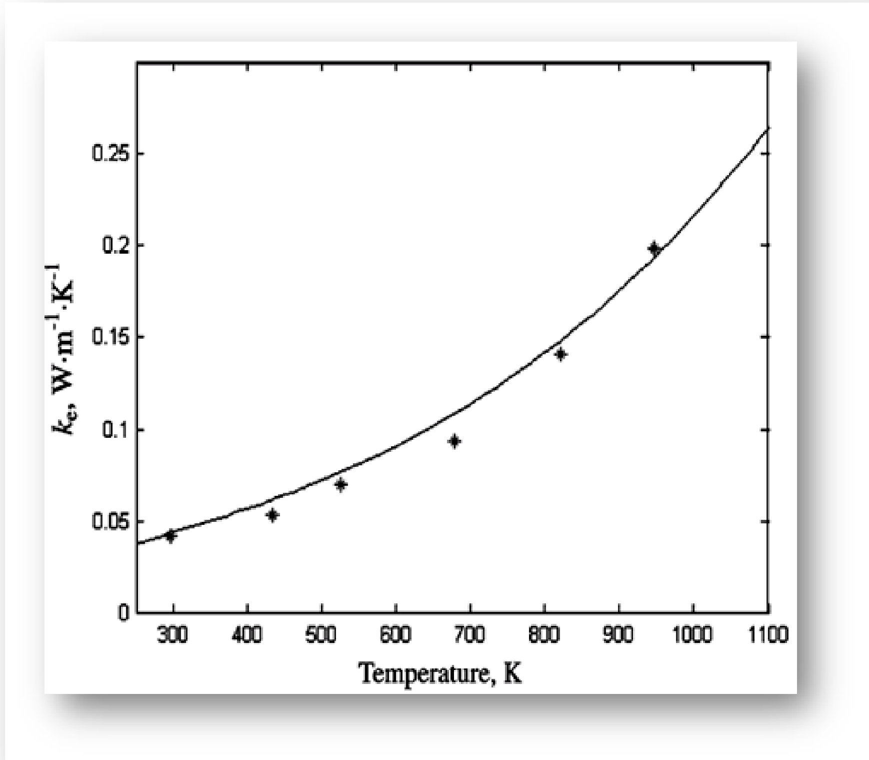
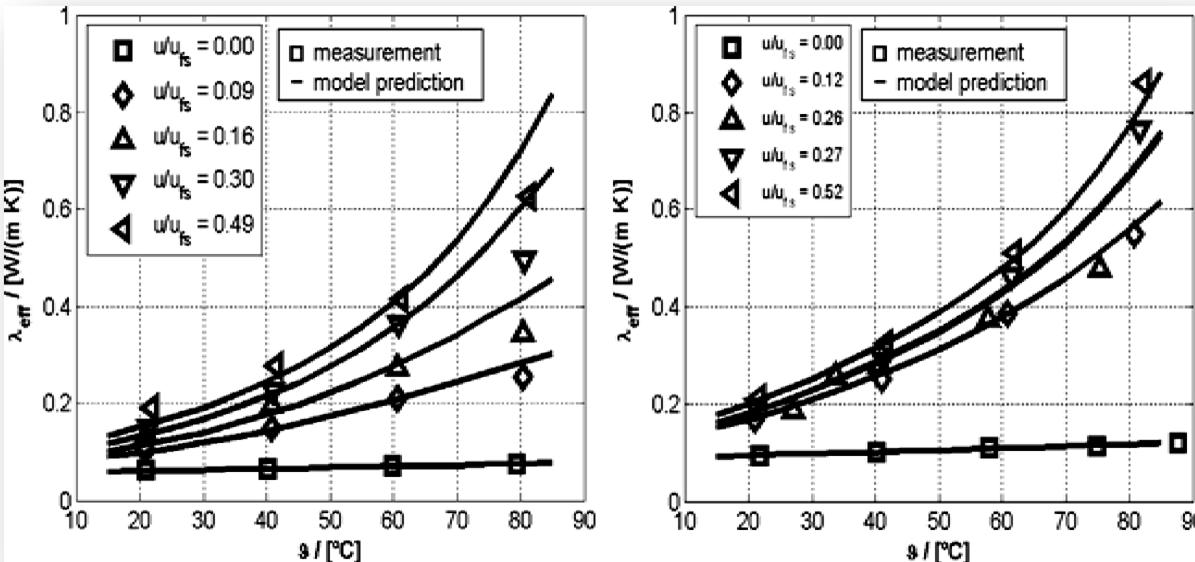


Figure 2-5 Temperature dependence of thermal conductivity of xonotlite [21].

In 2008 Mishra et al. [22] dealt with the solution of conduction–radiation heat transfer problems involving variable thermal conductivities. The discrete transfer method has been used for the determination of radiative information for the energy equation that has been solved using the lattice Boltzmann method. Lattice Boltzmann method and the discrete transfer method have been found to successfully deal with the complexities introduced due to variable thermal conductivity.

The effective thermal conductivities of open-cell foam materials were numerically calculated and the predictions were compared with the existing experimental data by **Wang and Pan in 2008 [23]**.

Oches et al. (2008)[24] gave a detailed description of modeling and measurement of the effective thermal conductivity of porous bulk materials at temperatures up to 80˚C and moisture contents below free water saturation as shown in fig. 2-6.



(a) (b)

Figure 2-6 Model predictions and measured data for the thermal conductivity of(a) expanded glass granules and (b) expanded clay as a function of the temperature with normalized water content as parameter [24].

In 2009 Veiseh et al. [25] studied three models from relationship between thermal conductivity and density of mineral wool insulations and the mean temperature used was 10°C. The thermal conductivity might be evaluated indirectly using the apparent density and the established mathematical correlation of these properties (reference curve) using a large amount of experimental data for finding the most appropriate reference curve. Different semi-empirical models were presented and statistically compared with the experimental results to evaluate the best one.

Tseng and Chu (2009) [26] examined the heat transfer mechanisms in 14 samples of vacuum insulation panels (VIPs) to reveal the influence of porous foam structure on VIP performance. Two parameters were proposed to describe the foam structure, namely, the broken cell ratio and the average cell size. It were concluded that radiation heat transfer was influenced predominantly by the broken cell ratio. Radiation heat transfer increases as the broken cell ratio (cell size) increases, but solid conduction decreases as the broken cell ratio (cell size) increases.

A numerical model combined radiation and conduction heat transfer was developed to predict the effective thermal conductivity of fibrous insulation at various temperatures and pressures. Effective thermal conductivities of the fibrous insulation were measured over a wide range of temperature (300–973 K) and pressure (10^{-2}–10^{5} Pa) using a developed apparatus given by **Zhao et al. (2009) [27]**.

In 2010 Domínguez-Muñoz et al. [28] presented polynomial fits for the average thermal conductivity and its standard deviation as functions of density for typical insulation materials. These functions were obtained by processing a large experimental data set, which was compiled in a previous European project headed by the BRE Scottish Laboratory.

Daryabeigi (2010)[6] modeled combined radiation/conduction heat transfer through unbounded alumina fibrous insulation using the diffusion approximation for modeling the radiation component of heat transfer in the optically thick insulation. The validity of the heat transfer model was investigated by comparison to previously reported experimental effective thermal conductivity data over the insulation density range of 24 to 96 kg/m^3 , with a pressure range of 0.001 to 750 torr, and test sample hot side temperature range of 530 to 1360 K as shown in fig. 2-7.

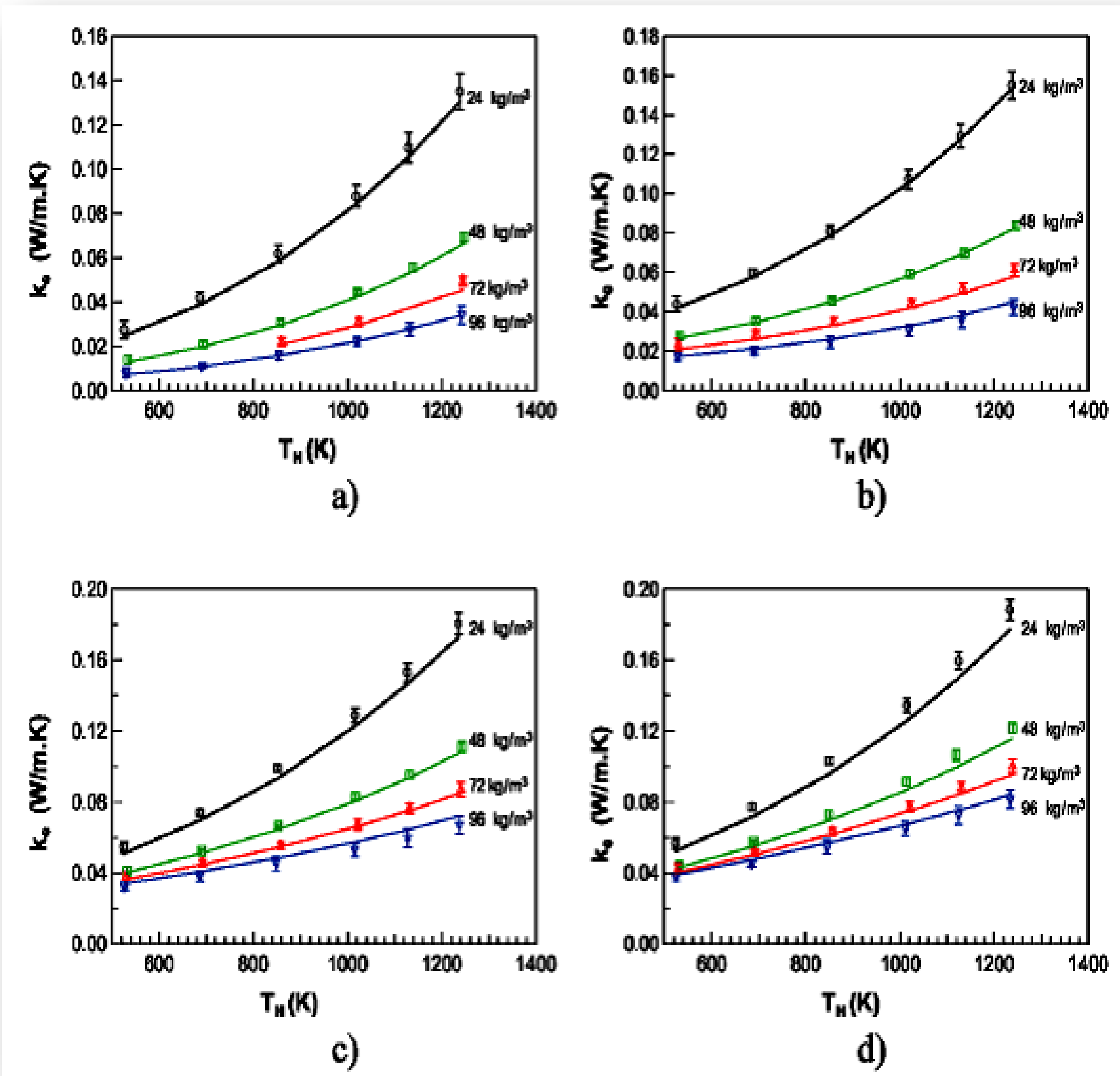


Figure 2-7 Variation of predicted and measured effective thermal conductivity with hot side temperature at various insulation densities at static pressures of: a) 0.1 torr b) 1torr c) 10 torr d) 100 torr [6].

Yüksel et al. (2010) [29] examined the temperature dependence of effective thermal conductivity of samples of glass wool reinforced with aluminum foil. The experiments were realized by the guarded hot plate in temperature differences of 5, 10 and 15 °C and the temperatures of 25 and 40 °C. The results revealed that in the case of reinforcing the aluminum foil, ETC increased with increasing the temperature or changing of temperature difference (5, and 15 °C).

Combined radiation and conduction heat transfer through various high-temperature, high-porosity, unbounded (loose) fibrous insulations was modeled by **Daryabeigi et al. (2010, 2002) [30, 31]** using the diffusion approximation for modeling the radiation component of heat transfer in the optically thick insulations and semi-empirical formulations for modeling the solid conduction contribution of heat transfer in fibrous insulations with the relevant parameters from experimental data.

Song and Yu (2010) [9] used Fourier transform infrared (FTIR) spectroscopy for the characterization of radiative heat properties of fiber assemblies with low bulk densities. Results showed that radiative heat conductivity decreases with bulk density, and increases with the increase of temperature.

2-2 Optimum economic design of insulation:

The economical design is an important issue in thermal engineering work. The optimum design, minimum cost and high efficiency, is the necessary requirement. In order to fulfill this requirement, there are several works tried to find out the criteria for the optimum design such as (Nelson et al. (2009) [32]

and Soponpongpipat et al. (2010) [33]). In case of thermal insulation, various works deal with the optimum thickness of insulation,

In 2002Ahmad [34] discussed the main insulation materials and types of walls structures used in residential buildings in Saudi Arabia. Then a systematic approach for optimization of insulation materials thickness, payback period and cost analysis for different wall structures was developed and applied to two main cities in Saudi Arabia (Riyadh and Dammam).

Kalyon and Sahin (2002) [35] studied a numerical solution for the optimum insulation thickness of a pipe subjected to convective heat transfer that minimizes the heat loss by using the control theory approach and steepest descent method. It was shown that obtaining an optimal thickness variation of insulation that minimizes the heat losses to the ambient using control theory can be executed in a systematic manner.

In 2003Çomaklı and Yüksel [36] investigated the optimum insulation thickness for the coldest cities of Turkey like Erzurum, Kars and Erzincan. The optimization was based on the life cycle cost analysis. As a result considerable energy saving was obtained when the optimum insulation thickness was applied. Savings up to 12.113 $\$/m^2$ of wall area could be maintained for Erzurum.

In 2004 Al-Khawaja [37] determined the optimum thickness of insulation for some insulating materials (wallmate, fiberglass, and polyethylene foam) used in order to reduce the rate of heat flow to the buildings in hot countries.

In 2005 Li and Chow [38] applied a thermo economic optimization analysis with a simple algebraic formula derived for estimating the optimum insulation thickness for tubes of different sizes. The optimization was based on a life-cycle cost analysis. Also the effects of design parameters on the optimum thickness were investigated.

In 2006 Öztürk et al. [39] presented four different thermo-economic techniques for optimum design of hot water piping systems. They were as follows: the first one is a sequential optimization of pipe diameter based on minimization of total cost without considering heat losses and then of insulation thickness based on minimization of cost of insulation and heat losses. The second is simultaneous optimization of pipe diameter and insulation thickness based on the first law of thermodynamics and cost. The third is simultaneous determination of pipe diameter and insulation thickness based on maximization of exergy efficiency without considering cost. Finally, the fourth is simultaneous determination of pipe diameter and insulation thickness based on maximization of exergy efficiency and cost minimization.

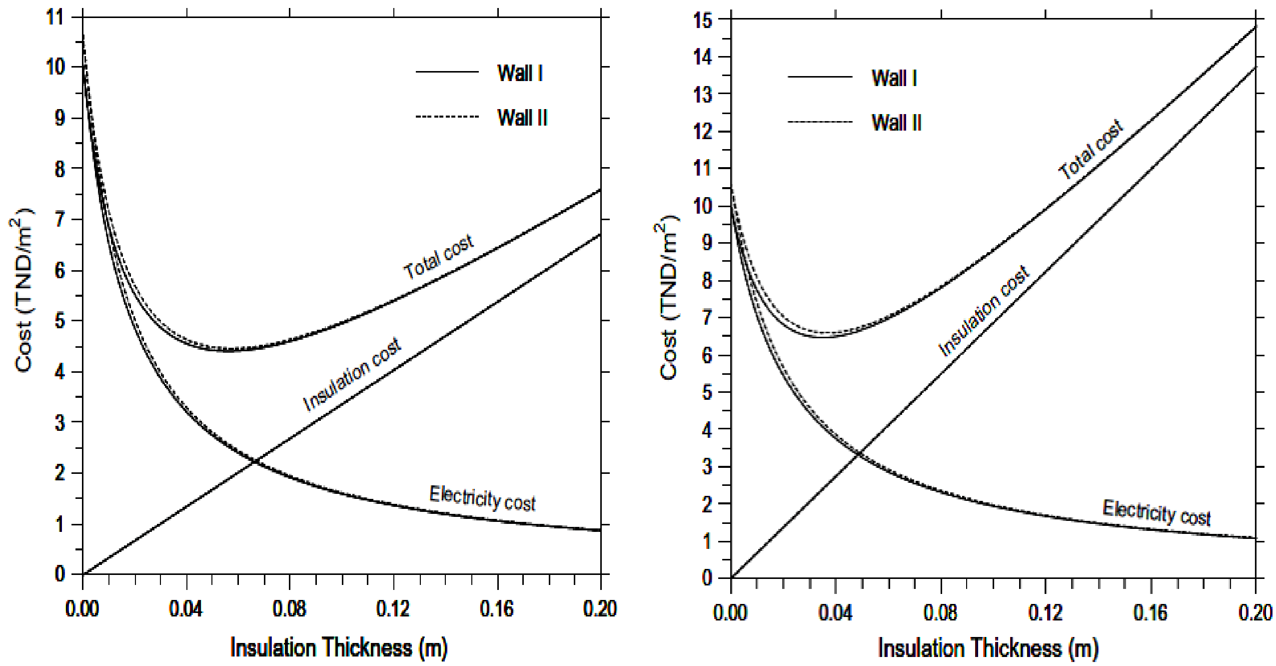
Dombayci et al. (2006) [40] calculated the optimum insulation-thickness of the external wall for the five different energy-sources (coal, natural gas, LPG, fuel oil and electricity) and two different insulation materials (expanded polystyrene, rock wool) for Denizli in Turkey. The optimization was based on a life-cycle cost analysis.

In 2007 Mahlia et al. [41] found that a relationship between the thermal conductivity (k) and optimum thickness (t_{opt}) of insulation material was nonlinear which obeys a polynomial function of $t_{opt} = 0.0818 - 2.973k + 64.6k^2$. This relationship would be very useful for practical use to estimate the optimum thickness of insulation material in reducing the rate of heat flow through building wall by knowing its thermal conductivity only.

In 2009 Jinghua et al. [42] calculated the optimum thicknesses of five insulation materials including expanded polystyrene, extruded polystyrene, foamed polyurethane, perlite and foamed polyvinyl chloride with a typical residential wall using solar-air cooling and heating degree-days analysis and P_1–P_2 economic model.

In 2009 Nelson et al. [32] evaluated the effect of the power generation unit (PGU) efficiency over the primary energy reduction when a Cooling, Heating, and Power (CHP) system is utilized. Regarding to the PGU efficiency, an increase of the efficiency reduces the primary energy use more than proportionally. For example, increasing the PGU efficiency from 0.25 to 0.35 (increase of 40%) can reduce the primary energy use from 5.4% to 16% (increase of 200%).

In 2010 Daouas et al. [43] presented study deals with the determination of optimum insulation thickness for external walls of buildings in Tunisia. Optimization was based on an economic model where a life-cycle cost analysis was carried out using two types of insulation materials and two typical wall structures as shown in fig. 2-8.



(a) **(b)**

Figure 2-8 Variations of insulation cost, electricity consumption cost and total cost with the insulation thickness of: (a) expanded polystyrene and (b) rock wool [43].

Soponpongpipat et al. (2010) [33] showed that the net saving value was the easiest criterion to indicate the optimum design, thus, it was studied that the optimum thickness analysis of air conditioning duct's insulation, which composes of the layer of rubber and fiber glass insulator, was conducted by means of thermo-economics method. In addition, the effects of heat transfer coefficient at inside and outside of duct on the optimum thickness of these insulators were also studied. From the results it was shown that the variation of inside and outside duct's heat transfer coefficient did not affect on the insulator's optimum thickness. In the case of a circular duct of 0.5 m in diameter and the inside and outside convective heat transfer coefficient of 6 and 22 W/m^2.K respectively, the optimum thickness was 0.0032 and 0.125 m for rubber and fiber glass insulator respectively. The net saving was 34,173.00 Baht per meter of duct as shown in fig. 2-9.

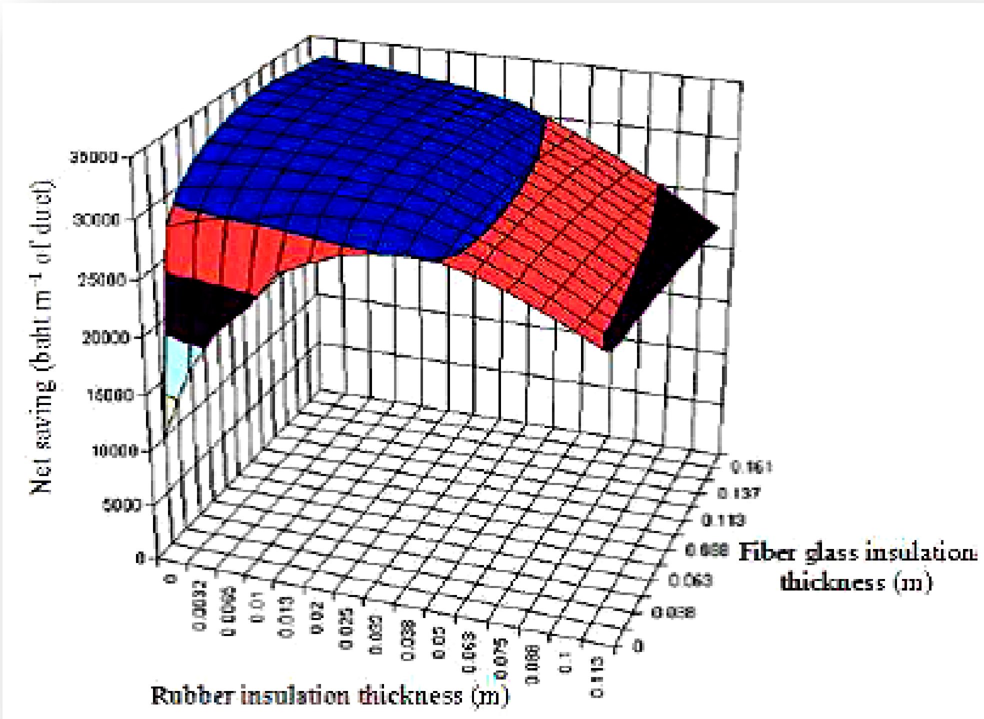


Figure 2-9 The calculation results of net saving of the case study [33].

Bahadori and Vuthaluru (2010)[44] worked, simple-to-use correlation, employing basic algebraic equations which were simpler than current available models involving a large number of parameters, requiring more complicated and longer computations, was formulated to arrive at the economic thickness of thermal insulation suitable for process piping and equipment. The correlation was as a function of steel pipe diameter and thermal conductivity of insulation for surface temperatures at 100 ˚C, 300 ˚C, 500 ˚C and 700 ˚C. They concluded that the average absolute deviation percent of proposed correlation for estimating the economic thickness of the thermal insulator was 2% demonstrating the excellent performance of proposed simple correlation as shown in fig. 2-10.

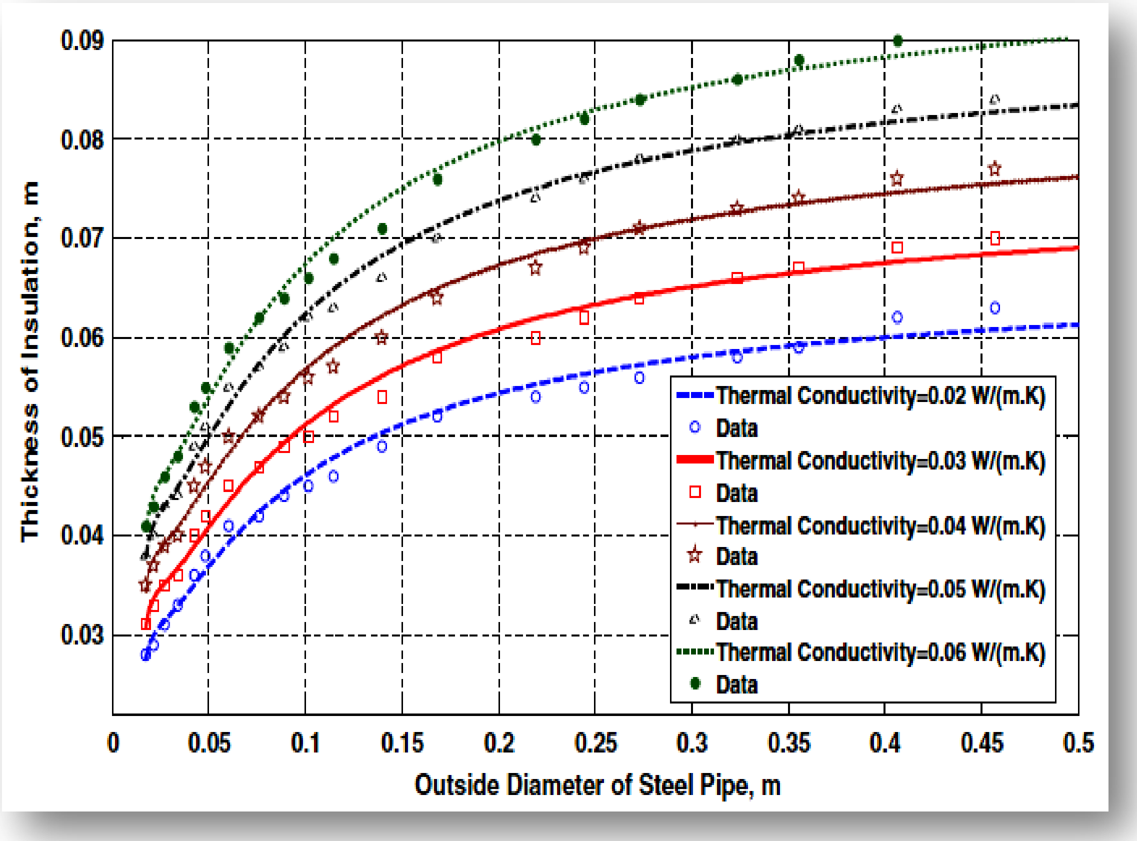


Figure 2-10 Comparison of predicted results from simple correlation with the reported data for surface temperature of 100 ˚C [44].

Aynur and Figen (2010) [45] calculated the optimum insulation thickness of the external wall for four cities from four climate zones of Turkey, energy savings over a lifetime of 10 years and payback periods for the five different energy types and four different insulation materials applied externally on walls.

In 2011 Daouas [46] calculated optimum insulation thickness, energy saving and payback period for a typical wall structure based on both cooling and heating loads. Yearly transmission loads were rigorously estimated using an analytical method based on Complex Finite Fourier Transform (CFFT).A sensitivity analysis shows that economic parameters, such as insulation cost, energy cost, inflation and discount rates and building lifetime, have a noticeable effect on optimum insulation and energy savings.

2-3 Single and multilayer thermal insulation:

The use of insulation materials to decrease heat transfer to/from surfaces has been in practice for many years. Recent concerns of energy conservation and awareness of the limited energy resources encouraged revisit the problem of thermal insulation. Most of the available studies focus on insulating air-conditioned buildings [47–50] and cold stores [51], because of the large potential for energy savings. These studies consider the flat plate or slab as the geometric configuration, presenting the large areas of roofs and facades. On the other hand, studies to improve thermal insulation for cylindrical geometry are few [52–58], in spite of the extensive use of pipelines and cylindrical heat exchangers in refineries, chemical industry, and power plants.

The common practice for piping systems is to use a single layer of an insulating material to reduce the heat transfer and maintain the insulation surface temperature below a prescribed safety limit [52–54]. Spreadsheets have been developed to calculate heat loss from insulated pipes [52]. A comprehensive literature review on the critical insulation thickness has been presented by **Aziz (1997) [53]**. A computer code to determine the optimum thickness of cylindrical insulation has been developed by **Petal and Mehta [54]**.The energy losses in Petal's model include the wind speed and thermal radiation effects on the thickness of the insulation material. Iterative solution of the energy equation on the basis of incremental increase in the insulation thickness results in an optimum thickness for minimum heat losses. This technique does not consider the most economic thickness, for which the annual cost is minimum.

The thermal insulation design economics have been reviewed by **Turner and Malloy (1989) [55]**, presenting extensive tables, and monographs to calculate the economic thickness for large number of parameters. These parameters include combinations of pipe sizes, cost, conductivity, and temperature differences [55]. In addition, the annual operation hours can be a critical parameter in determining the cost-effective insulation thickness [56].

A low thermal conductivity is desirable to achieve a maximum resistance to heat transfer. Therefore, for any given heat loss, a material of low thermal conductivity will be thinner than an alternative material of high conductivity. This is of particular advantage for process pipes because thinner layers of insulation reduce the surface area emitting heat and also reduce the outer surface that requires protection [59].

Multilayer insulation:

In 1988 Gathright and Reeve [60] measured the heat transfer rate, as a function of the number of layers of different types of multilayer insulation, for temperature differences of 284 K to 77 K and 77 K to 4.2 K. It was found that the insulation reduced heat transfer between the higher temperatures but generally had a detrimental effect at the lower temperatures. An exception was double aluminum coated NRC2, which also gave a slight reduction in heat transfer for up to ten layers, at the lower temperatures as shown in fig. 2-11.

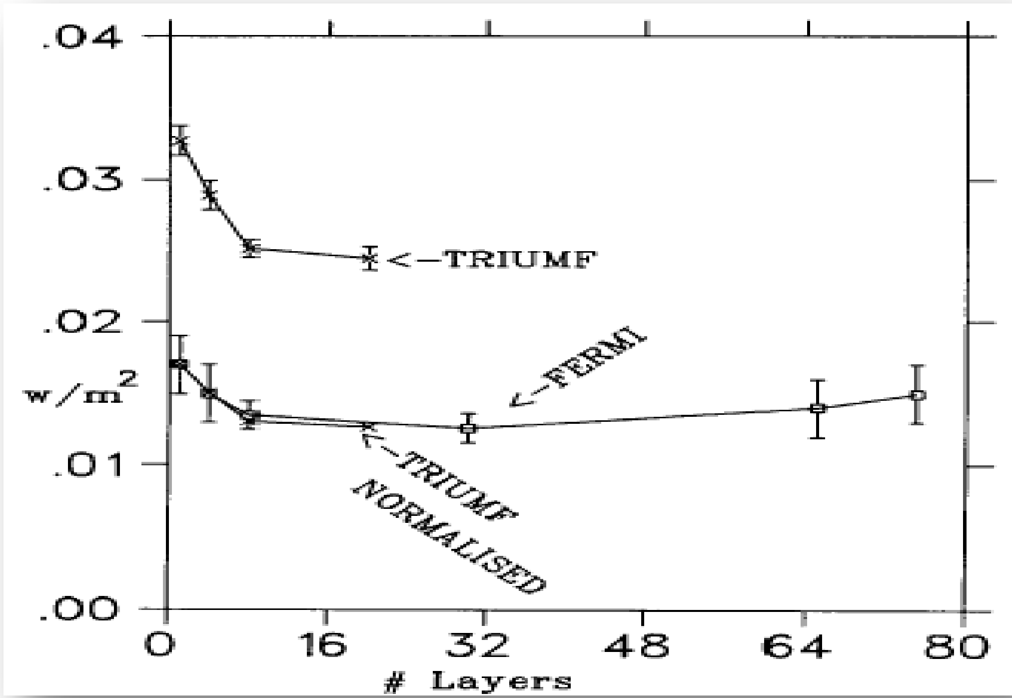


Figure 2-11 Heat transfer versus layers at 77K to 4.2K [60].

In 2000 Zaki and Turki [58] studied thermal insulation economic analysis for a system of pipelines insulated by different materials composite layers. The analysis was based on an explicit nonlinear cost function that includes the annual energy losses and the insulation initial costs.

The results showed clearly that the present insulation thicknesses were insufficient and far from being economical. An example was studied where cost function for the pipeline with superheated steam at 400˚C and 50-mm-thick rock wool was 95.55 $/m. This value drops to 41.8 $/m if the optimum thickness, 283 mm, of the same rock wool was used and reduces to 48 $/m when using a 195-mm calcium silicate layer. Further decrease in cost function was achieved by using two composite layers of calcium silicate, 39 mm thick, and rock wool, 244 mm thick as shown in fig. 2-12.

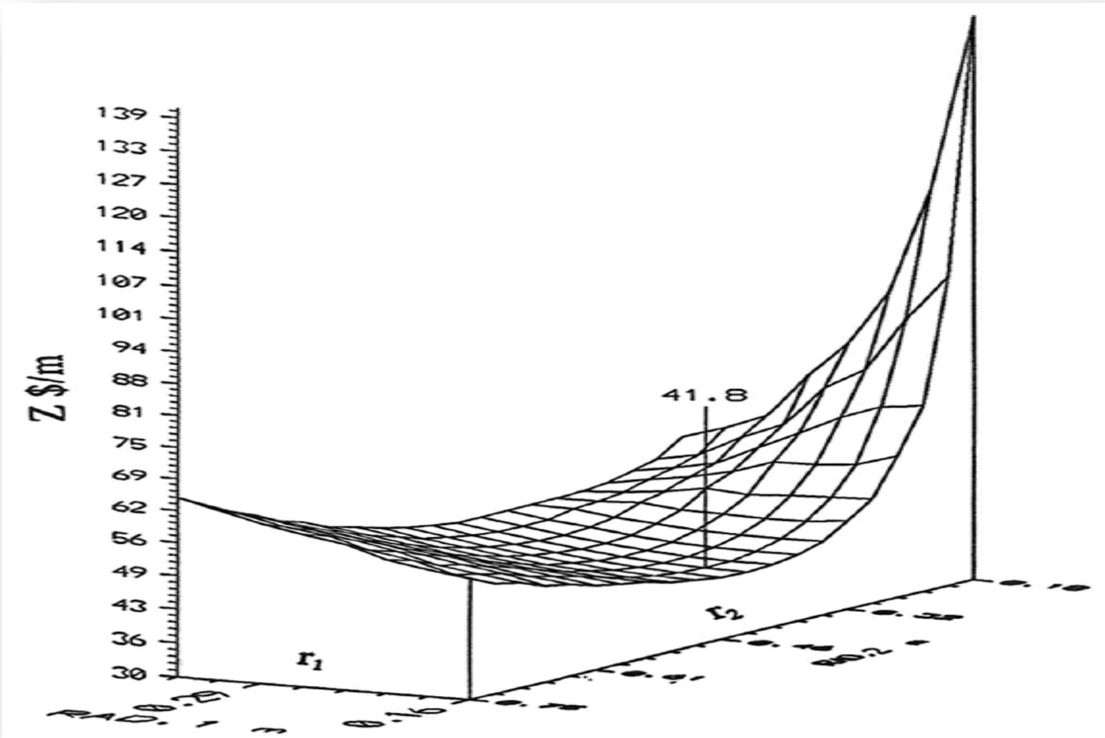


Figure 2-12 Cost function (Z) for a single pipeline with two insulation layers (RAD.1= calcium silicate and RAD.2= rock wool) for superheated steam pipeline [58].

Chorowski et al. (2000) [61] developed a mathematical model to describe the heat flux through multilayer insulation from 80 K to 4.2 K. The total heat flux between the layers was the result of three distinct heat transfer modes: radiation, residual gas conduction and solid conduction. The mathematical model enables prediction of MLI behavior with regard to different MLI parameters, such as gas insulation pressure, number of layers and boundary temperatures.

In 2004 Markus et al. [62] was concerned with multilayer thermal insulations (MTI) for application to high-temperature fuel cells operating at temperatures higher than 650 ℃.Therefore, solid and gas conduction as well as radiation had to be considered.

In 2006 Augusto et al. [63] made a thermodynamic analysis to minimize the power consumption of cryogenic systems containing refrigerated shields, considering all the design variables (temperatures, number of layers/cm present in the insulation and refrigeration efficiencies). It was concluded that the use of two shields yields lower refrigerating power consumption than using only one.

Peng and Cheng (2006) [64] investigated the combined radiation and conduction heat transfer in multilayer perforated insulation material (MLPIM) used in space. The effect of the main parameter, such as layer density, on the thermal performance has been analyzed. The study on the performance of MLPIM presented active instruction to improve the insulating performance and accomplish optimum design of MLPIM. The variation of conductive heat flux and radiant heat flux with layer density is shown in Fig. 2-13. Fig. 2-14 shows the variation of heat loss flux and effective thermal conductivity with layer density.

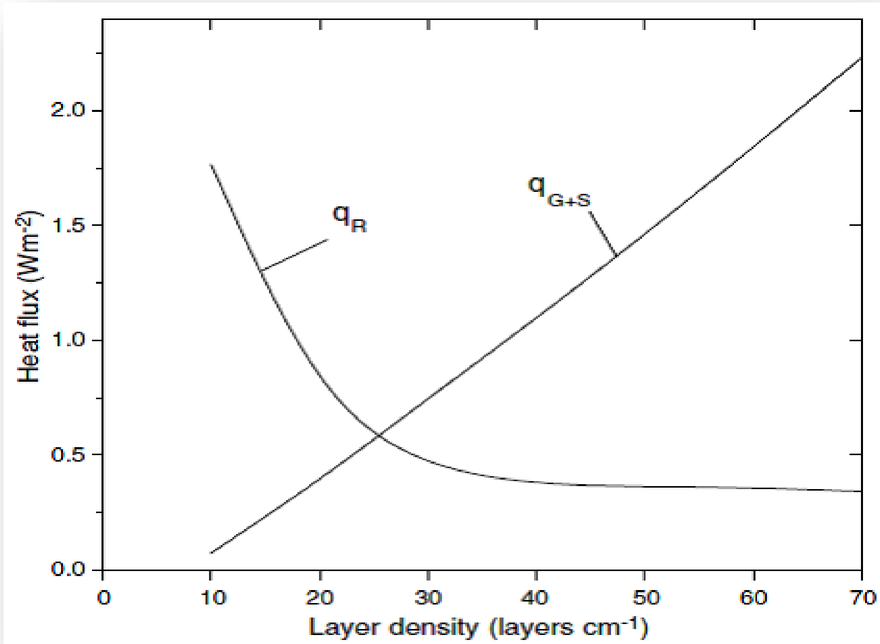


Figure 2-13 Variation of conductive heat flux and radiant heat flux with layer density [64].

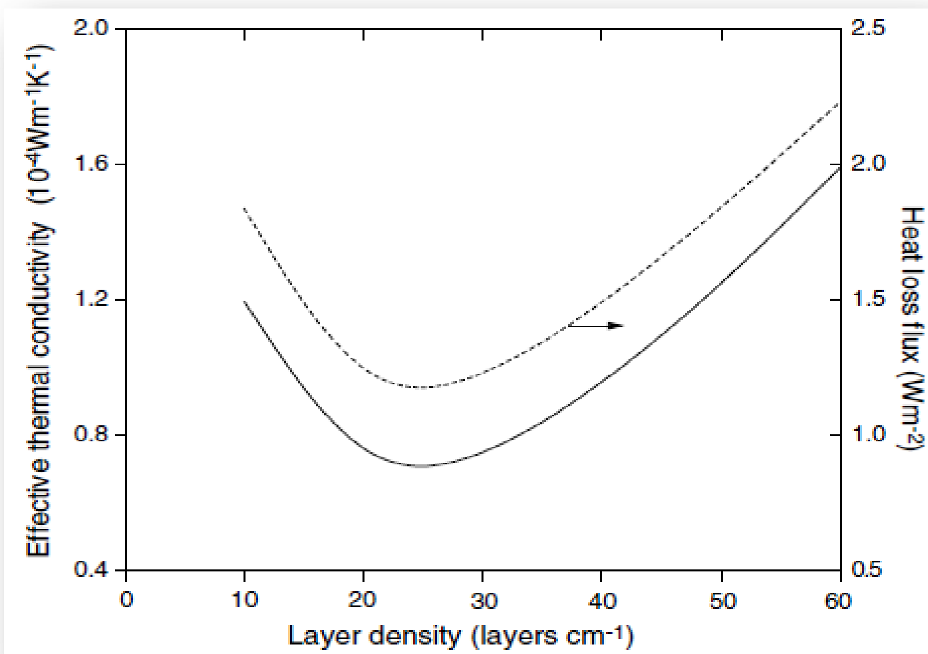


Figure 2-14 Variation of effective thermal conductivity and heat loss flux with layer density [64].

Ohmori (2006)[65] investigated the distribution of contact pressure between adjacent layers by using experimental data obtained by the laboratory scale calorimeter; the results of analysis have been applied to evaluate the thermal performance of multiplayer insulation (MLI) around a horizontal cylinder. And the non-dimensional contact pressure parameter has been introduced as a useful parameter to evaluate and compare the thermal performance among different kinds of MLI as shown in fig. (2-15). Thermal performance of MLI was affected by contact pressure between adjacent layers.

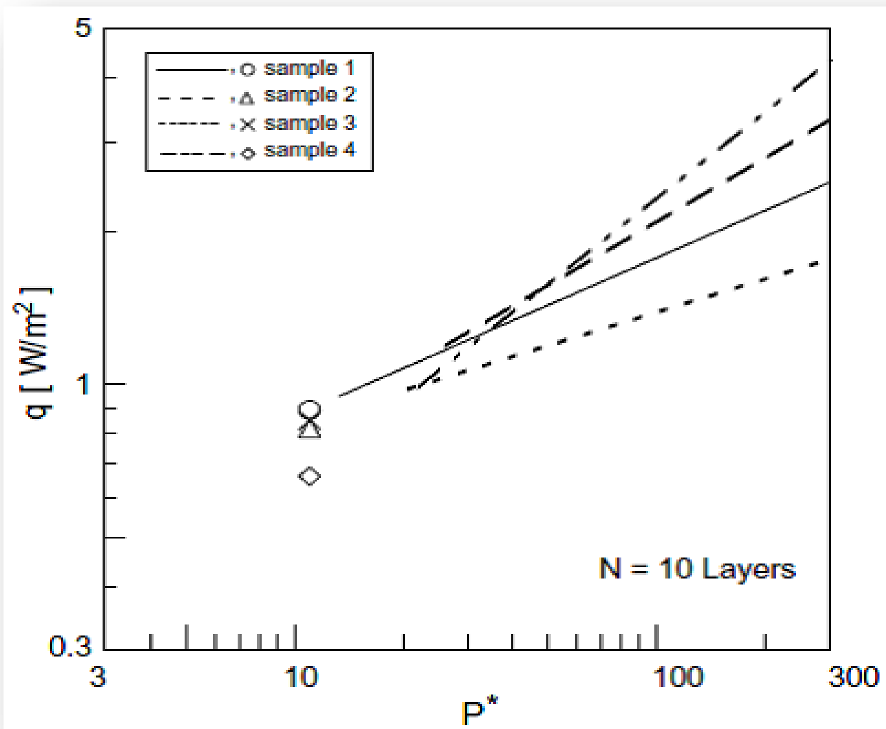


Figure 2-15 Heat flux q through MLI specimens as a function of applied non-dimensional contact pressure parameter P* [65].

In 2007 Choi et al. [66] studied the thermal properties of composite insulation materials using thermal conductivity test equipment by heat flux method, and performed quantitative evaluation on the measurement precision and uncertainty of composite materials.

In 2010 Chen and Yu [67] analyzed the theoretical heat transfer of flexible multilayer thermal insulation material for high–low temperature resistance. It was concluded that the numerical model of the flexible multilayer material can be applied in practical estimation. The model was greatly useful to design and improve the properties of the flexible multilayer thermal insulation material.

Johnson et al. (2010) [68] tested different combinations of aerogel blanket and multilayer insulation materials at the Cryogenics Test Laboratory of NASA Kennedy Space Center. The best performance was obtained with aerogel layers located in the middle of the blanket insulation system.

Numerous tests of various multilayer insulation systems have indicated that there are optimal densities for these systems. **Johnson and Weisend (2010) [69]** discovered that by converting the equations from heat flux to thermal conductivity using Fourier's Law, the equations became functions of layer density, temperatures, and material properties only. The thickness and number of layers of the blanket were merged into a layer density. These equations were then differentiated with respect to layer density. By setting the first derivative equal to zero, and solving for the layer density, the critical layer density was determined.

Chapter three

Theoretical background

This chapter is divided into three main sections. The first section briefly presents the models of thermal conductivity for three types of insulation materials (rock mineral wool, calcium silicate, cellular glass). It starts by introducing the effect of density and temperature on thermal conductivity of the mineral wool, and then effect of temperature on thermal conductivity at constant density for the calcium silicate and cellular glass. In the second section, the subject of heat loss in the presence of single and multilayer insulation is analyzed. The final part of the chapter introduces the economical thickness for insulation material.

3-1Properties of thermal insulation materials

Three types of insulation materials are used as follows:

- **Calcium Silicate Insulation:** Calcium silicate is a very rigid, high-density material used exclusively for applications above 120°C. This insulation material has been a standard for high temperature applications for many years. Compressive strengths are very good, and it is noncombustible. It is suitable for temperatures from 120°C up to 538°C. Calcium silicate is manufactured from slurry that is poured into molds to make various pipe covering shapes [1, 2].
- **Mineral wool insulation:** This is obtained by melting rocks and slag in furnaces at very high temperatures. The maximum temperature of use is

almost as high as that for calcium silicate, but its compressive strength is much lower. It is available in both rigid molded forms for piping or vessels and as flexible blankets for irregular surfaces [2].

- **Cellular Glass Insulation:** Cellular glass insulation is a high strength; versatile insulation used in temperature services up to 538˚C.Cellular glass is applied to hot piping in single or double layers [1].

3-2 Thermal conductivity models

The conductivity of three insulations as well as heat loss has been studied for each case:

1- Mineral wool with nine values of density (40, 50, 70, 90, 100, 110, 120, 150 and 200 kg/m^3) for 31 thicknesses (from 5 through 155 mm) with temperature range (from 10 to 400 ˚C).

2- Calcium Silicate, with constant density 256 kg/$m^3$for the same thicknesses with temperature range (from -17.75 to 537.75 ˚C).

3- Cellular glass, with constant density 120 kg/m^3 for the same thicknesses with temperature range (from -184.4 to 482.2 ˚C).

3-2-1Effect of density on thermal conductivity

The data for mineral wool as in table A-1indicate the general trend associated with loose-fill thermal insulation. That is, as density increases from the minimum possible value upwards, k_{ins} decreases to a minimum and then increases. Therefore, the k_{ins} variation with density should satisfy the general empirical relationship associated with this characteristic behavior of materials as given in eq. (3-1) [25, 28, and 70].

$$k_{ins} = A + B\rho + C/\rho \qquad \dots (3\text{-}1)$$

Where A, B and C are constant. Using the Method of Least Squares in STATISTICA program, the data for mineral wool for each test condition were fitted in the form of eq. (3-1) and the empirical constants were determined. The resulting equations are as follows in table 3-1:

Table 3-1 Thermal conductivity equations (isothermal equations) and correlation coefficients as function of mineral wool insulation density for each temperature.

Temperature, ˚C	Thermal conductivity equations, W/m. K	R^2
10	$k_{ins} = 0.019405 + 5.43538 * 10^{-5}\rho + \frac{0.491447}{\rho}$ … (3-2)	0.90721
20	$k_{ins} = 0.026590 + 3.79660 * 10^{-5}\rho + \frac{0.341145}{\rho}$ … (3-3)	0.85195
30	$k_{ins} = 0.029280 + 2.68896 * 10^{-5}\rho + \frac{0.287181}{\rho}$ … (3-4)	1.00000
40	$k_{ins} = 0.034770 + 6.08480 * 10^{-6}\rho + \frac{0.140478}{\rho}$ … (3-5)	0.63076
50	$k_{ins} = 0.038560 - 7.16950 * 10^{-6}\rho + \frac{0.087856}{\rho}$ … (3-6)	0.77757
60	$k_{ins} = 0.041980 - 1.66702 * 10^{-5}\rho - \frac{0.002912}{\rho}$ … (3-7)	0.84039
100	$k_{ins} = 0.039908 - 1.53470 * 10^{-5}\rho + \frac{0.411917}{\rho}$ … (3-8)	0.83491
200	$k_{ins} = 0.069620 - 9.89330 * 10^{-5}\rho + \frac{0.022189}{\rho}$ … (3-9)	0.90075
300	$k_{ins} = 0.084810 - 1.08030 * 10^{-4}\rho + \frac{0.019871}{\rho}$ … (3-10)	0.96610
400	$k_{ins} = 0.117430 - 1.85580 * 10^{-4}\rho - \frac{0.37675}{\rho}$ … (3-11)	0.99741

3-2-2 Effect of temperature on Thermal conductivity

When temperature increases, changes in the thermal conductivity take place for each heat transfer mechanism [17]. In general, thermal conductivity equations of insulation are as follows:

- **Mineral wool insulation:** the data for mineral wool insulation as in table A-1 at constant density show increasing k_{ins} with temperature. In order to determine k_{ins} variation with mean test temperature, the insulation data were fitted to a relationship of the general form shown by eq.(3-12);

$$k_{ins}=a+b\,T_m+c\,T_m^2 \qquad \ldots (3\text{-}12)$$

Where a, b and c are constants. The resulting equations are as follows in table 3-2.

Table 3-2 Thermal conductivity equations and correlation coefficients function of mineral wool insulation temperature for each density.

Density, kg/m^3	Thermal conductivity equations, W/m. K	R^2
40	k_{ins} = -0.003170+1.13986*$10^{-4}T_m$+5.9239*$10^{-8}T_m^2$ … (3-13)	0.99940
50	k_{ins} = -0.005450+1.25860*$10^{-4}T_m$+4.4683*$10^{-8}T_m^2$ … (3-14)	0.99952
70	k_{ins} = -0.024830+2.18250*$10^{-4}T_m$–5.2866*$10^{-8}T_m^2$ … (3-15)	0.99897
90	k_{ins} = 0.001851+8.26570*$10^{-5}T_m$+8.5611*$10^{-8}T_m^2$ … (3-16)	0.99580
100	k_{ins} = 0.013570+2.48120*$10^{-5}T_m$+1.4142*$10^{-7}T_m^2$ … (3-17)	0.99724
110	k_{ins} = 0.001673+8.40740*$10^{-5}T_m$+7.4634*$10^{-8}T_m^2$ … (3-18)	0.99634
120	k_{ins} = 0.020818-2.66735*$10^{-6}T_m$+1.5995*$10^{-7}T_m^2$ … (3-19)	0.99746
150	k_{ins} = 0.028205 -3.02630*$10^{-5}T_m$+1.7678*$10^{-7}T_m^2$ … (3-20)	0.99753
200	k_{ins} = 0.022652+1.19136*$10^{-5}T_m$+1.0469*$10^{-7}T_m^2$ … (3-21)	0.99831

- **Calcium silicate insulation:** The data of calcium silicate in table A-2 at ρ=256 kg/m^3 were fitted to a relationship of the general form in eq. (3-22);

$$k_{ins} = 0.0304+1.17*10^{-4}T_m-1.42*10^{-7}T_m^2+1.71*10^{-10}T_m^3 \qquad \dots (3\text{-}22)$$

Correlation coefficient=0.999999

- **Cellular glass insulation:** The data of cellular glass in table A-3 at ρ=120 kg/m^3 were fitted to a relationship of the general form in eq.(3-23);

$$\mathrm{k_{ins}} = 9.66184*10^{-3}+3.7803*10^{-5}\mathrm{T_m}+3.53567*10^{-7}\mathrm{T_m^2}-$$

$$5.47487*10^{-10}\mathrm{T_m^3}+5.82075*10^{-13}\mathrm{T_m^4} \qquad \dots (3\text{-}23)$$

Correlation coefficient=0.999984

3-2-3 Effect of temperature and density on Thermal conductivity:

To determine k_{ins} variation with mean test temperature, the respective coefficients of A, B, and C, from the isothermal equations in table 3-1were fitted to relationships of the general forms;

$$\mathrm{A\ (T)} = a_1 + b_1\, T_m + c_1\, T_m^2 \qquad \dots (3\text{-}24)$$

$$\mathrm{B\ (T)} = a_2 + b_2\, T_m + c_2\, T_m^2 \qquad \dots (3\text{-}25)$$

$$\mathrm{C\ (T)} = a_3 + b_3\, T_m + c_3\, T_m^2 \qquad \dots (3\text{-}26)$$

Example: A (10) =0.019405, A (20) =0.026590

B (10) = $5.43538 * 10^{-5}$, B (20) = $3.79660 * 10^{-5}$

C (10) = 0.491447, C (20) = 0.341145 …..etc.

Where A (T), B (T) and C (T) are an expression for temperature dependence; a_1 through to c_3 is constants and T_m is the mean temperature. In general empirical relationships are formed for determining k_{ins} in terms of temperature and density, eq. (3-27)

$$k_{ins} = (a_1+b_1T_m+c_1\,T_m^2) + (a_2+b_2T_m+c_2T_m^2)\,\rho + (a_3+b_3T_m+c_3\,T_m^2)/\rho \qquad \ldots (3\text{-}27)$$

The resulting equation is as follows, i.e. eq. (3-28);

$$k_{ins} = (-0.02734-1.7*10^{-4}T_m+6.10802*10^{-8}T_m^2) + (2.896*10^{-4}-1.0401*10^{-6}T_m+5.2235*10^{-10}T_m^2)\,\rho + (0.293269+5.76*10^{-4}T_m-2.20441*10^{-6}T_m^2)/\rho \qquad \ldots (3\text{-}28)$$

Correlation coefficient= 0.99645

3-3 Case studies

Three case studies are used to illustrate how the heat loss changes with thickness of single and multilayer insulation:

- **First case:** hot water at 98°C flows through a 2 -inch Schedule 40 horizontal steel pipe (D_i =0.0525m, D_0 =0.0603m) and is exposed to atmospheric air at 20˚C. The water velocity is 0.25 m/s [73].
- **Second case:** A 3-inch Schedule 40 pipe (D_i =0.0779m, D_0 =0.0889m) contains gas oil at 190˚C, assuming that the film convection coefficient for the internal side is 200 W/m^2 ·K and the ambient temperature is 20˚C. The metal conductivity is 80 W/ m ·K [2].
- **Third case:** A steam flowing through 6-inch Schedule 40 steel pipe (D_i =0.154m, D_0=0.168m) where the outside surface temperature is 200˚C and is exposed to atmospheric air at 20˚C [1].

3-4 Calculations:

3-4-1 Introduction:

For computation of heat losses, economical thickness of insulating material and to decide on the quantity of the materials required, the following data are required

1. Operating temperature.
2. Surface temperature.
3. Ambient temperature.
4. Existing thickness of insulation, etc.

Using **MATLAB** code (Appendix B and C) to calculate the three cases of different insulations in single layer and multilayer. Figure 3-2 shows the block diagram of the program steps for single and two-layer insulation.

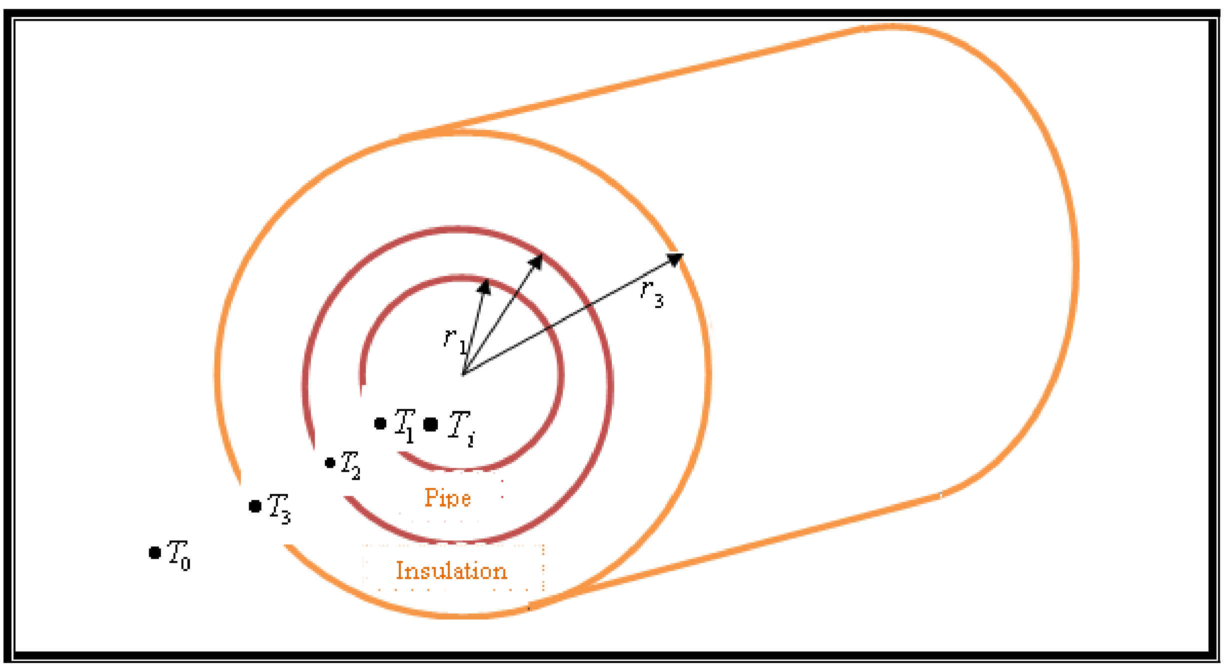


Figure 3-1 Sketch of steel pipe with single layer insulation.

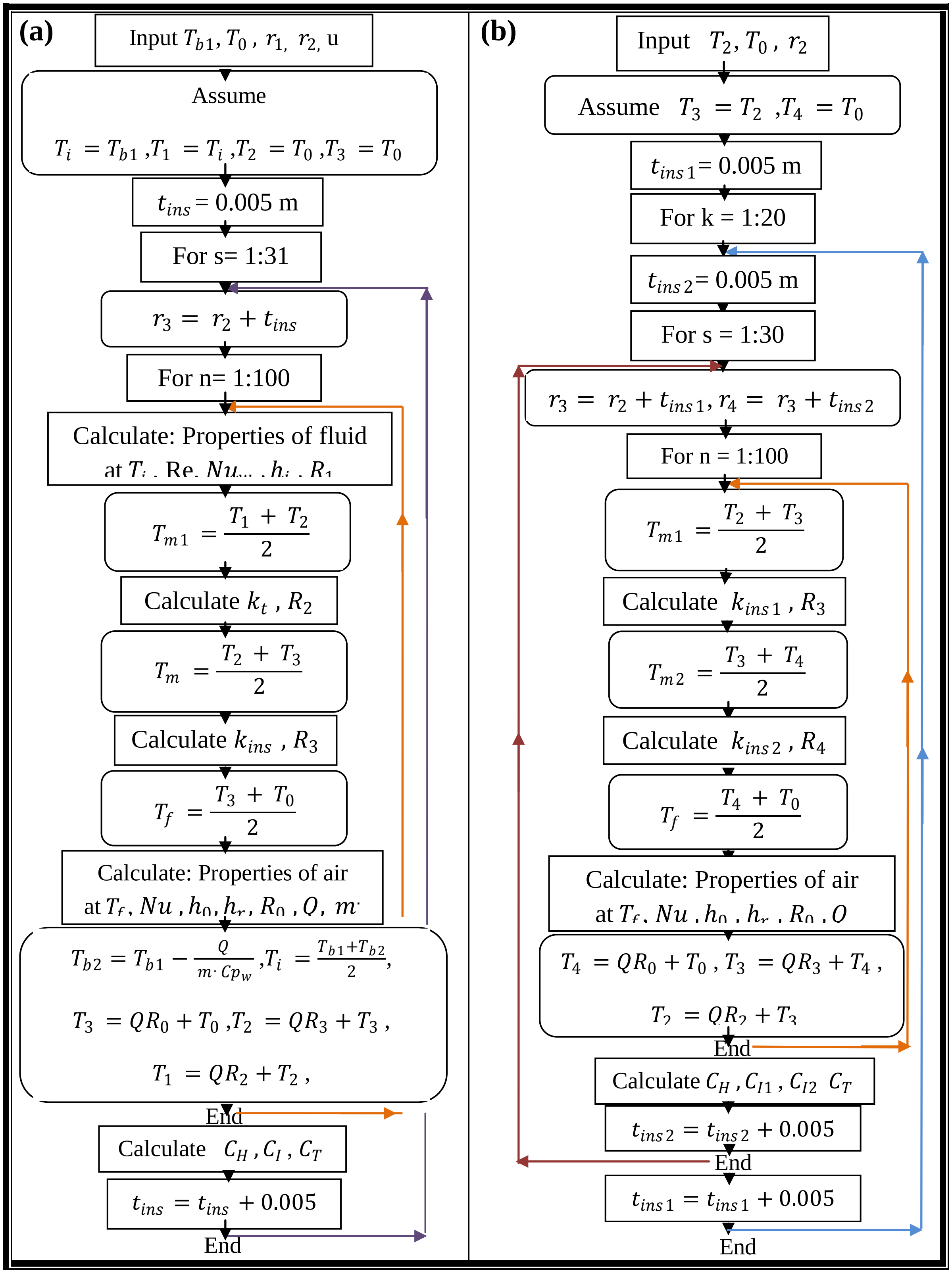


Figure 3-2 Block diagram illustrates the steps of program for (a) single and (b) two- layer insulation.

3-4-2 Heat transfer from pipe with and without insulation

Two case studies for heat transfer with the influence of heat radiation being neglected and considered, these two cases investigated for comparison of the results between them.

A. Cases with the influence of heat radiation being neglected

While the influence of outside surface heat radiation is not considered, it can be seen from fig. 3-1 that the total thermal resistance per unit length is:

$$\sum R_{th} = \frac{1}{2\pi r_1 h_i} + \frac{\ln(r_2/r_1)}{2\,\pi k_t} + \frac{\ln(r_3/r_2)}{2\,\pi k_{ins}} + \frac{1}{2\pi r_3 h_0} \quad \ldots (3\text{-}29)$$

And the relative heat transfer rate with heat convection and heat conduction terms is:

$$Q = \frac{T_i - T_0}{\sum R_{th}} = \frac{T_3 - T_0}{\frac{1}{2\pi r_3 h_0}} \quad \ldots (3\text{-}30)$$

Equation (3-30) implies that heat transfer rate Q and insulted surface temperature T_3 for situations without considering the influence of outside surface heat radiation can be readily obtained. The heat transfer rate of a pipe without insulation of situations not considering the influence of outside surface heat radiation can be written as:

$$Q_0 = \frac{T_i - T_0}{\frac{1}{2\pi r_1 h_i} + \frac{\ln(r_2/r_1)}{2\,\pi k_t} + \frac{1}{2\pi r_3 h_0}} \quad \ldots (3\text{-}31)$$

Equations (3-30) and (3-31) lead to find the percentage of energy saving ($\%Q_{sav}$) of an insulated pipe while neglecting heat radiation is being:

$$\%Q_{sav} = \frac{Q_0 - Q}{Q_0} * 100 \quad \ldots (3\text{-}32)$$

B. Cases with the influence of heat radiation being considered

While the influence of outside surface heat radiation is considered, the complete heat transfer rate of an insulated pipe can be expressed as:

$$Q_a = \frac{T_i - T_{3a}}{\frac{1}{2\pi r_1 h_i} + \frac{\ln(r_2/r_1)}{2\pi k_t} + \frac{\ln(r_3/r_2)}{2\pi k_{ins}}} \quad \dots (3\text{-}33)$$

Where T_{3a} is the actual surface temperature after considering heat radiation (T_{3a} calculated depending on Q_a). Then the surface convective heat transfer rate becomes:

$$q_c = 2\pi r_3 h_0 (T_{3a} - T_0) \quad \dots (3\text{-}34)$$

The surface radiation heat transfer rate is:

$$q_r = 2\,\pi\, r_3 \varepsilon\, \sigma\, (T_{3a}^4 - T_0^4) = 2\,\pi\, r_3 h_r (T_{3a} - T_0) \quad \dots (3\text{-}35)$$

Where σ = Stefan Boltzmann constant= $5.67*10^{-8}$W/m^2.K^4

ε = 0.1for low emissivity [78]

From equations (3-34) and (3-35), under the conditions of ≠0, $T_{3a} \neq T_3$, $q_c \neq 2\pi r_3 h_0 (T_3 - T_0)$, the complete heat transfer rate of an insulated pipe can be written as:

$$Q_a = q_c + q_r \quad \dots (3\text{-}36)$$

Therefore, the complete heat transfer rate Q_a and the surface temperature T_{3a} can be readily deduced from equations (3-33) to (3-36).

The external radiation heat convection coefficient is defined from eq. (3-35) as:

$$h_r = \varepsilon\, \sigma\, (T_{3a}^2 + T_0^2)(T_{3a} + T_0) \quad \dots (3\text{-}37)$$

With both Q and Q_a being available, we can now further define the error of heat transfer rate (%EQ) generated by neglecting heat radiation effect as:

$$\%EQ = \frac{Q_a - Q}{Q_a} * 100 \qquad \dots (3\text{-}38)$$

While the influence of outside surface heat radiation is considered, the complete heat transfer rate of a pipe can be written as:

$$Q_{a0} = \frac{T_i - T_{2a}}{\frac{1}{2\pi r_1 h_i} + \frac{\ln(r_2/r_1)}{2\pi k_t}} \qquad \dots (3\text{-}39)$$

Where T_{2a} is the actual surface temperature of a pipe with the consideration of the influence of heat radiation (T_{2a} calculated depends on Q_{a0}). Additionally, its surface convective heat transfer rate is:

$$q_{c0} = 2\pi r_2 h_0 (T_{2a} - T_0) \qquad \dots (3\text{-}40)$$

And the surface radiation heat transfer rate is:

$$q_{r0} = 2\pi r_2 \varepsilon_0 \sigma (T_{2a}^4 - T_0^4) = 2\pi r_2 h_r (T_{2a} - T_0) \qquad \dots (3\text{-}41)$$

ε_0 = 0.8 emissivity for steel pipe [78]

From equations (3-40) and (3-41), the complete heat transfer rate of pipe can be expressed as:

$$Q_{a0} = q_{c0} + q_{r0} \qquad \dots (3\text{-}42)$$

The complete heat transfer rate Q_{a0} and surface temperature T_{2a} of pipe in situations considering the influence of outside surface heat radiation can be readily obtained from equations (3-39) to (3-42). And finally, percentage energy saving ($\%Q_{asav}$) of an insulated pipe while considering heat radiation can be expressed by:

$$\%Q_{asav} = \frac{Q_{a0} - Q_a}{Q_{a0}} * 100 \qquad \dots (3\text{-}43)$$

3-4-3 Economical Thickness

Economic thickness of insulation is that thickness of insulation, which when applied to a heated surface will provide the minimum total dollar cost per year of the combined insulation cost and energy cost [71]. When the reason for the use of thermal insulation is to reduce energy expenses, optimal thickness results from an economic calculation that considers insulation and energy costs. This calculation is greatly influenced by many parameters such as ambient temperatures, process conditions, maintenance and installation cost, etc. [2].

For calculations, total cost must comprise cost of heat loss and installing cost (material and labor) of insulation as follows:

- **Cost of heat loss, \$/m. yr.:** the data [72] of heat loss and cost were fitted to a relationship of the general form given by eq. (3-44);

$$C_H = (0.0527596*Q+0.00097868)*3.2808 \quad \ldots (3\text{-}44)$$

Correlation coefficient= 1.00000

- **Installing cost (material and labor) of insulation, \$/m. yr.:** the data [72] of thickness and cost for each insulation and pipe diameter (2, 3 and 6 in)were fitted to a relationship of the general forms shown in equations (3-45) to (3-53);

- **Mineral wool:**

NPS=2 in

$$C_{I1} = (19.0551* t_{ins} +0.809)*3.2808 \quad \ldots (3\text{-}45)$$

Correlation coefficient= 0.998984

NPS=3 in

C_{I1}= (21.4623* t_{ins} +0.943)*3.2808 … (3-46)

Correlation coefficient= 0.9977

NPS=6 in

C_{I1}= (26.8744* t_{ins} +1.11456)*3.2808 … (3-47)

Correlation coefficient= 0.991568

- **Calcium silicate:**

NPS=2 in

C_{I2}= (20.36* t_{ins} +0.88)*3.2808 … (3-48)

Correlation coefficient= 0.990045

NPS=3 in

C_{I2}= (24.1057* t_{ins} +0.968)*3.2808 … (3-49)

Correlation coefficient= 0.985555

NPS=6 in

C_{I2}= (28.1777* t_{ins} +1.18500)*3.2808 … (3-50)

Correlation coefficient= 0.996252

- **Cellular glass:**

NPS=2 in

C_{I3}= (22.6884*t_{ins} +0.972)*3.2808 … (3-51)

Correlation coefficient= 0.992087

NPS=3 in

C_{I3}= (24.9044*t_{ins} +1.144)*3.2808 … (3-52)

Correlation coefficient= 0.993018

NPS=6 in

C_{I3}= (31.2036*t_{ins} +1.32900)*3.2808 … (3-53)

Correlation coefficient= 0.998593

❖ **Total cost, \$/m. yr.:** Simulation of installing and heat loss cost as given by eq. (3-54);

$$C_T = C_H + C_I \qquad \dots (3\text{-}54)$$

When the reason for thermal insulation is personal protection, calculations are performed to have a temperature on the outer insulation face not higher than 60°C [2].

3-5 Solved examples

To clarify the applications, two solved examples are reported below:

Example 1:

For a single layer calculate the optimum thickness of insulation for the first case using mineral wool insulation at a density of 40 kg/m^3.

Solution: Evaluate Reynolds number at the bulk temperature to determine the flow regime. It is assumed that $T_i = T_{b1}$, the true or correct (trial & error) assumption of $T_i = 97.995$˚C (371.145 K) from table 3-3 to calculate the properties of water (ρ_w, μ_w, k_w, Pr_w, and Cp_w)

Where:

ρ_w = 959.7850 kg/m^3 $\quad$ μ_w = 3.0202*10^{-4} kg/m. s $\quad$ k_w = 0.68024 W/m. K

Pr_w = 1.7935 $\quad$ Cp_w = 4.2089*10^3 J/kg. K

$$Re = \frac{\rho_w u\ (2r_1)}{\mu_w} = \frac{959.7850 * 0.25 * 0.0525}{3.0202 * 10^{-4}} = 4.1710 * 10^4$$

Table 3-3 Properties of water are function of temperature calculated by fitting data [73] where the temperature in K.

Properties of water equations,	Units	R^2
ρ_w= -0.002235T^2+ 0.952T + 914.323 … (3-55)	kg/m^3	0.999945
μ_w= 3.395*$10^{-12}T^4$ - 5.325*$10^{-9}T^3$ + 3.15*$10^{-6}T^2$ 0.000835T + 0.08412 … (3-56)	kg/m. s	0.999629
k_w=-7.05256*$10^{-6}T^2$+0.005647T-0.444139 … (3-57)	W/m. K	0.994396
Pr_w=-1.70626*$10^{-6}T^3$+0.00214T^2-0.9035T+129.573 … (3-58)	-	0.999491
Cp_w=6.8107*$10^{-5}T^3$ - 0.06609T^2+21.9623T+1679.55 … (3-59)	J/kg. K	0.999568

Note: Properties of water illustrates in figs. A- 1 through A-5.

Inside heat transfer coefficient:

As the flow is turbulent, therefore use eq. (3-60) to calculate inside heat transfer coefficient [73]

$$Nu_w = \frac{\mathrm{hi}\ (2\mathrm{r}_1)}{\mathrm{k_w}} = 0.023\ \mathrm{Re}^{0.8}\mathrm{Pr}_\mathrm{w}^{0.4} = 144.3444 \qquad \ldots (3\text{-}60)$$

$$hi = \frac{k_w}{2r_1} Nu_w = \frac{0.68024}{0.0525} * 144.3444 = 1870.3 \mathrm{W}/m^2.\mathrm{K}$$

Thermal resistance:

For unit length of the pipe the thermal resistance on the inside (R_1) is

$$R_1 = \frac{1}{2\pi r_1 h_i} = \frac{1}{\pi\ (0.0525)\ (1870.3)} = 0.0032\ \mathrm{K/W}$$

To calculate mean temperature of pipe (T_{mp}): assume inside pipe temperature (T_1) is equal bulk temperature (T_i) and so outside pipe temperature (T_2) equals ambient temperature (T_0), the true or correct (trial & error) assumption of T_1=371.07 K (97.92˚C) & T_2=371.06 K (97.91˚C).

$T_{mp} = \frac{(T_1+T_2)}{2} = \frac{(371.072+371.0624)}{2}$ =371.0672 K

From eq. (3-61) calculate thermal conductivity of pipe at T_{mp} = 371.0672 K, see fig. A-9:

k_t=1.58234*10^{-12}*T^4+ 1.5668*10^{-8}*T^3-2.76413*10^{-5}*T^2-

0.0194177*T+62.0529 … (3-61)

k_t=51.8722 W/m. K

Again, on a unit length the thermal resistance of pipe (R_2) is

$R_2 = \frac{\ln(r_2/r_1)}{2\pi k_t} = \frac{\ln(0.0302/0.02625)}{2*\pi*51.8722}$ =0.0004 K/W

Now try to increase insulation thickness until reaching optimum thickness based on economic cost, the true or correct (trial & error) assumption of (t_{ins} = 0.035 m) where radius of pipe with single layer insulation is given by

$r_0 = r_3 = r_2 + t_{ins}$ =0.0302+0.035= 0.0652 m

To calculate mean temperature of insulation for a particular density: assume outside surface insulation temperature (T_3) equals ambient temperature (T_0). The true or correct (trial & error) assumption of T_3=307.1K (33.1˚C)

$T_m = \frac{(T_3+T_2)}{2} = \frac{(307.1+371.06)}{2}$ =339.08 K

From eq. (3-28) calculate of thermal conductivity at T_m = 339.08 K, i.e.

k_{ins} = 0.0431W/m. K

For unit length the thermal resistance of insulation (R_3) is

$R_3 = \frac{\ln(r_3/r_2)}{2\pi k_{ins}} = \frac{\ln(0.0652/0.0302)}{2*\pi*0.0431}$ = 2.8463K/W

To calculate outside resistance (R_4) the outside heat transfer coefficient is needed as in eq. (3-62)

$R_4 = \frac{1}{2\pi r_3 h_0}$ … (3-62)

Calculation of outside heat transfer coefficient:

Outside heat transfer coefficient h_0 is calculated from the empirical correlation for natural convection over a horizontal cylinder surface using eq. (3-63) [74, 75, and 76] at film temperature.

$$Nu = \frac{h_0(2r_0)}{k_a} = [0.6 + \frac{0.387(Gr_D Pr)^{(1/6)}}{[1+(0.559/Pr)^{(9/16)}]^{(8/27)}}]^2 \qquad \text{… (3-63)}$$

From table 3-4 calculate the properties of air (ν, Pr, k_a) at $T_f = \frac{T_3+T_0}{2} = 300.125K$.

Table 3-4: The properties of air (ν, Pr, k_a) are function of film temperature calculated by fitting data [73] where the temperature in K.

Properties of air equations,	Units	R^2
υ =[-3.01*$10^{-8}T_f^3$+1.25*$10^{-4}T_f^2$+2.45*$10^{-2}T_f$-1.98]*10^{-6} … (3-64)	m^2/s	1.0000
Pr=-1.9*$10^{-18}T_f^6$+7.69*$10^{-15}T_f^5$-1.22*$10^{-11}T_f^4$+9.30*$10^{-9}T_f^3$– 3.06*$10^{-6}T_f^2$+ 1.11*$10^{-4}T_f$+ 0.781 … (3-65)	-	1.0000
k_a=1.3*$10^{-11}T_f^3$-4.70*$10^{-8}T_f^2$+1.02*$10^{-4}T_f$- 5.31*10^{-4} … (3-66)	W/m. K	1.0000

Note: Properties of air illustrates in figs. A- 6 through A-8.

Where at T_f = 300.125K:

υ = 1.5818*10^{-5} m^2/s Pr= 0.7085 k_a= 0.0262 W/m. K

β = $\frac{1}{T_f} = \frac{1}{300.9123}$ = 0.0033 K^{-1}

$$Gr_D = \frac{g\beta(T_3-T_0)(2r_3)^3}{\upsilon^2} = 4.0288*10^6$$

$$Nu = [0.6 + \frac{0.387(Gr_D Pr)^{(1/6)}}{[1+(0.559/Pr)^{(9/16)}]^{(8/27)}}]^2 = 19.5876$$

$$h_0 = \frac{Nu\ k_a}{2r_3} = 3.9385\ W/m^2.K$$

Substitute h_0= 3.9385 W/m^2.K in eq. (3-62):

$$R_4 = \frac{1}{2\pi * 0.0652 * 3.9385} = 0.6203 \text{ K/W}$$

Calculated critical radius as:

$$r_c = \frac{k_{ins}}{h_0} = \frac{0.0431}{3.9385} = 0.0109 \text{ m}$$

The value of the critical radius is less than the outside radius of the pipe (0.0302 m), so addition of any mineral wool insulation would cause a decrease in the heat transfer [73].

Calculate heat loss:

$$Q = \frac{T_i - T_0}{R_1 + R_2 + R_3 + R_4} = \frac{371.1449 - 293.15}{0.0032 + 0.0004 + 2.8463 + 0.6203} = 22.4755 \text{ W/m}$$

For the calculation of heat loss without insulation use the same procedure where:

T_{b1}=98˚C, $T_{b2} = 97.9537$˚C, $T_i = 97.9769$˚C, $T_1 = 97.6490$˚C, T_2= 97.6060˚C, R_0=0.7673 W/m, k_t=51.882 W/m. K , h_0=0.7673W/m^2.K, then

$$Q_0 = \frac{T_i - T_0}{\frac{1}{2\pi r_1 h_i} + \frac{\ln(r_2/r_1)}{2\pi k_t} + \frac{1}{2\pi r_3 h_0}} = \frac{371.1269 - 293.15}{0.0032 + 0.0004 + 0.7673} = 101.1390 \text{W/m}$$

Calculate total cost (C_T) of heat loss and installing insulation is by substituting Q = 22.4755W/m in eq. (3-44) and t_{ins} = 0.035 m in eq. (3-45):

Cost of heat loss:

$$C_H = (0.0527596*22.4755+0.00097868)*3.2808 = 3.8935 \$/\text{m. yr.}$$

Installing cost for insulation:

$$C_{I1} = (19.0551*0.035+0.809)*3.2808 = 4.8422 \$/\text{m. yr.}$$

Total cost (C_T) of heat loss and installing insulation:

$$C_T = 3.8935 + 4.8422 = 8.7357 \$/\text{m. yr.}$$

Where C_T =8.7357 \$/m. yr. is the minimum total cost then t_{ins} = 0.035 m is the optimum thickness for this insulation.

Calculate percentage energy saving: from eq. (3-32)

$$\%Q_{sav} = \frac{Q_0 - Q}{Q_0} * 100 = \frac{101.1390 - 22.47556}{101.1390} * 100 = 77.7776\ \%$$

Note: the same procedure is repeated for each insulation and case studies.

Example 2:

For multilayer calculate the optimum thickness of insulation for the third case using calcium silicate as the first insulation and cellular glass as second insulation (with influence of radiation).

Solution: Assume the steam is superheated. The thermal conductivity of the steel pipe is many orders of magnitude larger than that of the insulation material, so it can be assumed that the temperature drop through the thickness of the pipe is negligible, and that the temperature at the outside surface of the pipe (hence, the inside surface of the insulation) is T_2.

T_2=200+273.15=473.15 K

T_0=20+273.15=293.15 K

Thermal resistance:

To calculate mean temperature of insulation for first and second layer, assume outside surface insulation temperature for first layer (T_3) is equal to inside surface insulation temperature (T_2). Also assume outside surface insulation temperature for second layer (T_4) is between outside pipe surface temperature (T_2) and air temperature (T_0).The true or correct (trial & error) assumption is of T_3=414.72K (141.57˚C), & T_4=310.51 K (37.36˚C).

Mean temperature calculation:

T_{m1}=mean temperature for the first layer, K

$$T_{m1} = \frac{(T_2+T_3)}{2} = \frac{(473.15+414.72)}{2} = 443.93\text{K}$$

T_{m2}=mean temperature for the second layer, K

$$T_{m2} = \frac{(T_3+T_4)}{2} = \frac{(414.72+310.51)}{2} = 362.61\text{K}$$

Thermal conductivity at T_{m1} and T_{m2}:

Use eq. (3-22) to calculate thermal conductivity for the first layer ($k_{ins\,1}$) and eq. (3-23) for the second layer ($k_{ins\,2}$):

$k_{ins\,1}$ = 0.069316 W/m. K

$k_{ins\,2}$ = 0.053819 W/m. K

To calculate first and second optimum thickness based on economic cost using (trial & error), it is assumed that the thickness for second layer ($t_{ins\,2}$) is constant while changing the first layer thickness ($t_{ins\,1}$). The true or correct (trial & error) assumption is $t_{ins\,1}$=0.03 m, and $t_{ins\,2}$=0.06 m.

r_2=0.168/2=0.084 m

$r_3=r_2+t_{ins\,1}$=0.084+0.03=0.114 m

$r_0=r_4=r_3+t_{ins\,2}$= 0.114+0.06= 0.174 m

For unit length the thermal resistance of first layer (R_3) and second layer (R_4) is

$$R_3 = \frac{\ln(r_3/r_2)}{2\,\pi k_{ins\,1}} = \frac{\ln(0.114/0.084)}{2*\pi*0.069316} = 0.70118\text{K/W}$$

$$R_4 = \frac{\ln(r_4/r_3)}{2\,\pi k_{ins\,2}} = \frac{\ln(0.174/0.114)}{2*\pi*0.053819} = 1.2505\text{K/W}$$

Calculation of outside thermal resistance (R_0):

The pipe will lose heat to the surroundings by radiation as well as by natural convection [73, 77]. To calculate outside thermal resistance (R_0) the outside heat transfer coefficient (h_0+h_r) is needed using eq. (3-63) and table (3-4) as before to calculate (h_0), and eq. (3-37) at low emissivity (ε = o.1) for radiation [13].

$h_0 = \frac{\text{Nu } k_a}{2r_4} = \frac{49.795 * 0.026331}{2 * 0.174}$ =3.7677W/m^2.K

h_r=ε σ $(T_4^2 + T_0^2)(T_4 + T_0)$

h_r= 0.1 *5.67*10^{-8} $(310.51^{\,2} + 293.15^2)(310.51+293.15)$ =0.62414W/m^2.K

For unit length, the outside thermal resistance (R_0) is

$R_0 = \frac{1}{2\pi r_4 (h_0 + h_r)} = \frac{1}{2\pi 0.174 * (3.7677 + 0.62414)}$=0.20827K/W

Calculation of heat loss:

$Q_a = \frac{T_2 - T_0}{R_3 + R_4 + R_0} = \frac{473.15 - 293.15}{0.70118 + 1.2505 + 0.20827}$ = 83.336W/m

q_r=2πr_0εσ$(T_4^4 - T_0^4)$=2π*0.174*0.1*5.67*$10^{-8}$$(310.51^{\,4} - 293.15^4)$=11.8431 W/m

$q_c = 2\pi r_0 h_0 (T_4 - T_0)$ =2π*0.174* 3.7677(310.5062 − 293.15) =71.4924 W/m

Substitute q_r and q_c in eq. (3-36)

$Q_a = q_r + q_c$ =11.8431+71.4924 =83.336 W/m

For calculation of heat loss without insulation (Q_{a0}) at the exterior surface of the pipe (ε = 0.8) by using the same procedure then the result:

T_2= 200˚C (473.15 K), h_0=7.2255 W/m^2.K, and h_r= 10.7687 W/m^2.K

To calculate R_0 :

$R_0 = \frac{1}{2\pi r_2 (h_0 + h_r)} = \frac{1}{2\pi 0.084 * (7.2255 + 10.7687)}$ = 0.105295 K/W

$Q_{a0} = \frac{T_2 - T_0}{R_0} = \frac{473.15 - 293.15}{0.105295}$ =1709.5 W/m

q_{r0}=2π$r_2 \varepsilon_0$σ $(T_2^4 - T_0^4)$=2π*0.084*0.8*5.67*$10^{-8}$$(473.15^{\,4} - 293.15^4$=1023.05W/m

$q_{c0} = 2\pi r_2 h_0 (T_2 - T_0)$ =2π*0.084* 7.2255 (473.15 − 293.15) = 686.439 W/m

Substitute q_{r0} and q_{c0} in eq. (3-42)

$Q_{a0} = q_{r0} + q_{c0}$ =1023.048+686.439W =1709.5 W/m

Calculated total cost (C_T) of heat loss and install insulation by substitute Q = 83.336 W/m in eq. (3-44), $t_{ins\,1}$= 0.03 m in eq. (3-46) and $t_{ins\,2}$= 0.06 m in eq. (3-47):

Cost of heat loss:

C_H= (0.0527596*83.336+0.00097868)*3.2808 =14.428 $/m. yr.

Installing cost for first layer:

C_{I2}= (28.1777*0.03+1.18500)*3.2808 = 6.6611 $/m. yr.

Installing cost for second layer:

C_{I3}= (31.2036*0.06+1.32900)*3.2808=10.503 $/m. yr.

Total cost (C_T) of heat loss and installing insulation:

C_T =C_H+ C_{I2}+C_{I3}=14.428 +6.6611+10.503=31.592 $/m. yr.

Where C_T =31.592 $/m. yr. is the minimum total cost, then $t_{ins\,1}$= 0.03 m and $t_{ins\,2}$= 0.06 m is the optimum thickness for this case (case 1 in table 3-5).

Calculated percentage of energy saving: from eq. (3-32)

$$\%Q_{asav} = \frac{Q_{a0}-Q_a}{Q_{a0}} *100 = \frac{1709.5\ -83.336}{1709.5} *100 = 95.1251\%$$

Note: the same procedure is repeated for each case in table 3-5.

Table 3-5 Exhibits first and second layer for each case study of multilayer insulation.

Case	First layer	Second layer
1	Calcium silicate	Cellular glass
2	Calcium silicate	Mineral wool
3	Cellular glass	Calcium silicate
4	Cellular glass	Mineral wool
5	Mineral wool	Calcium silicate
6	Mineral wool	Cellular glass

Chapter four

Results and discussion

4.1 Effective thermal conductivity of insulation

The thermal conductivity of insulations is affected by temperature and density as follows:

4.1.1 Temperature effect

Three operating temperatures, 98, 190 and 200˚C using three nominal pipe size 2, 3 and 6 inch respectively. These are investigated in the present work to show their effects on thermal conductivity performance in the presence of three different insulations (Mineral wool, Calcium Silicate and Cellular glass). Figures 4-1 through 4-3 show the variation of thermal conductivity with mean temperature for water, gas oil, and steam flowing into the three different pipes considered in this investigation with above insulations.

The total thermal conductivity of insulation is a result of convection, conduction due to air and solid phase and radiation in the space separating particles composing the insulation material as in eq. (4-1); Thermal conductivity due to convection is negligible under normal conditions. Conductive thermal conductivity, where k_{cond} is the sum of the thermal conductivity of air and fibers [6, 17 and 25].

$$k_{ins} = k_{conv.} + k_{cond.} + k_{rad.} \quad \ldots (4\text{-}1)$$

The results indicate clearly that thermal conductivity increases as mean temperature increases for various insulation studies. And figures 4-1 through 4-3 indicate that the lowest thermal conductivity was obtained in the mineral wool insulation at 200 kg/m^3.

It is obvious that the thermal conductivity generally increases with temperature for the three insulations. This behavior can be interpreted from the fact that the thermal conductivity of air and fiber increase with temperature as well as radiation of air, which contributes to the overall conductivity as it increases with forth power of temperature as clear from the basic radiation heat transfer equation; eq. (4-2),

$$q_r = \varepsilon \, 2 \, \pi r_0 \sigma \, (T_s^4 - T_0^4) \quad \ldots (4\text{-}2)$$

Figure 4-4 illustrates the behavior of thermal conductivity with range of temperature from 255-700 K for insulation types studied (equations 3-13 through 3-23). In fig. 4-5 is depicted a similar diagram produced by another research work [3], in order to allow comparisons with respect to the described model's accuracy.

These results agree well with the results of **Tseng and Kuo (2002) [15], Wang et al. (2003) [16], Karamanos et al. (2004) [17], Manohar et al. (2006) [20], Wei et al. (2007) [21], Oches et al. (2008) [24], Veiseh et al. (2009) [25], Daryabeigi (2010) [6]** and **Yüksel et al. (2010) [29]** as shown in chapter two.

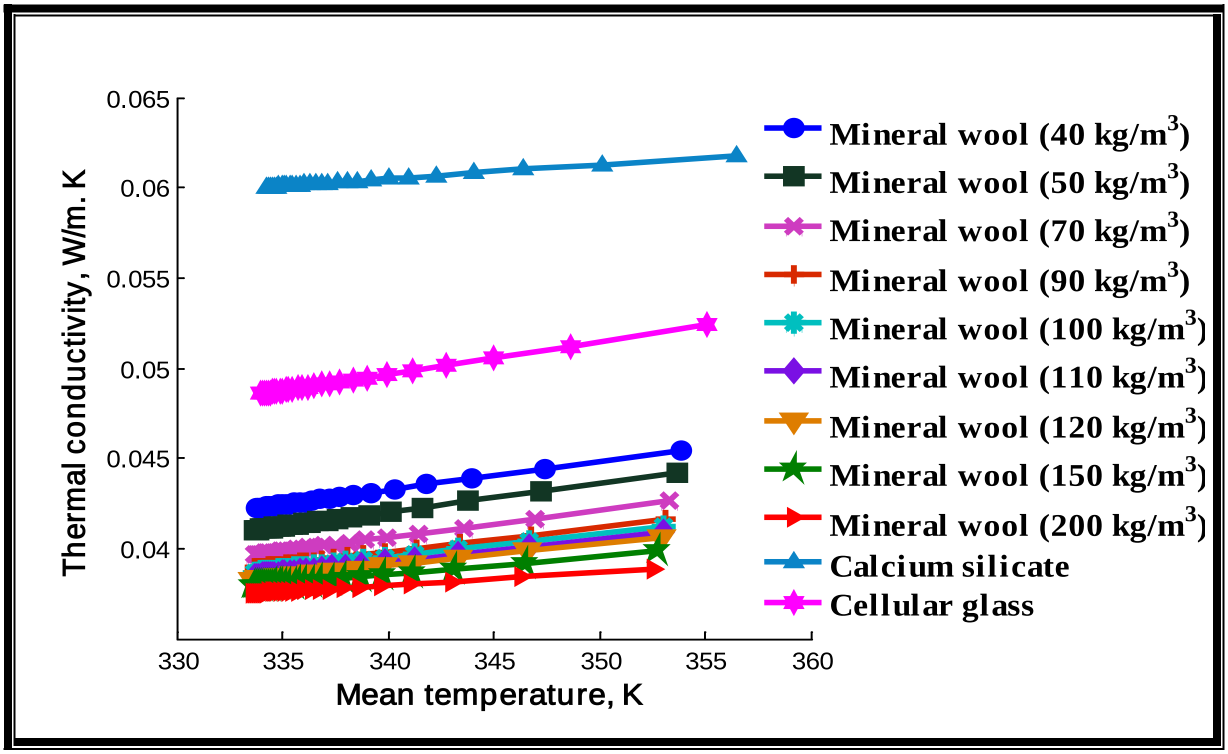


Figure 4-1 Effect of mean temperature on thermal conductivity for three insulations at 98˚C water temperature, T_0= 20˚C and NPS = 2 inch (case 1).

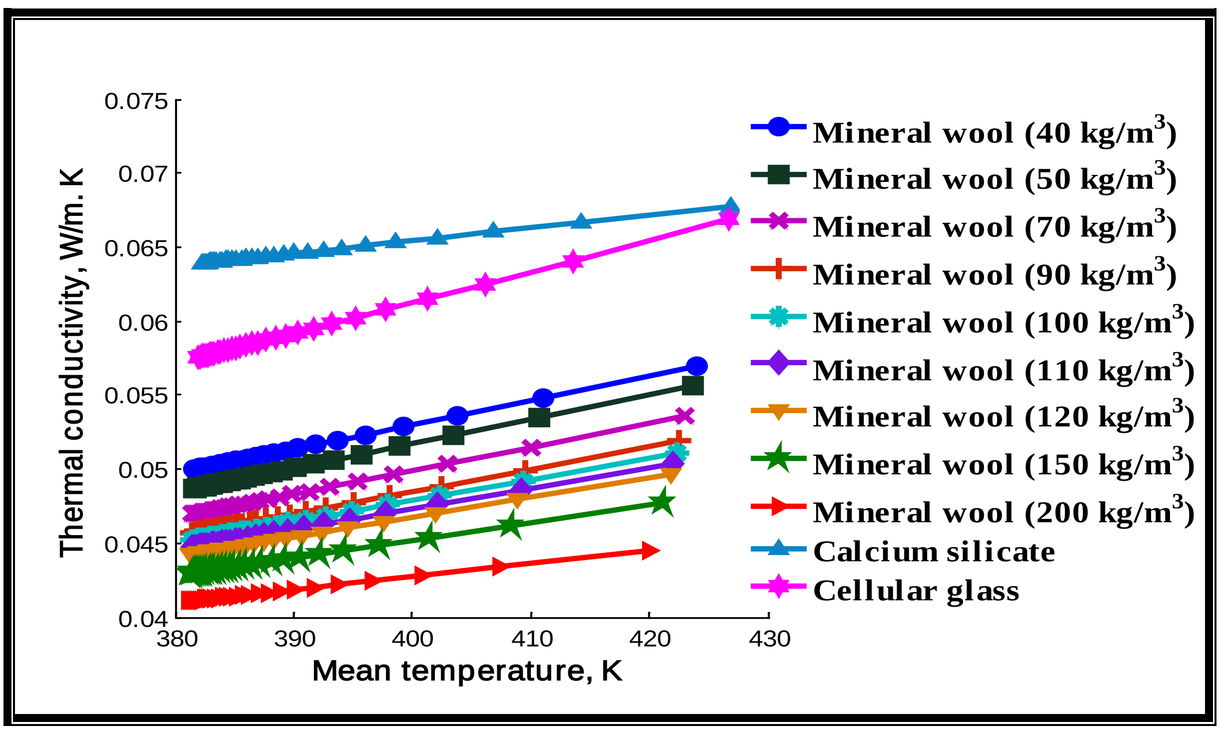


Figure 4-2 Effect of mean temperature on thermal conductivity for three insulations at 190˚C gas oil temperature, T_0= 20˚C and NPS = 3 inch (case 2).

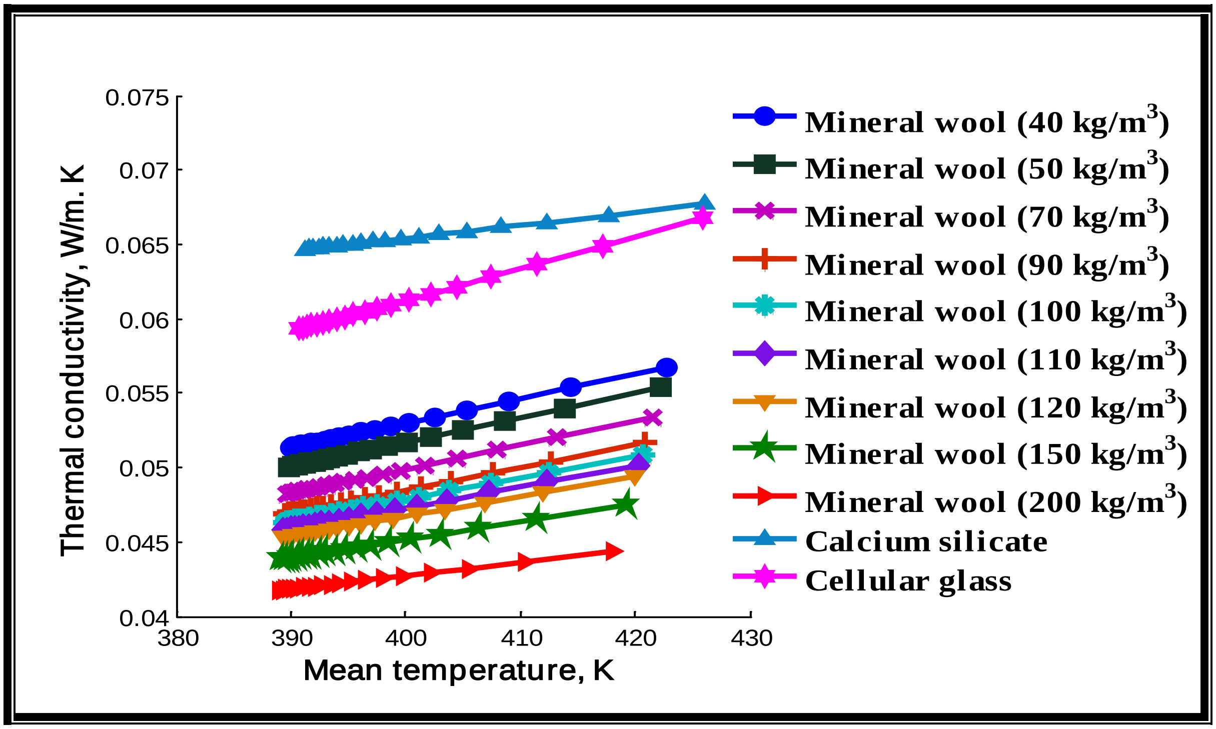


Figure 4-3 Effect of mean temperature on thermal conductivity for three insulations at 200˚C steam temperature, T_0= 20˚C and NPS = 6 inch (case 3).

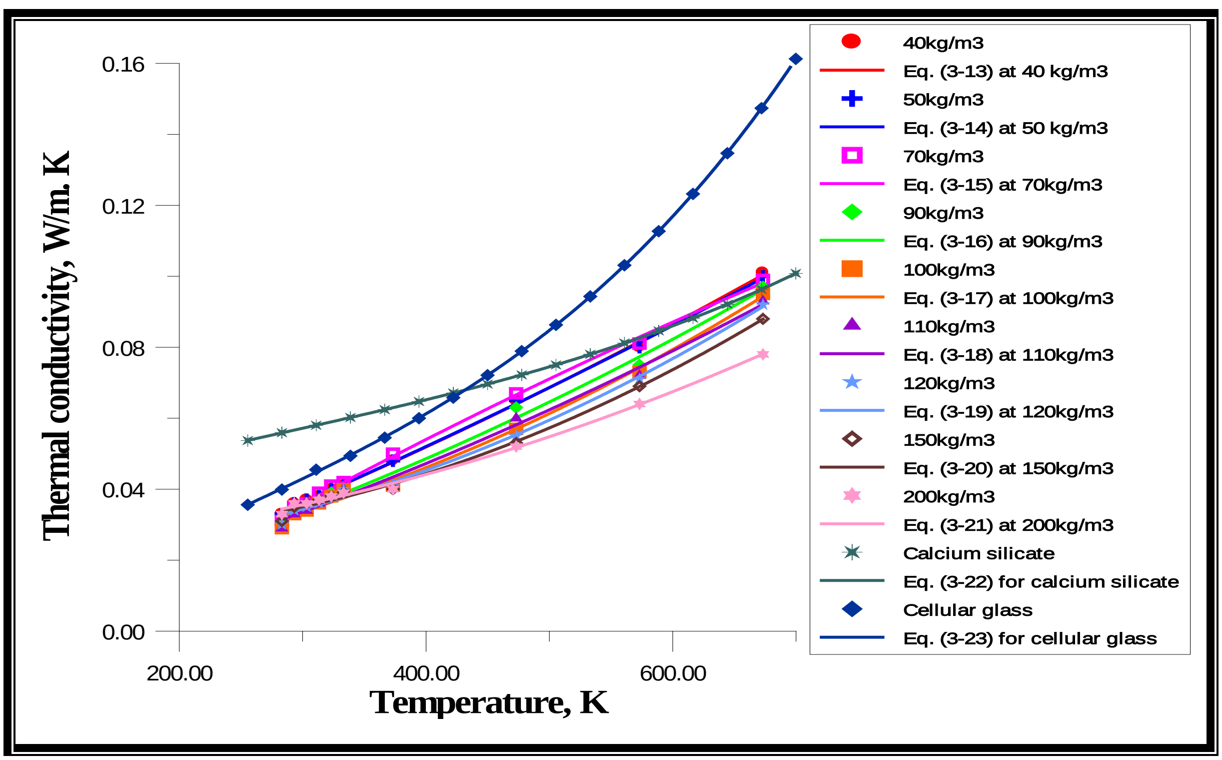


Figure 4-4 Illustrates the behavior of thermal conductivity with changing temperature at various densities for the three insulations.

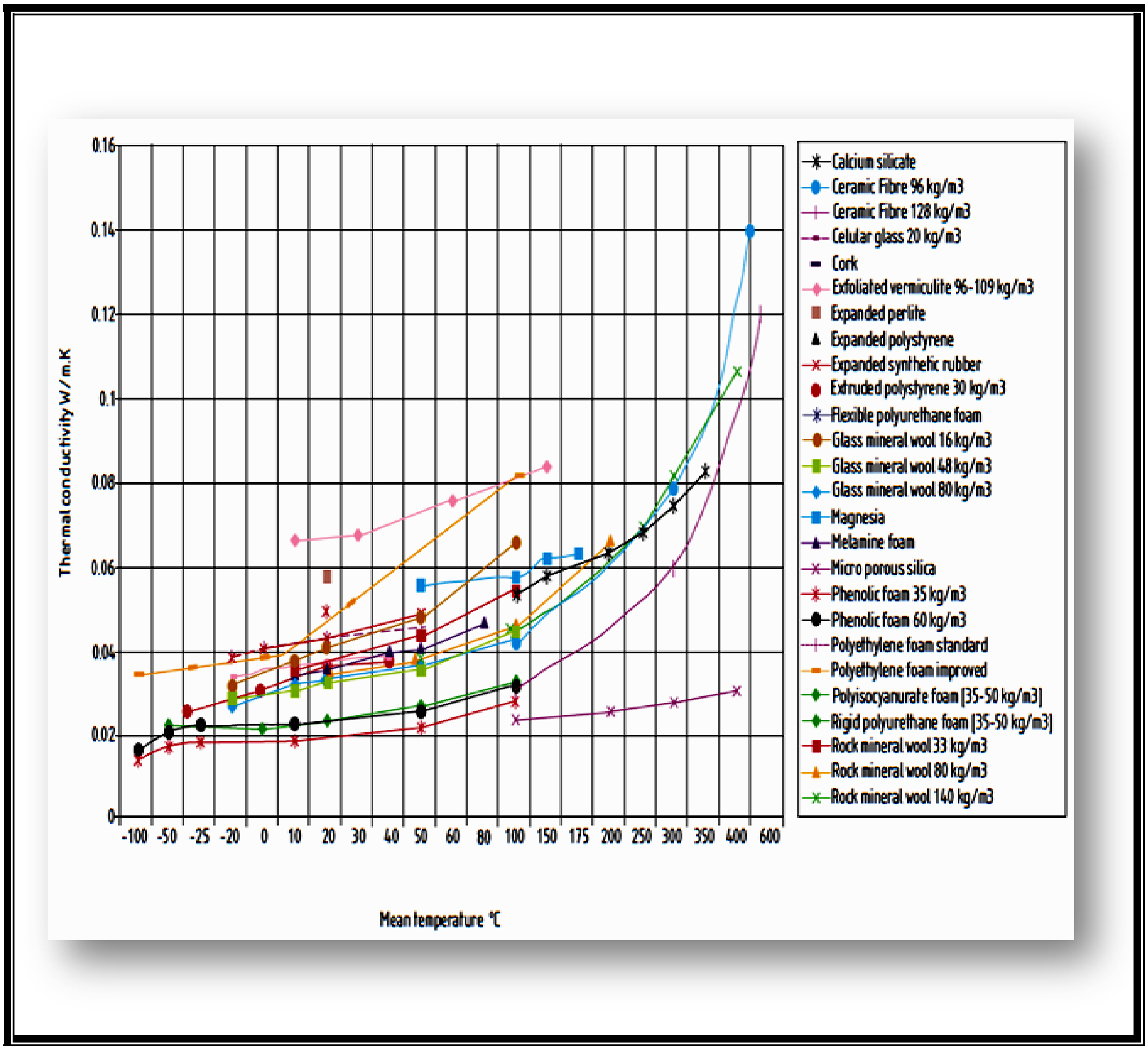


Figure 4-5 Typical thermal conductivity values for insulating materials used at above and below ambient temperatures [3].

4.1.2 Density effect

On the other hand, the behavior of thermal conductivity with density is not as simple as it appears in figs. 4-6 and 4-7 ,where at low temperatures the thermal conductivity decreases with density for a particular range and then increases, because the effect of radiation is slight at low temperature and this effect is slighter at high density as the void of the insulation decreases, while the conductivity of the fibers is the major contributor to the overall conductivity which increases with density because the coherence between fiber particles increases.

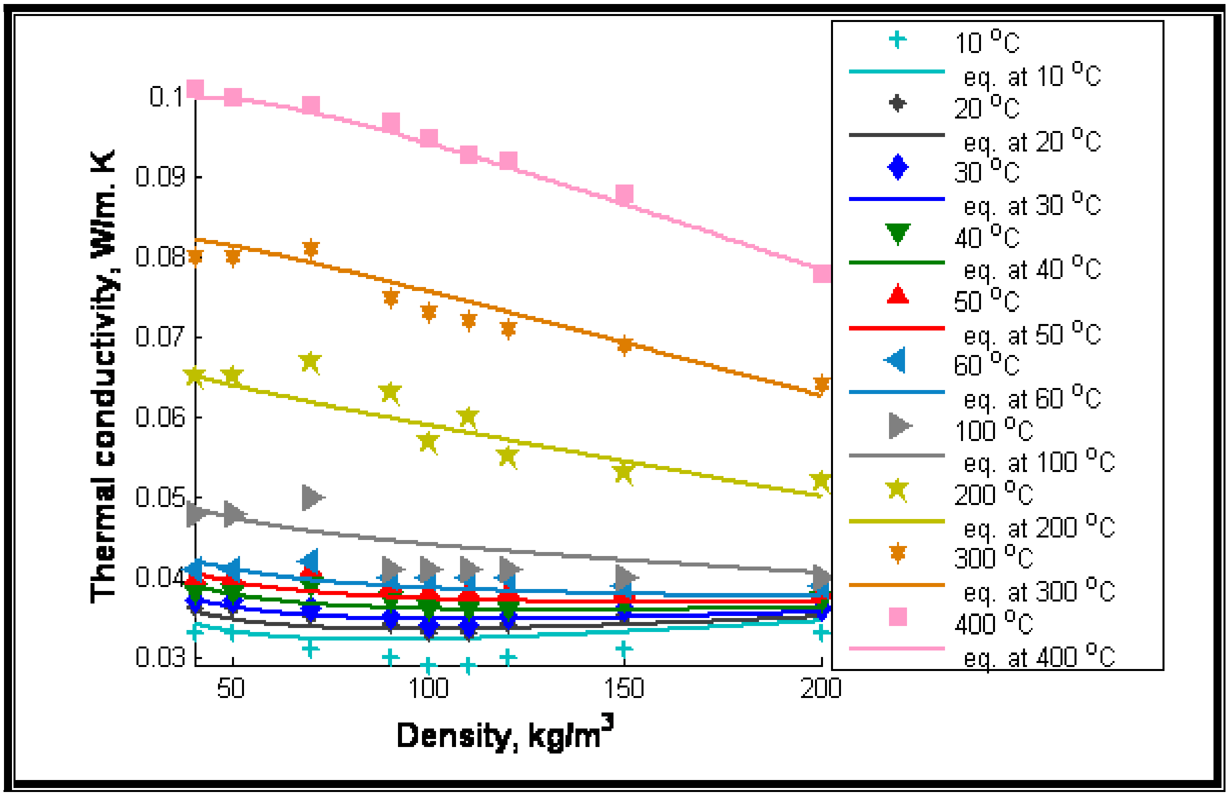


Figure 4-6 Illustrates the behavior of thermal conductivity with changing density at various temperatures for mineral wool insulations (Comparison between data [79] and eq. (3-28)).

At high temperatures, the effect of radiation arises because it is raised to the forth power in the radiation heat transfer equation, especially at low densities for the reason that the void is large where the effect of radiation can be considered the only factor in the conductivity phenomena. At high densities the void decreases and that leads to reduce the radiation and reduce the conductivity and this agrees with [2].

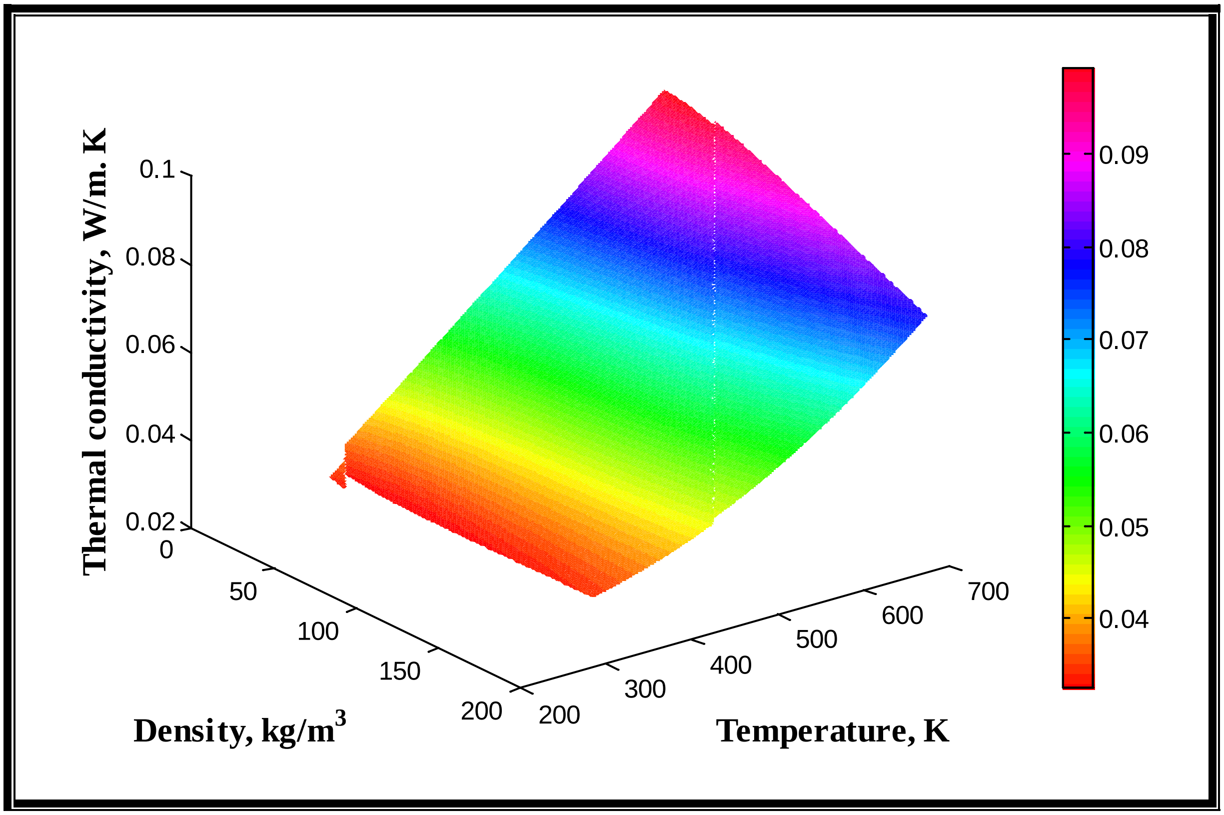


Figure 4-7 Illustrates the behavior of thermal conductivity with changing density and temperature for mineral wool insulations.

The interpretation above will help us to choose the optimum case of the same insulation (with different densities) and choosing the density for particular working temperature. Furthermore the different insulations are optimized according to the behavior of their thermal conductivities with temperature and density. The optimum thermal conductivity is obtained at 200 kg/m^3 mineral wool insulation for three cases studied. The results show a lowering of thermal conductivity with increasing densities [16].

4-2 Heat transfer from bare pipe with and without influence of radiation

The results for three cases studied as shown in fig. 4-8 are plotted as heat transfer from bare pipe versus ambient temperature. It exhibits that the heat transfer with radiation (ε = 0.8) is higher than that when influence of radiation is neglected (ε = 0). This effect increases with increased surface area and operating temperature where case 3 at (NPS= 6 inch and 200°C operating temperature) is larger than case 2 at (NPS= 3 inch and 190°C operating temperature) and the latter case is larger than case 1 at (NPS= 2 inch and 98°C operating temperature). While the heat transfer decreases with increased ambient temperature, this refers to reduced temperature difference between operating and ambient temperature and as such agrees with [1, 80].

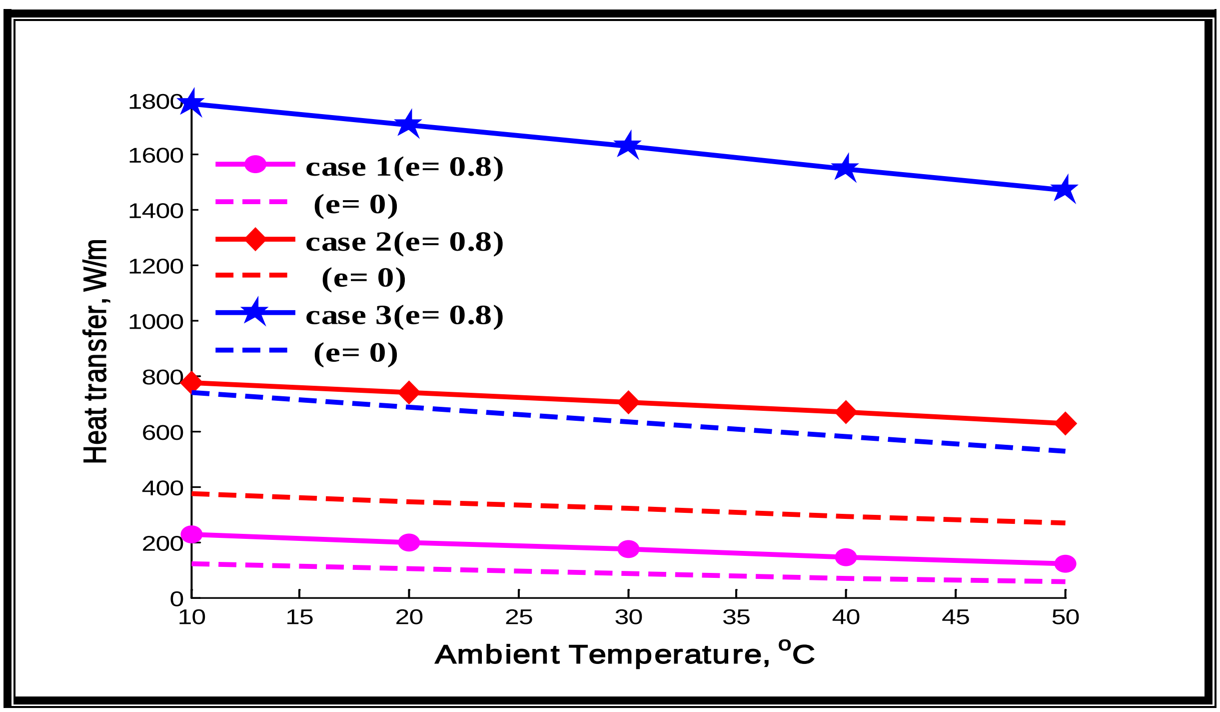


Figure 4-8 Heat transfer from bare pipe with neglected and considered influence of radiation for the three cases studied.

4-3 Optimum thickness using single layer insulation

The case of an insulated pipe containing hot water with T_i=98˚C, T_0=20˚C, D_i=0.0525 m, and D_0=0.0603 m, using single layer of mineral wool, calcium silicate, and cellular glass insulation are firstly examined. The results are shown in figs. 4-9 through 4-13. These figures show that the heat loss (Q_a), annual cost and percentage energy saving ($\%Q_{sav}$) are affected by the insulated thickness (t_{ins}), operating temperature (T_i), and ambient temperature (T_0).

Figure 4-9 shows the diminishing heat loss with increasing insulation thickness. The cost of heating decreases by diminishing increments due to increasing thickness of insulation, whereas the initial construction cost increases linearly. The total cost is the sum of cost of energy and insulation material. The total cost of energy and insulation material will show a minimum when plotted versus the insulation thickness as shown in fig. 4-10 for **rock mineral wool at 40 kg/$\boldsymbol{m^3}$**. It is evident that the minimum cost (8.8074 $/m) occurs at a thickness of about (0.035 m). The insulation thickness at the minimum of total cost curve is taken as the optimum insulation thickness. As shown in fig. 4-9 the heat loss decreases with increasing insulation thickness where heat loss at the optimum insulation thickness is Q_a=22.8890 W/m ($q_c = 19.7470\text{W/m}$ & $q_r = 3.142\ \text{W/m}$) at T_s=T_{3a}= 32.5982˚C, while Q_a= 45.8326 W/m at 0.01 m thickness of personal protection at (T_s=T_{3a}= 50.8010˚C). Figure 4-11 shows $\%Q_{sav}$ versus t_{ins} where $\%Q_{sav}$ increases with increasing thickness of insulation and at optimum insulation thickness of 88.4846% with considered influence of radiation.

When the influence of heat radiation is being neglected, Q and T_s=T_3 are 22.4755 W/m and 33.9408˚C respectively occurs at the optimum thickness of about 0.035 m. And the error from the influence of heat radiation being neglected (%EQ) is 1.8065 %.

Details here are explained only to **rock mineral wool at 40 kg/m^3,** while others are presented in the tables in appendix D. Figures 4-12 and 4-13 show a comparison among three insulations for heat loss and total cost versus thickness of insulation for first case respectively. Both figures show that the rock mineral wool insulation at 200 kg/m^3 has the best performance (low thermal conductivity). Also for different ambient temperatures, the rock mineral wool insulation at 200 kg/m^3 is the best choice. This is illustrated in table 4-1 where optimum thickness is shown at various ambient temperatures.

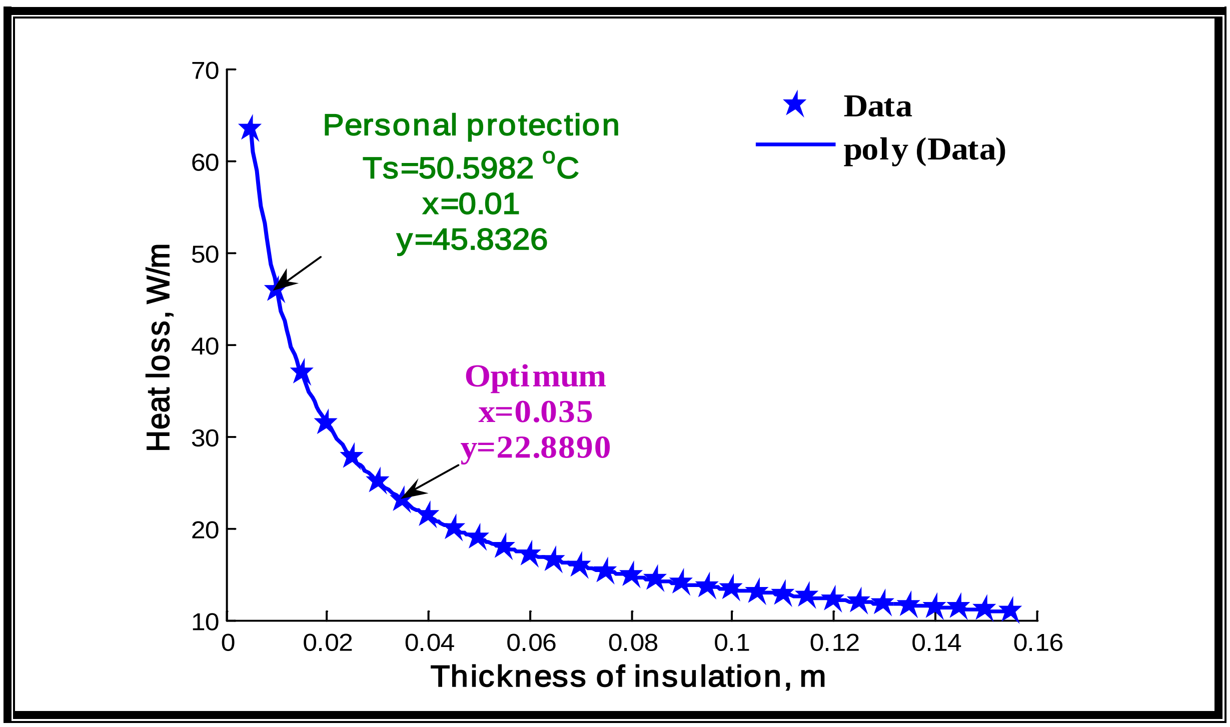


Figure 4-9 Effect of insulation thickness on the heat loss in the case of 98˚C water temperature, T_0= 20˚C (case 1) and ε = 0.1 for mineral wool insulation at 40 kg/m^3.

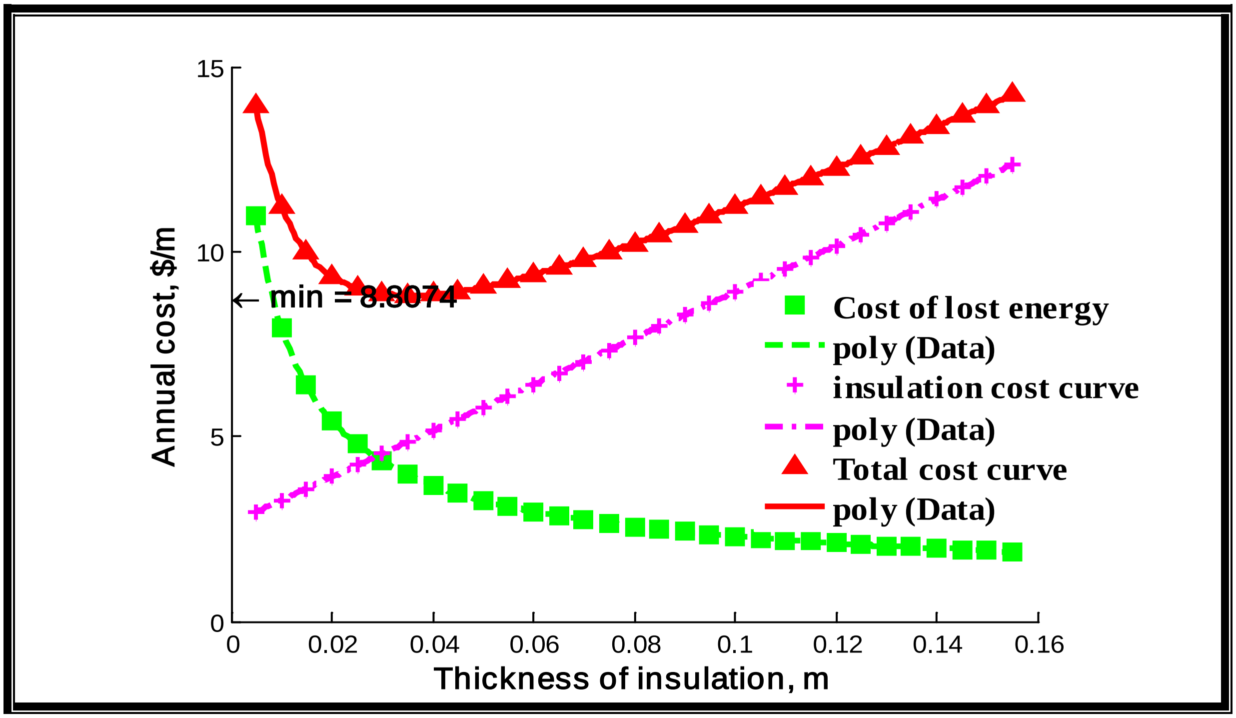


Figure 4-10 Effect of insulation thickness on the annual cost in the case of 98˚C water temperature, T_0= 20˚C (case 1) and ε = 0.1 for mineral wool insulation at 40 kg/m^3.

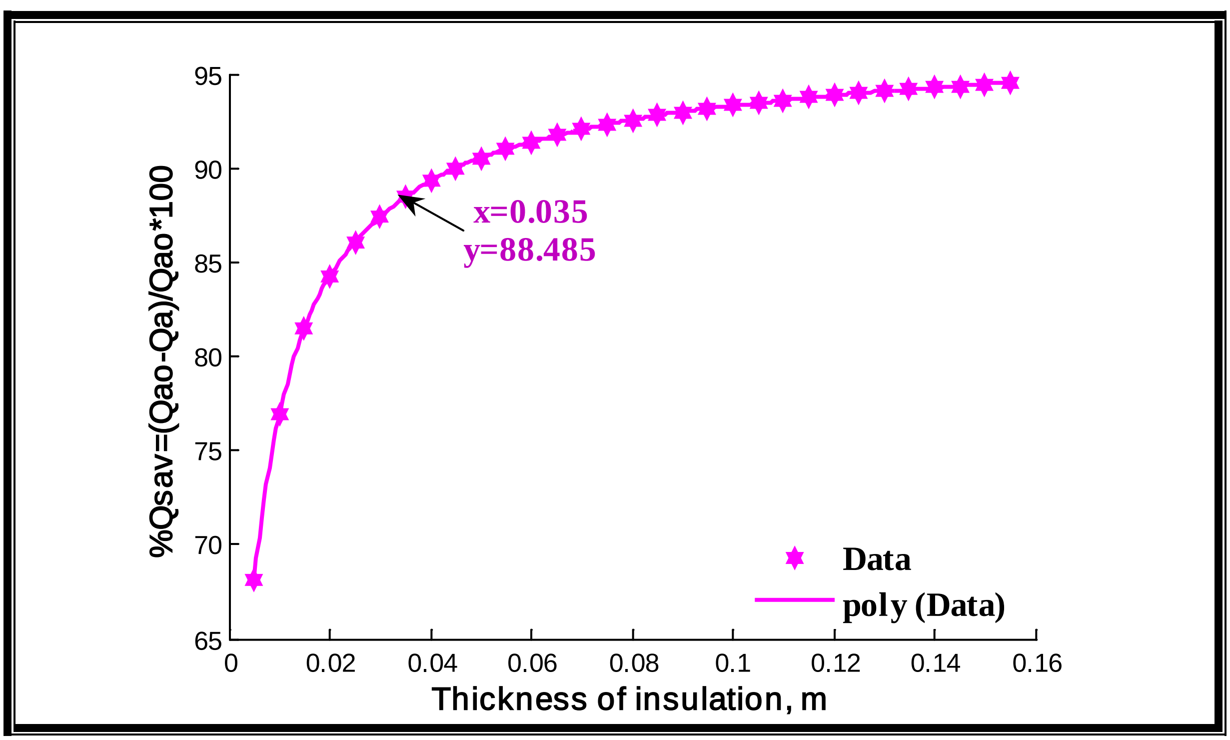


Figure 4-11 Relation between $\%Q_{sav}$ and thickness of insulation in the case of 98˚Cwater temperature, T_0= 20˚C (case 1) and ε = 0.1 for mineral wool insulation at 40 kg/m^3.

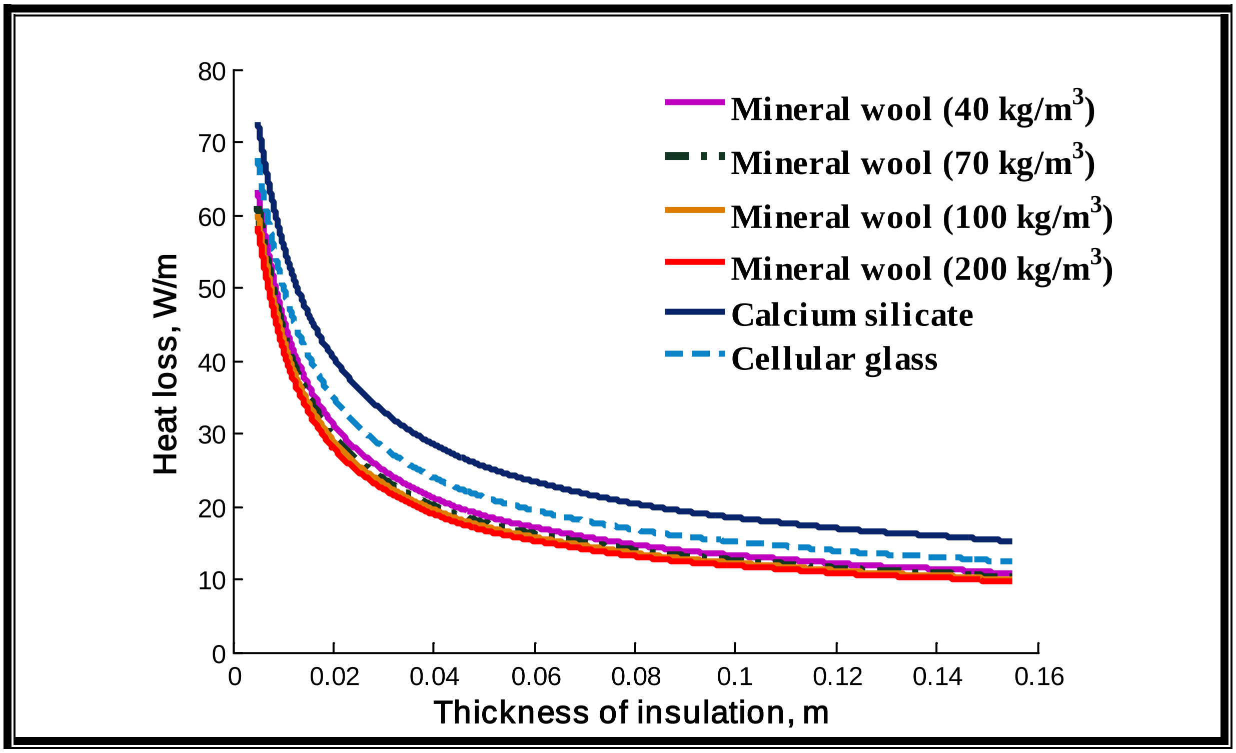


Figure 4-12 Relationship between heat loss and thickness of insulation for 98˚Cwater temperature, ε = 0.1 and T_0=20˚C (case 1).

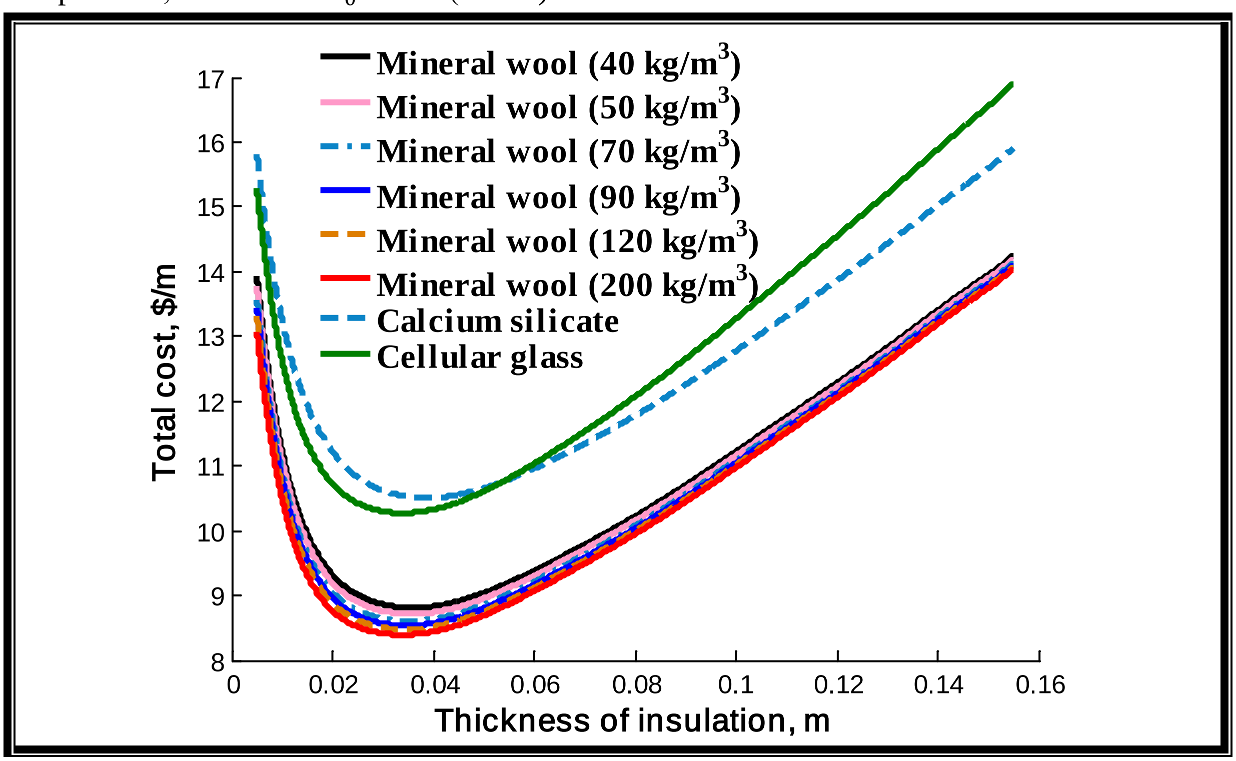


Figure 4-13 Comparison of total cost between Mineral wool, Calcium Silicate, and Cellular Glass insulations for 98˚C water temperature, ε = 0.1 and T_0=20˚C (case 1).

Table 4-1 Optimum thickness of insulation at different ambient temperatures for first case studied (using water).

Insulation types		Ambient temperature, ˚C				
		10	20	30	40	50
Rock mineral wool, kg/m^3	40	0.040	0.035	0.030	0.030	0.025
	50	0.035	0.035	0.030	0.030	0.025
	70	0.035	0.035	0.030	0.030	0.025
	90	0.035	0.035	0.030	0.030	0.025
	100	0.035	0.035	0.030	0.030	0.025
	110	0.035	0.035	0.030	0.030	0.025
	120	0.035	0.035	0.030	0.030	0.025
	150	0.035	0.035	0.030	0.030	0.025
	200	**0.035**	**0.035**	**0.030**	**0.025**	**0.025**
Calcium silicate		0.040	0.040	0.035	0.030	0.025
Cellular glass		0.035	0.035	0.030	0.025	0.025

The second case examined is of an insulated pipe containing gas oil with T_i=190˚C, T_0=20˚C, D_i=0.0779 m, *and* D_0=0.0889 m using single layer of mineral wool, calcium silicate, and cellular glass insulation. The results are shown in figs. 4-14 through 4-19.These figures show that the heat loss (Q_a), annual cost and percentage energy saving ($\%Q_{sav}$) are affected by the insulated thickness (t_{ins}), emissivity (ε), operating temperature (T_i), and ambient temperature (T_0).

Details are explained in the following only for **rock mineral wool at 200 kg/$\boldsymbol{m^3}$,** while other insulation materials are presented in the tables in appendix D.

Figure 4-14 for **rock mineral wool insulation at 200 kg/m^3** shows the heat loss decreased with increased thickness of insulation and selected optimum thickness equals 0.065 m where Q_a= 44.5648 W/m($q_c = 38.2935$ W/m & $q_r = 6.2713$W/m) at T_s=T_{3a}= 34.80˚C depending on the minimum cost (15.3878 $/m) as shown in fig. 4-15. The selected thickness for personal protection is equal 0.02 m depending on (T_s=T_{3a}= 58.774˚C) which is the outside insulation surface temperature being approximately below 60˚C where Q_a= 92.75 W/m.

When the influence of heat radiation being neglected; Q and T_s=$T_3$are 44.15 W/m and 36.53˚C respectively occur at the optimum thickness of about 0.065 m. The error from the influence of heat radiation being neglected (%EQ) is 0.94 %.

Figures 4-16 and 4-17 show a comparison among three insulations for heat loss and total cost versus thickness of insulation respectively. Both figures show that the rock mineral wool insulation at 200 kg/m^3 has the best performance (low thermal conductivity). Also for different ambient temperatures, the rock mineral wool insulation at 200 kg/m^3 is the best choice. This is illustrated in table 4-2 where optimum thickness is shown at various ambient temperatures.

Figure 4-18 shows rock mineral wool insulation at 200 kg/m^3at different ambient temperatures from plotting heat loss versus thickness of insulation. It is evident that the heat loss decreases with increasing thickness of insulation and increased ambient temperature. This decrease occurs because of reduced temperature difference between external surface insulation temperature and ambient temperature.

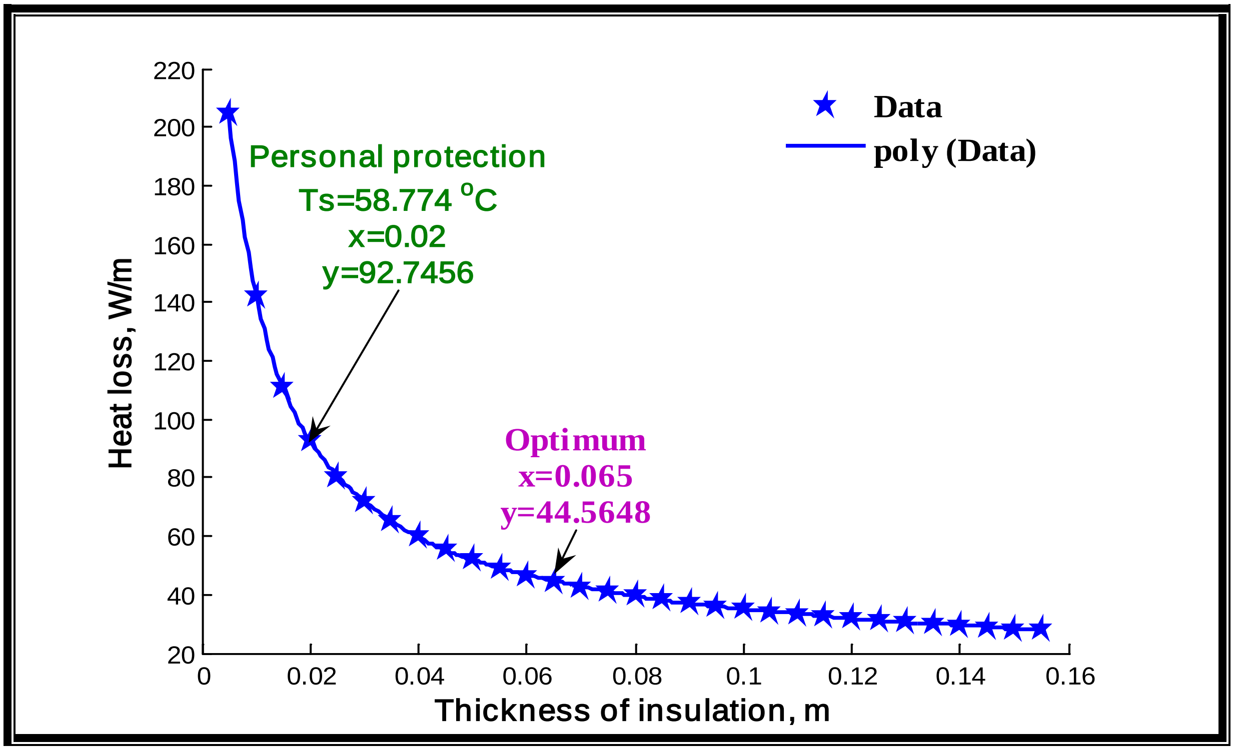


Figure 4-14 Effect of insulation thickness on the heat loss in the case of 190˚C gas oil temperature, T_0= 20˚C and ε = 0.1 (case 2) for mineral wool insulation at 200 kg/m^3.

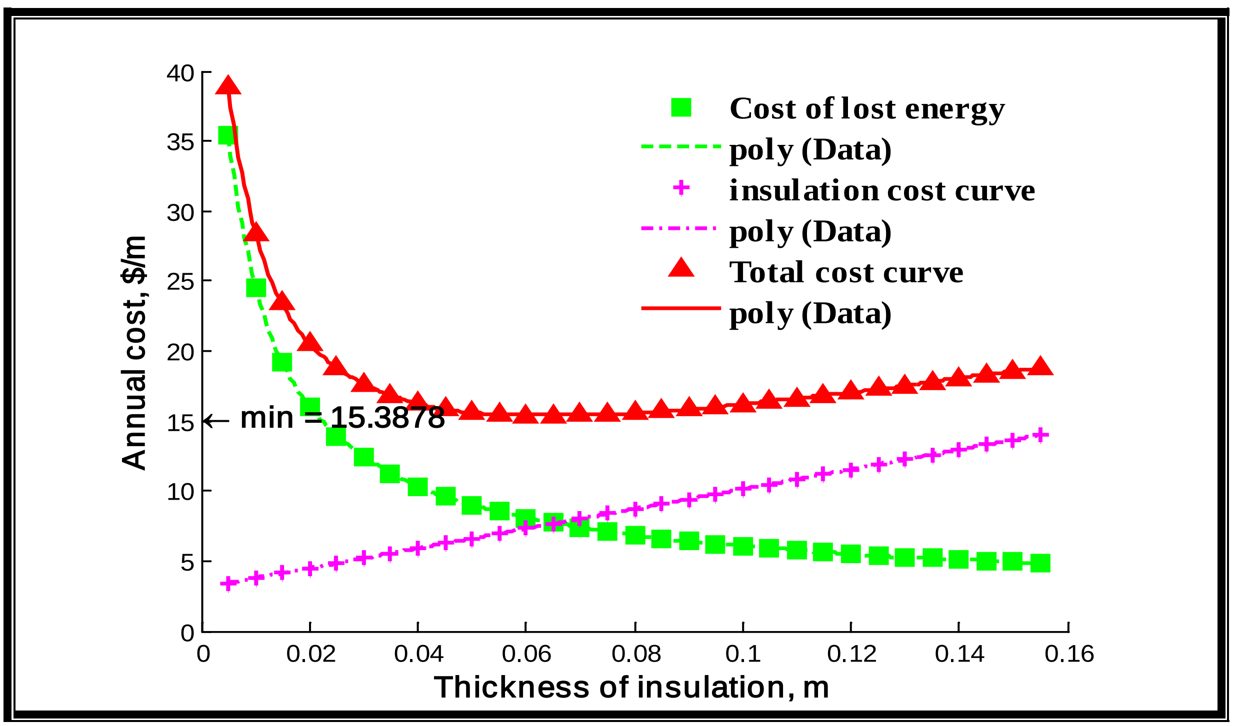


Figure 4-15 Effect of insulation thickness on the annual cost in the case of 190˚C gas oil temperature, T_0= 20˚C and ε = 0.1 (case 2) for mineral wool insulation at 200 kg/m^3.

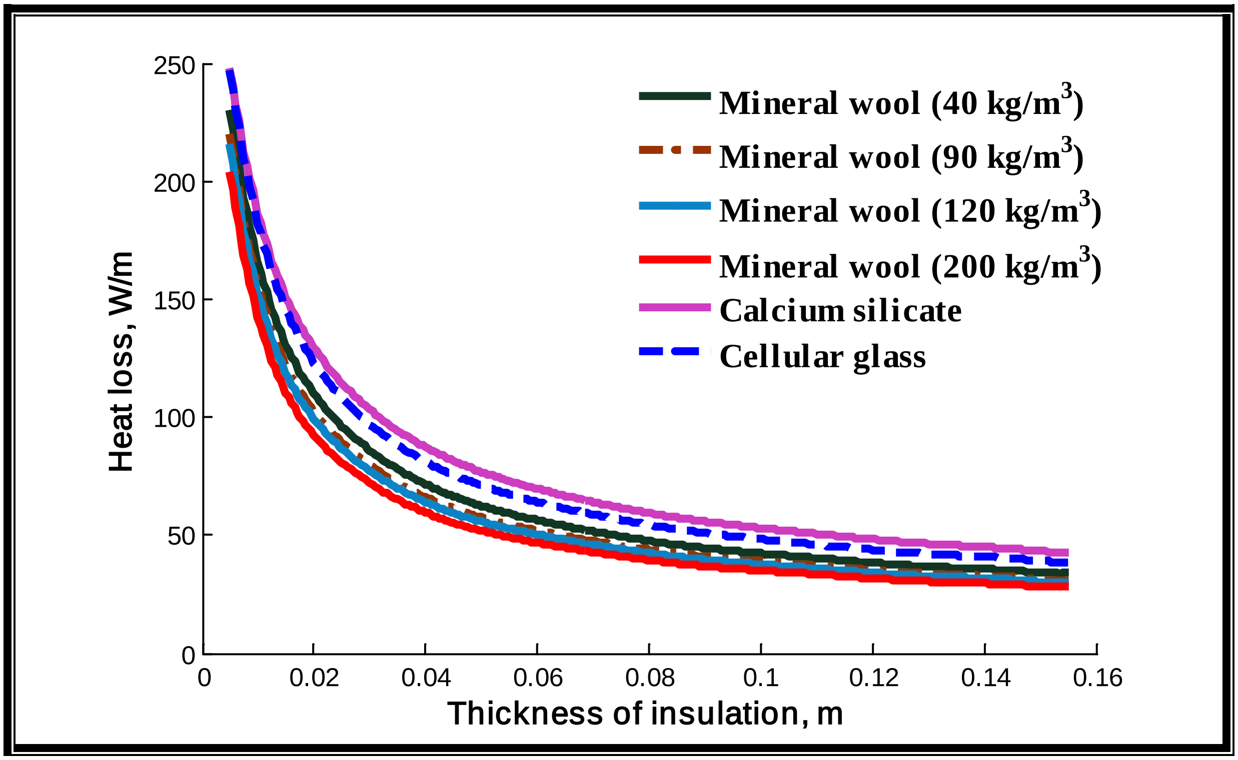


Figure 4-16 Relationship between heat loss and thickness of insulation for 190˚C gas oil temperature, ε = 0.1 and T_0=20˚C (case 2).

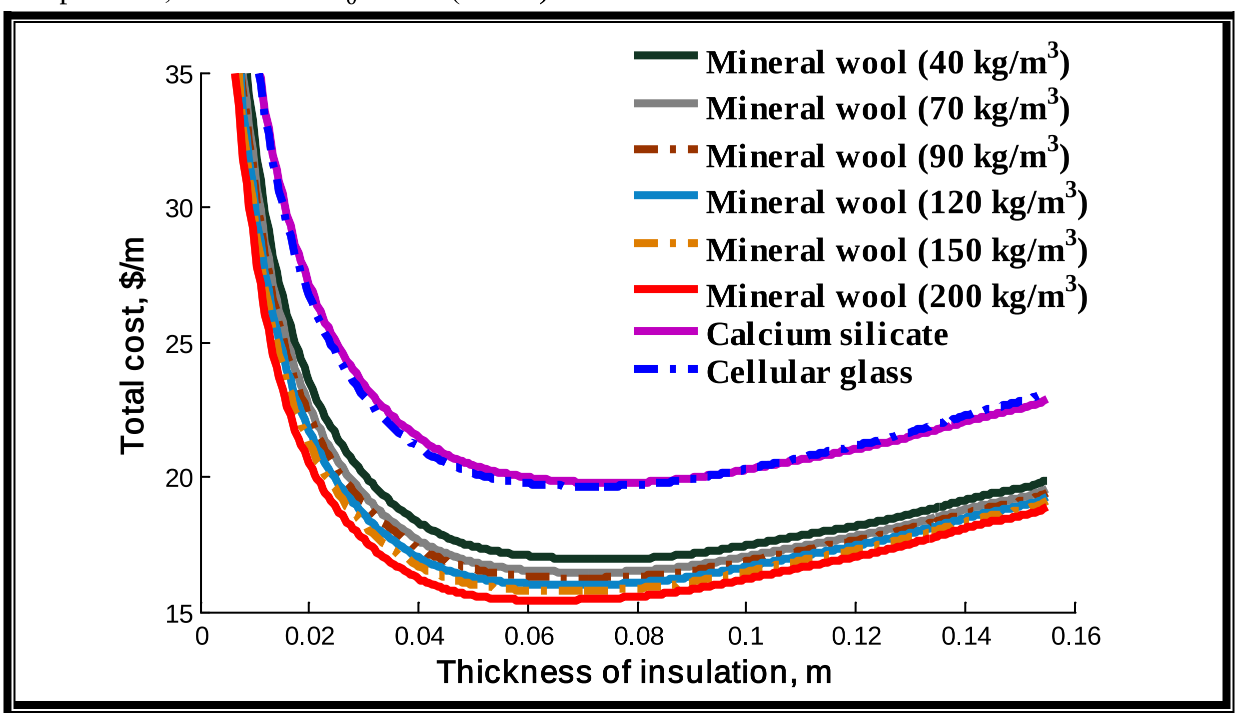


Figure 4-17 Comparison of total cost between Mineral wool, Calcium Silicate, and Cellular Glass insulations for190˚C gas oil temperature, ε = 0.1 and T_0=20˚C (case 2).

Figure 4-19 shows the error of heat transfer rate (%EQ) generated by neglecting heat radiation effect (ε = 0) for rock mineral wool insulation at 200 kg/m^3. It exhibits that %EQ is 0.94 % at 0.065 m thickness of insulation when (ε = 0.1) is used for pipe insulation and %EQ increases to 4.25% at the same thickness when (ε = 0.8) is used because of increased radiation heat transfer as shown in table 4-3.

Table 4-2 Optimum thickness of insulation at different ambient temperatures of second case studied (using gas oil).

Insulation types		**Ambient temperature, ˚C**				
		10	**20**	**30**	**40**	**50**
Rock mineral wool, kg/m^3	**40**	0.075	0.070	0.070	0.065	0.065
	50	0.070	0.070	0.070	0.065	0.065
	70	0.070	0.070	0.065	0.065	0.065
	90	0.070	0.070	0.065	0.065	0.060
	100	0.070	0.065	0.065	0.065	0.060
	110	0.070	0.065	0.065	0.065	0.060
	120	0.070	0.065	0.065	0.065	0.060
	150	0.070	0.065	0.065	0.060	0.060
	200	**0.065**	**0.065**	**0.060**	**0.060**	**0.060**
Calcium silicate		0.075	0.075	0.070	0.070	0.065
Cellular glass		0.070	0.070	0.070	0.065	0.065

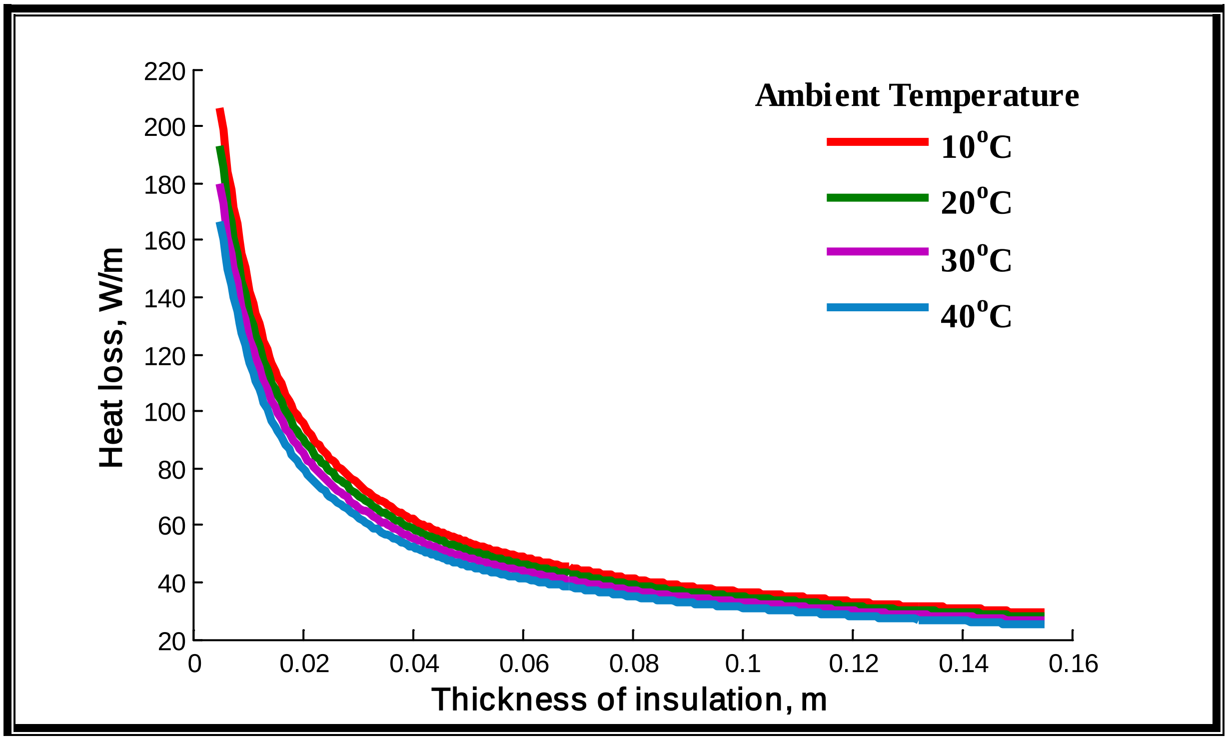


Figure 4-18 Relationship between heat loss and thickness of mineral wool insulation at 200 kg/m^3 for 190˚C gas oil temperature and ε = 0.1.

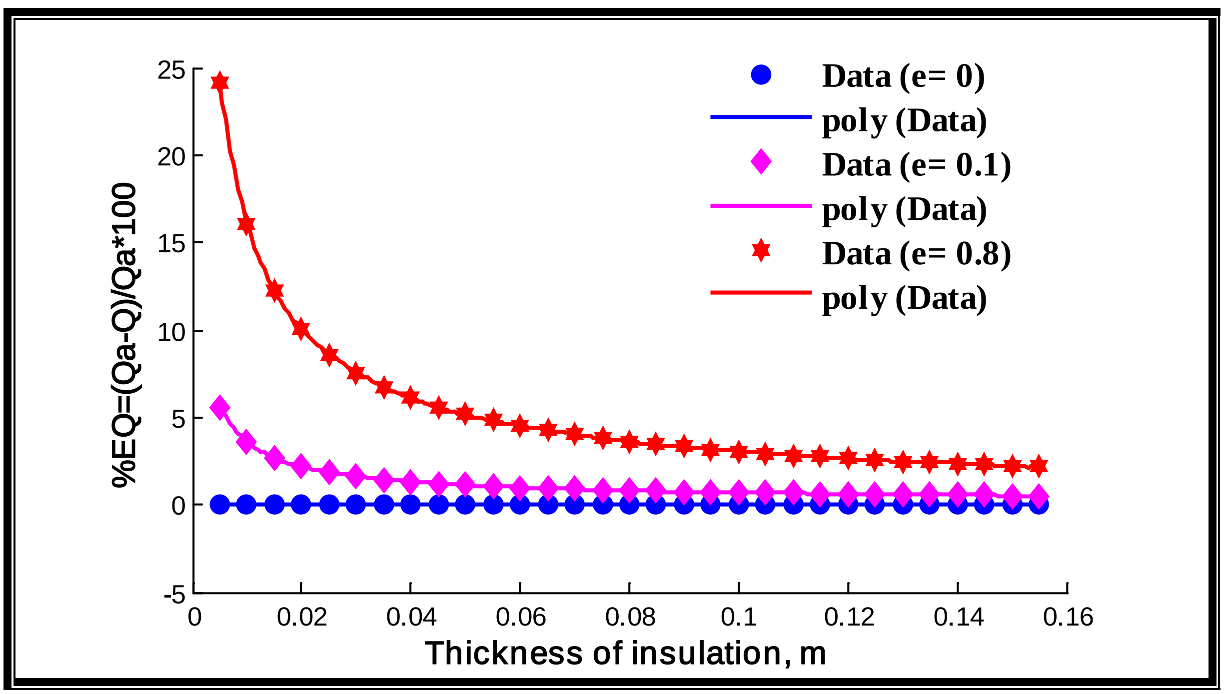


Figure 4-19 The relationship between %EQ and thickness of insulation in the case of 190˚C gas oil temperature and T_0 = 20˚C (case 2) for mineral wool insulation at 200 kg/m^3.

Table 4-3 The influence of neglecting heat radiation using rock mineral wool insulation at 200 kg/m^3, 190˚C gas oil temperature and T_0=20˚C.

emissivity	t_{opt}, **m**	T_s, **˚C**	q_c, **W/m**	q_r, **W/m**	**Heat loss (q_c+q_r), W/m**	**%EQ**
0	0.065	36.5330	44.1448	0	44.1448	0
0.1	0.065	34.8013	38.2935	6.2713	44.5648	0.9424 %
0.8	0.065	28.4112	18.5020	27.5996	46.1016	4.2445%

From the cases investigated in this study, it has been demonstrated that neglecting the influence of heat radiation effect, especially in the situations of low external ambient air convection coefficients, thin insulated thickness and large surface emissivity are likely to produce inaccurate results and this in agreement with [80].

The third case examined is of an insulated pipe containing steam with T_2=200˚C, T_0=20˚C, D_i=0.154 m, $and\ D_0$=0.168 m, using the same single layer of mineral wool, calcium silicate, and cellular glass insulation. The results are shown in figs. 4-20 and 4-21where a comparison among the three insulations for heat loss and total cost versus thickness of insulation is made respectively. Both figures show that the rock mineral wool insulation at 200 kg/m^3 has the best performance (low thermal conductivity). Also for different ambient temperatures, the rock mineral wool insulation at 200 kg/m^3 is the best choice. This is illustrated in table 4-4 where optimum thickness is shown at various ambient temperatures.

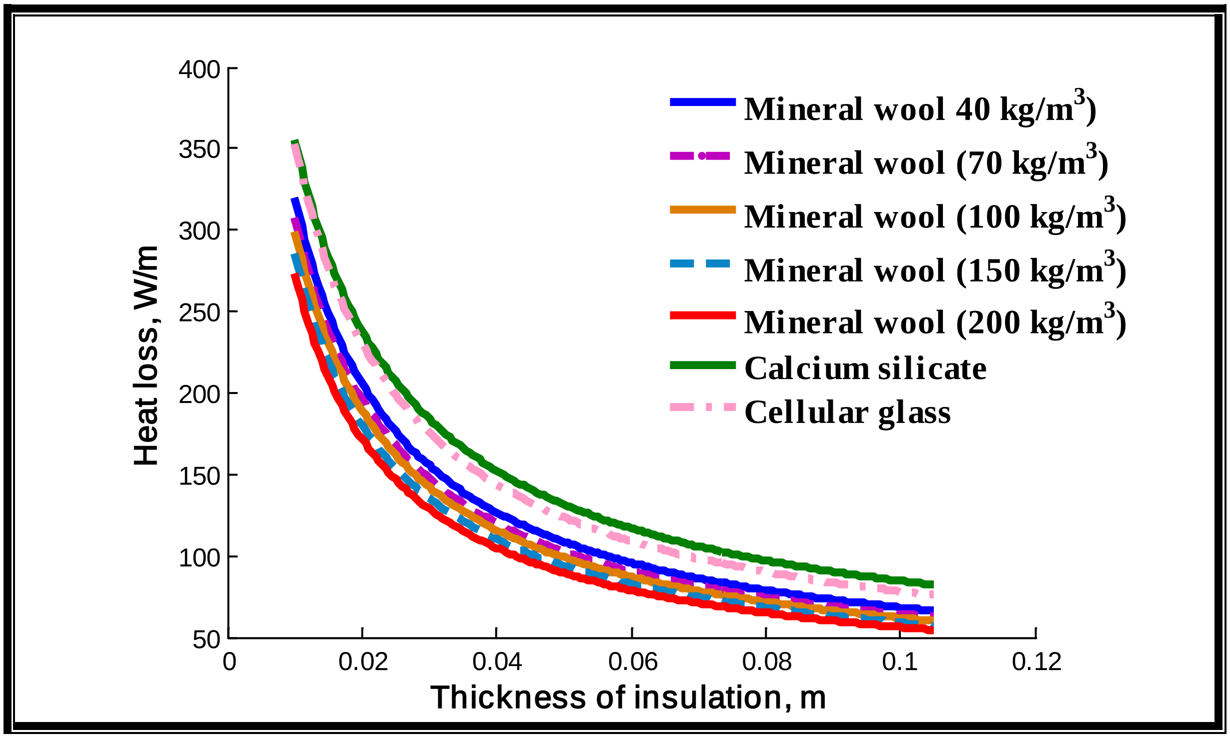


Figure 4-20 Relationship between heat loss and thickness of insulation at 200˚C steam temperature, ε = 0.1 and T_0=20˚C (case 3).

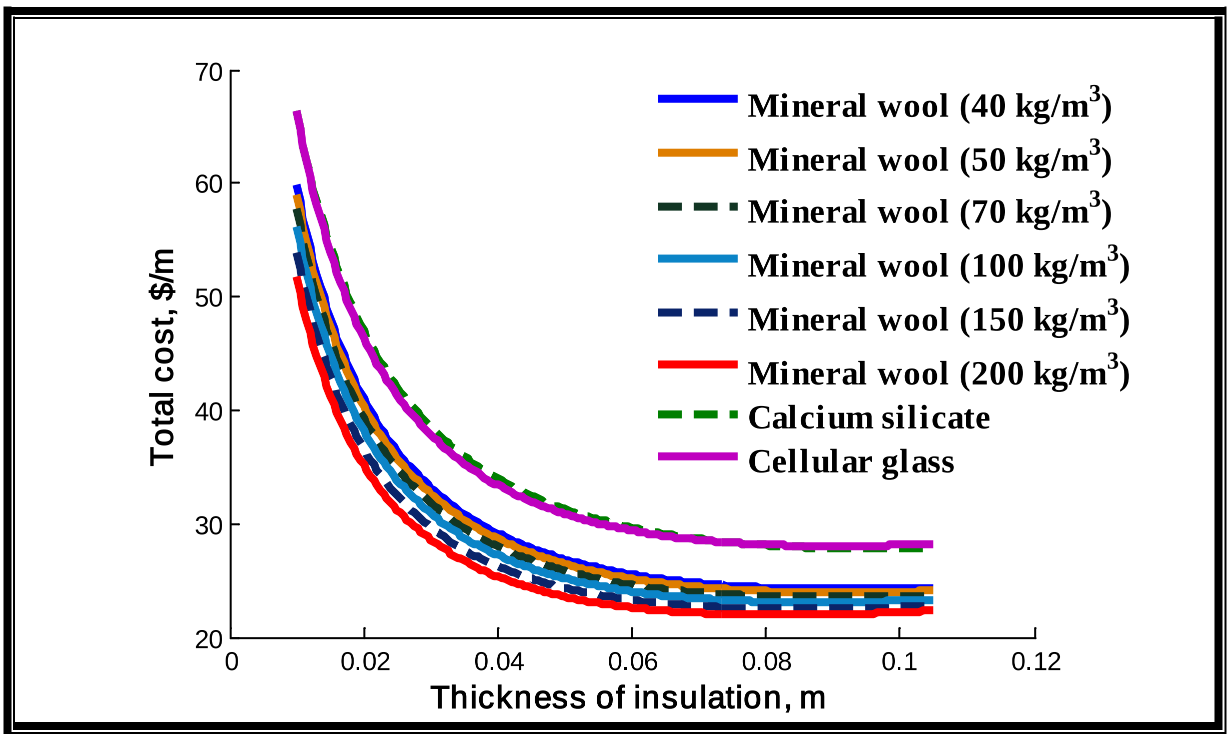


Figure 4-21 Comparison of total cost between Mineral wool, Calcium Silicate, and Cellular Glass at 200˚C steam temperature, ε = 0.1 and T_0=20˚C (case 3).

Table 4-4 Optimum thickness of insulation at different ambient temperatures of third case studied (using steam).

Insulation types		**Ambient temperature, °C**				
		10	**20**	**30**	**40**	**50**
Rock mineral wool, kg/m^3	**40**	0.095	0.090	0.090	0.085	0.085
	50	0.090	0.090	0.090	0.085	0.085
	70	0.090	0.090	0.085	0.085	0.080
	90	0.090	0.085	0.085	0.080	0.080
	100	0.090	0.085	0.085	0.080	0.080
	110	0.090	0.085	0.085	0.080	0.080
	120	0.090	0.085	0.085	0.080	0.080
	150	0.085	0.085	0.080	0.080	0.075
	200	**0.085**	**0.085**	**0.080**	**0.075**	**0.075**
Calcium silicate		0.100	0.100	0.095	0.090	0.090
Cellular glass		0.090	0.090	0.085	0.085	0.085

Figure 4-22 shows optimum insulation thicknesses (t_{opt}) for three cases studied versus ambient temperature from 10 through 50 ℃. It is evident that optimum insulation thickness for (case 3) is higher than (case 2) and the latter is higher than (case1) that refers to increased optimum insulation thickness with increasing operating temperature and nominal pipe diameter. This is because of increased thermal conductivity and then increase of heat loss and its cost as found by [44] where increased optimum insulation thickness with increased thermal conductivity and outside pipe diameter were also realized.

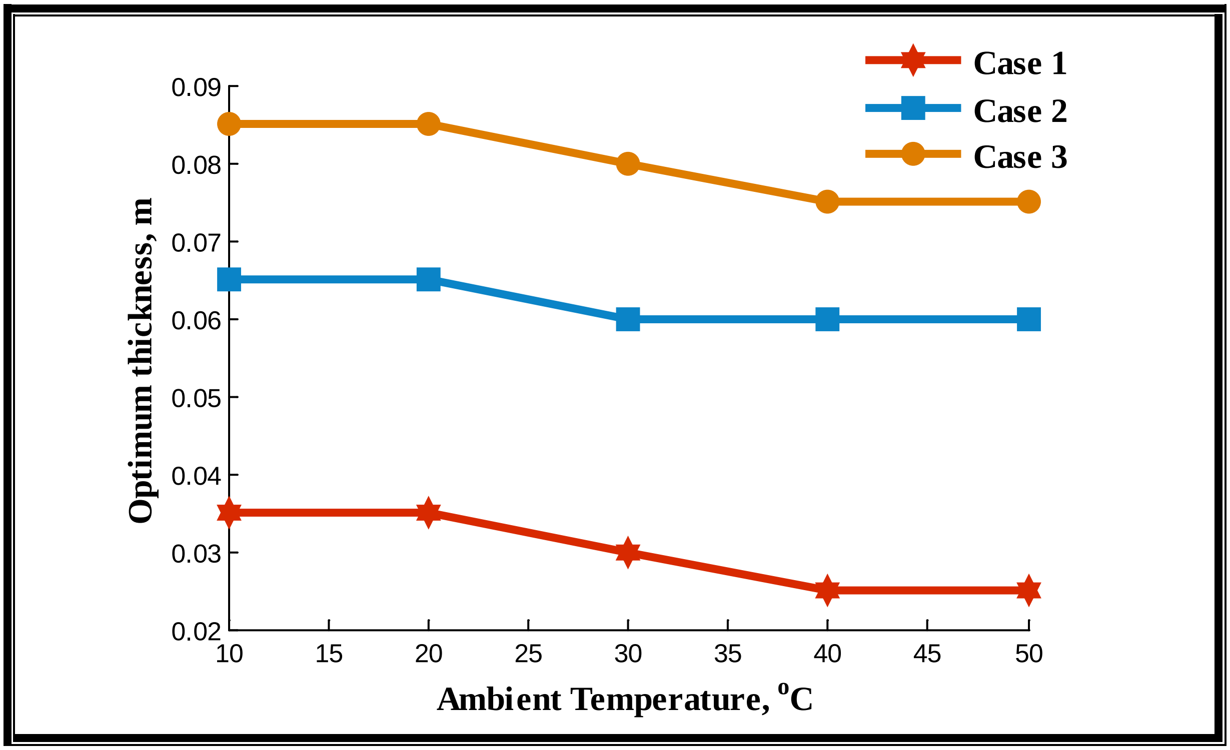


Figure 4-22 Comparison between optimum thicknesses for three cases studied versus ambient temperature

These results agree well with the results of **Al-Khawaja (2004) [37], Li and Chow (2005) [38], Öztürk et al. (2006) [39], Daouas et al. (2010) [43], Chen et al. (2009) [80]** and **Bahadori and Vuthaluru (2010) [44].**

4-4 Optimum insulation thicknesses in the multilayer insulation

The optimum economic insulation thicknesses calculated by linear programming (Matlab7) for composite two –layer insulation are presented in figs. 4-23 through 4-34 of third case studied (NPS= 6 inch, T_2=200˚C, T_0=20˚C and $\varepsilon = 0.1$).

Figure 4-23 shows the calculation results of heat loss of above information. The vertical axis shows heat loss in W/m of pipe. The primary horizontal axis displays the calcium silicate insulator thickness in meters and the secondary horizontal axis shows the cellular glass insulator thickness in meters. It can be seen that if only calcium silicate insulator is used for pipe insulation, the optimum thickness of calcium silicate insulator is 0.1 m and the heat loss at this thickness is 84.5922W/m of pipe and if only cellular glass insulator is used for pipe insulation, the optimum thickness of cellular glass insulator is 0.09 m and the heat loss at this thickness is 83.5414 W/m of pipe. On the other hand, if calcium silicate and cellular glass insulator are used as multilayer for pipe insulation, the optimum thickness of calcium silicate and cellular glass insulator is 0.03 and 0.06 m, respectively. The heat loss is 83.336W/m of pipe.

The results from fig. 4-23 shows that the heat loss decreases with increasing thicknesses of two –layer where the optimum thicknesses selected are depending on the minimum total cost as shown in fig. 4-24.

Figure 4-24 shows the calculation results of minimum total cost of case 1 (in table 4-5). It can be seen that if only calcium silicate insulator is used for pipe insulation, the optimum thickness of calcium silicate insulator is 0.1 m and the minimum total cost of this insulation thickness is 27.7779 $/m of pipe and if only cellular glass insulator is used for pipe insulation, the optimum thickness of cellular glass insulator is 0.09 m and the minimum total cost of this insulation thickness is 28.0374$/m of pipe. On the other hand, if calcium silicate and cellular glass are used as multilayer insulator for pipe insulation, the optimum thicknesses of calcium silicate and cellular glass insulator are 0.03 and 0.06 m, respectively. The minimum total cost is 31.5918 $/m of pipe.

The detailed results for cases studied are presented in tables 4-6 and 4-5 for single and composite two-layer insulation respectively.

Figures 4-26 and 4-30 show that the minimum total cost decreases with first layer of insulation being reduced, while it decreases and then increases with increased second layer of insulation. All this refers to the insulation material cost of first layer being higher than that of second layer insulation. There verse of this is shown in figures 4-28 and 4-34

The results from tables 4-5 and 4-6 show that the minimum total cost of double-layer insulation for case 1 and case 3are illustrated in figs. 4-24 and 4-28 respectively, which is higher than that of only calcium silicate insulation and only cellular glass insulation. This is because the materials cost of calcium silicate and cellular glass is extremely high. In case2 and case 5 in figs. 4-26 and 4-32 respectively, show that the minimum total cost of double-layer insulation is lower than that of only calcium silicate insulation. In contrast, the minimum total cost of double-layer insulation is higher than that of only mineral wool insulation. This is because the material cost of mineral wool is extremely lower than that of calcium silicate insulator. Cases 4 and 6 in figs. 4-30 and 4-34 respectively, show that the minimum total cost of double-layer insulation is lower than that of only cellular glass insulation. In contrast, the minimum total cost of double-layer insulation is higher than that of only mineral wool insulation. This is because the material cost of mineral wool is extremely lower than that of cellular glass insulator.

These results agree generally with the results of **Soponpongpipat et al. (2010) [33] & Zaki and Turki (2000) [58].** The thermo-economics analysis of optimum thickness of double-layer insulation was recommended when the cost of main insulator was higher than that of auxiliary insulator otherwise the thermo-economics analysis of optimum thickness of single-layer insulation was sufficient [33].

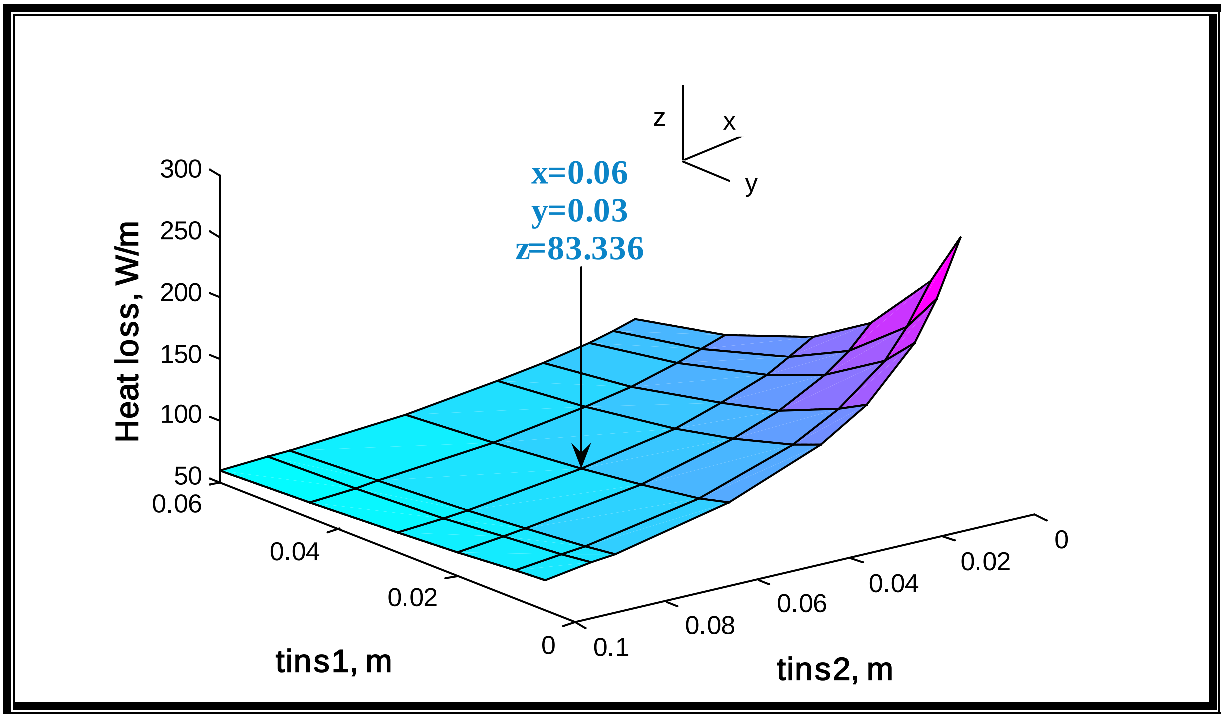


Figure 4-23 The calculation results of heat loss in case of first layer=Calcium silicate and second layer=Cellular glass (case 1 in table 4-5).

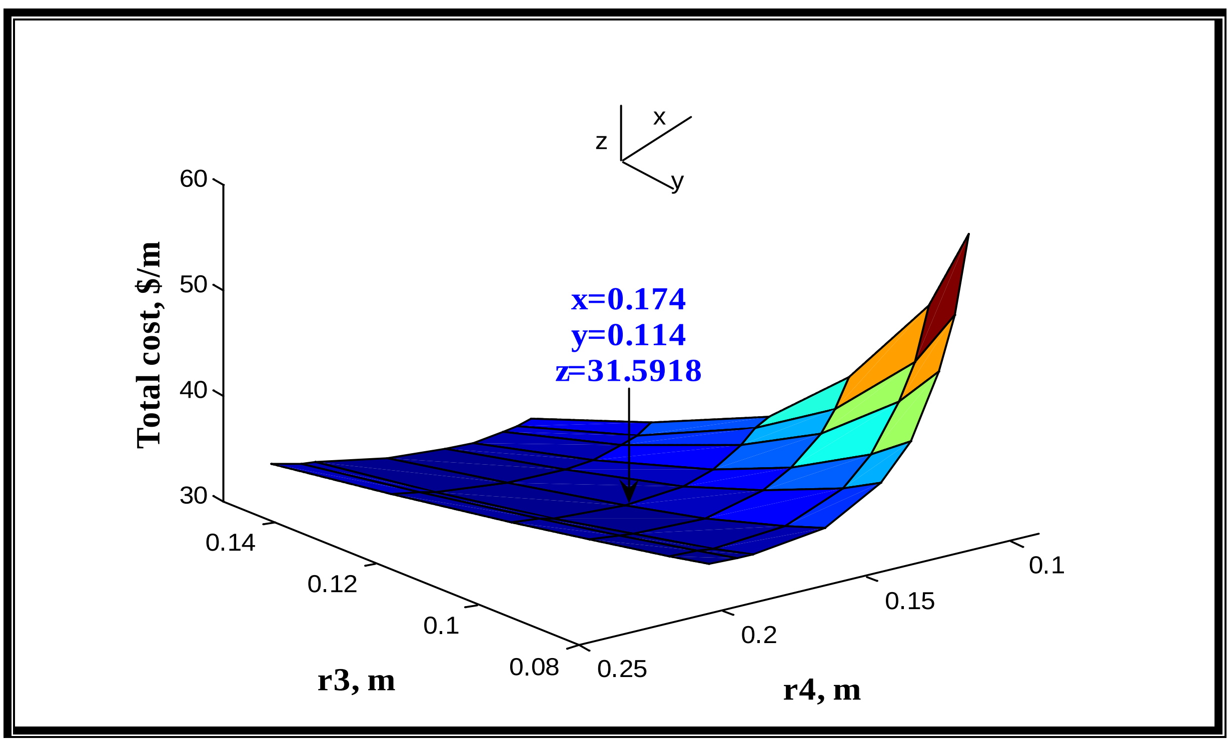


Figure 4-24 The calculation results of total cost in case of first layer=Calcium silicate and second layer=Cellular glass (case 1 in table 4-5).

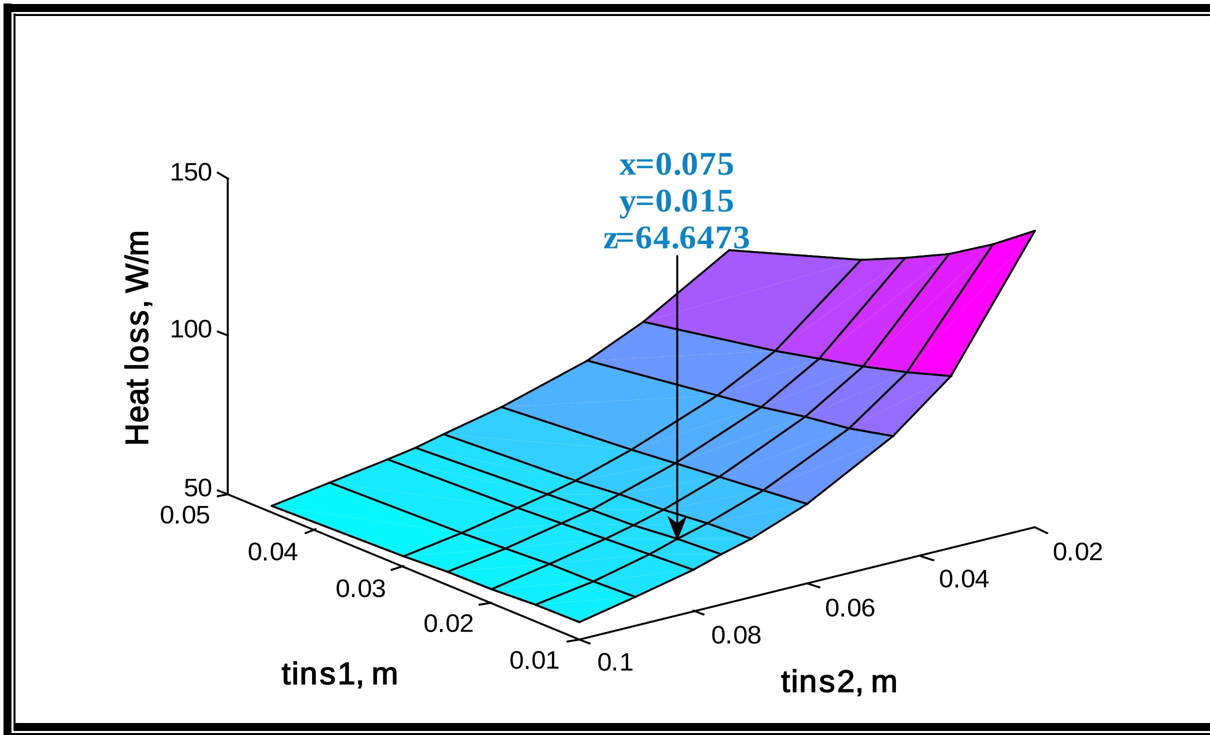


Figure 4-25 The calculation results of heat loss in case of first layer=Calcium silicate and second layer=Mineral wool at 200 kg/m^3(case 2 in table 4-5).

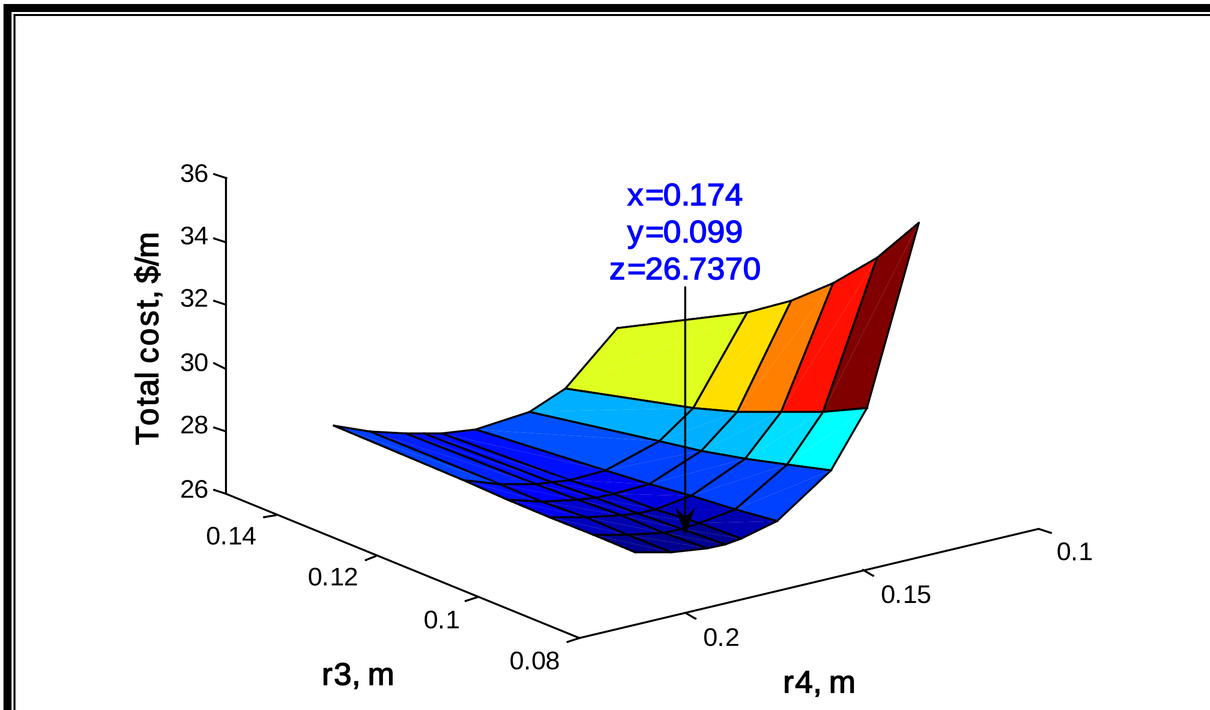


Figure 4-26 The calculation results of total cost in case of first layer=Calcium silicate and second layer=Mineral wool at 200 kg/m^3(case 2 in table 4-5).

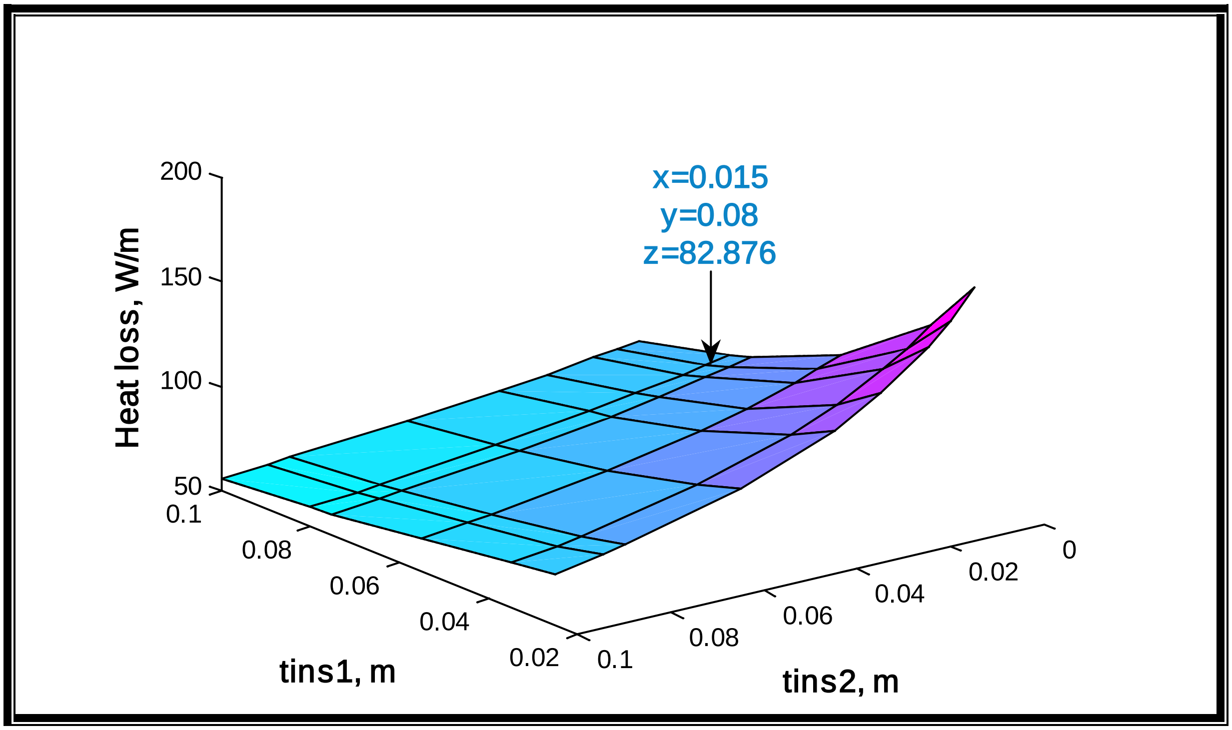


Figure 4-27 The calculation results of heat loss in case of first layer=Cellular glass and second layer=Calcium silicate (case 3 in table 4-5).

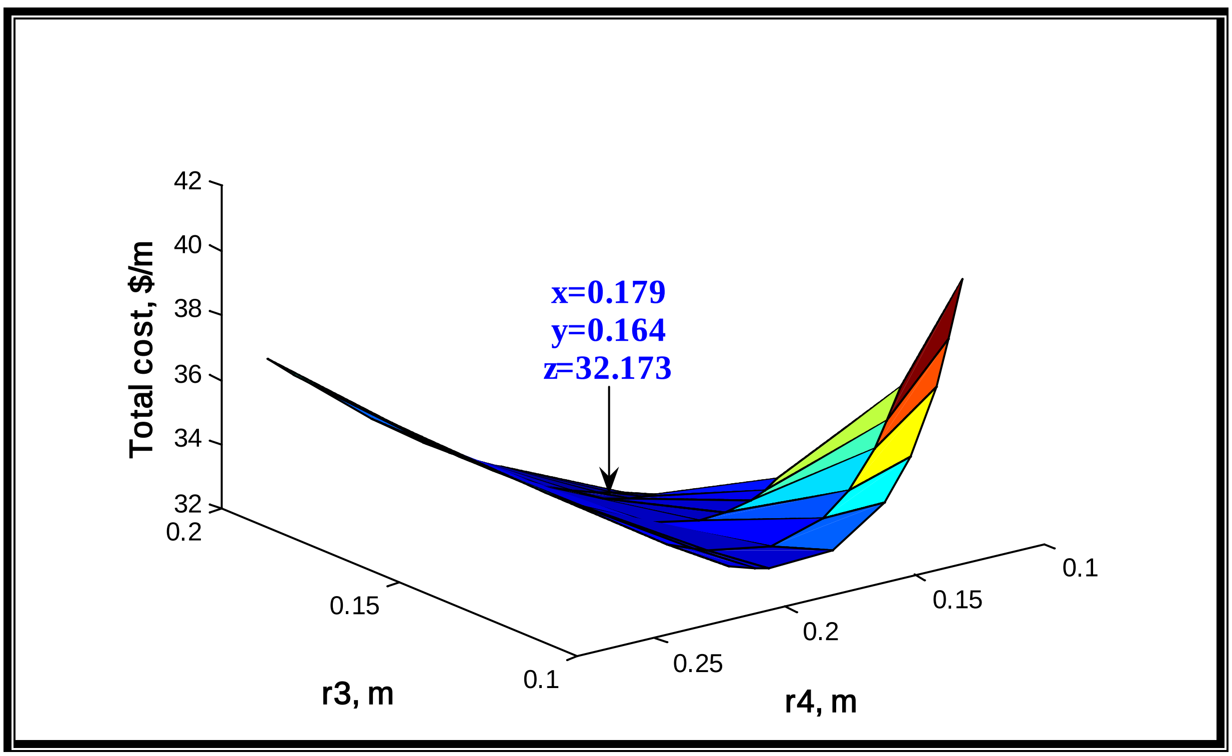


Figure 4-28 The calculation results of total cost in case of first layer=Cellular glass and second layer=Calcium silicate (case 3 in table 4-5).

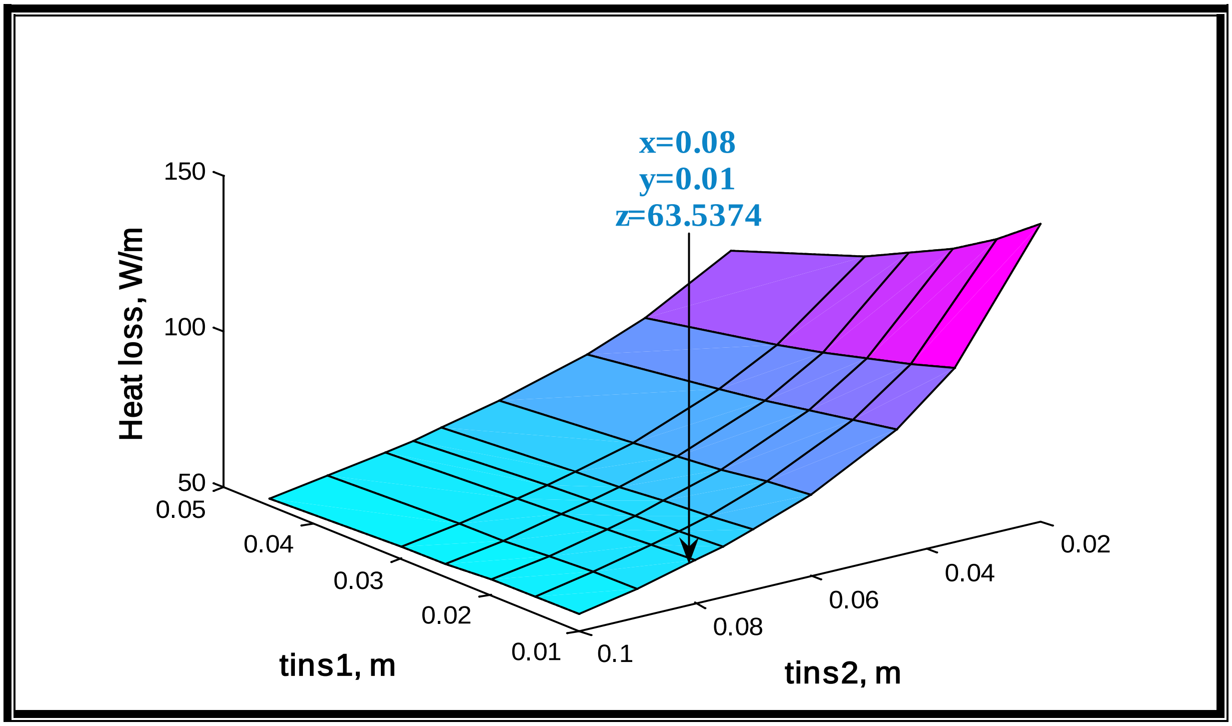


Figure 4-29 The calculation results of heat loss in case of first layer=Cellular glass and second layer=Mineral wool at 200 kg/m^3(case 4 in table 4-5).

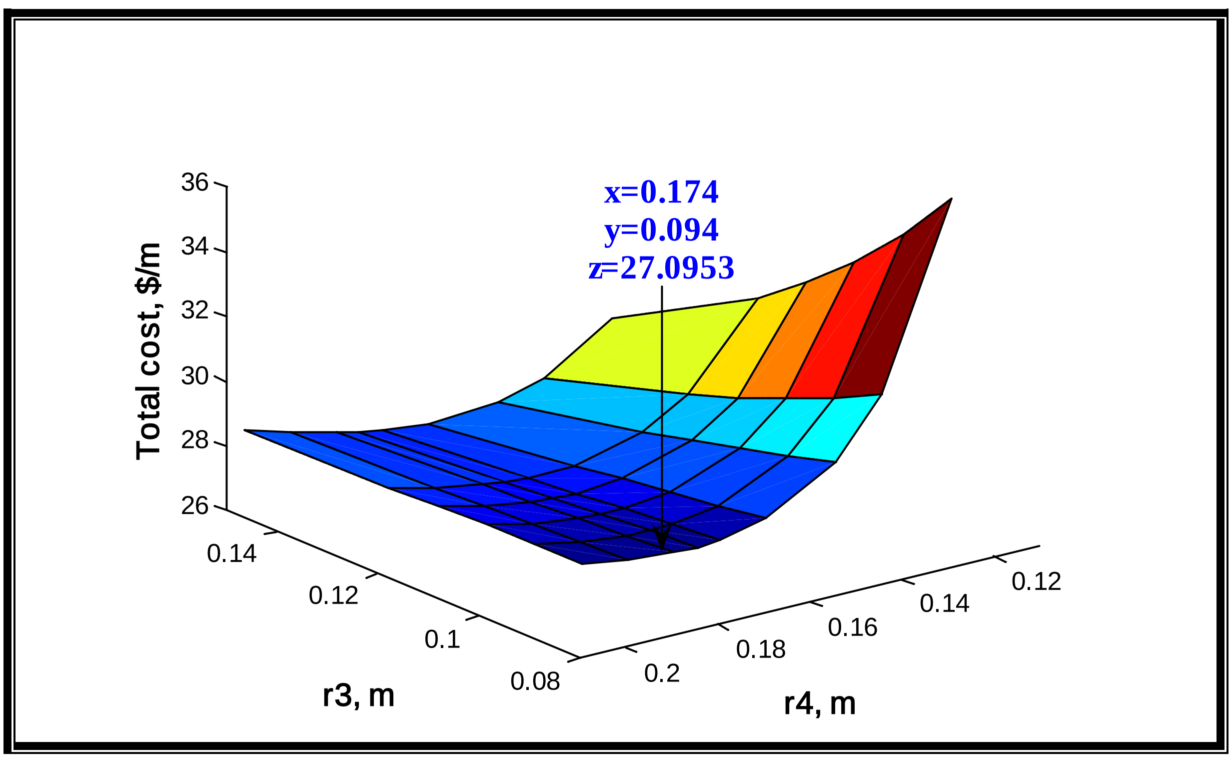


Figure 4-30 The calculation results of total cost in case of first layer=Cellular glass and second layer=Mineral wool at 200 kg/m^3(case 4 in table 4-5).

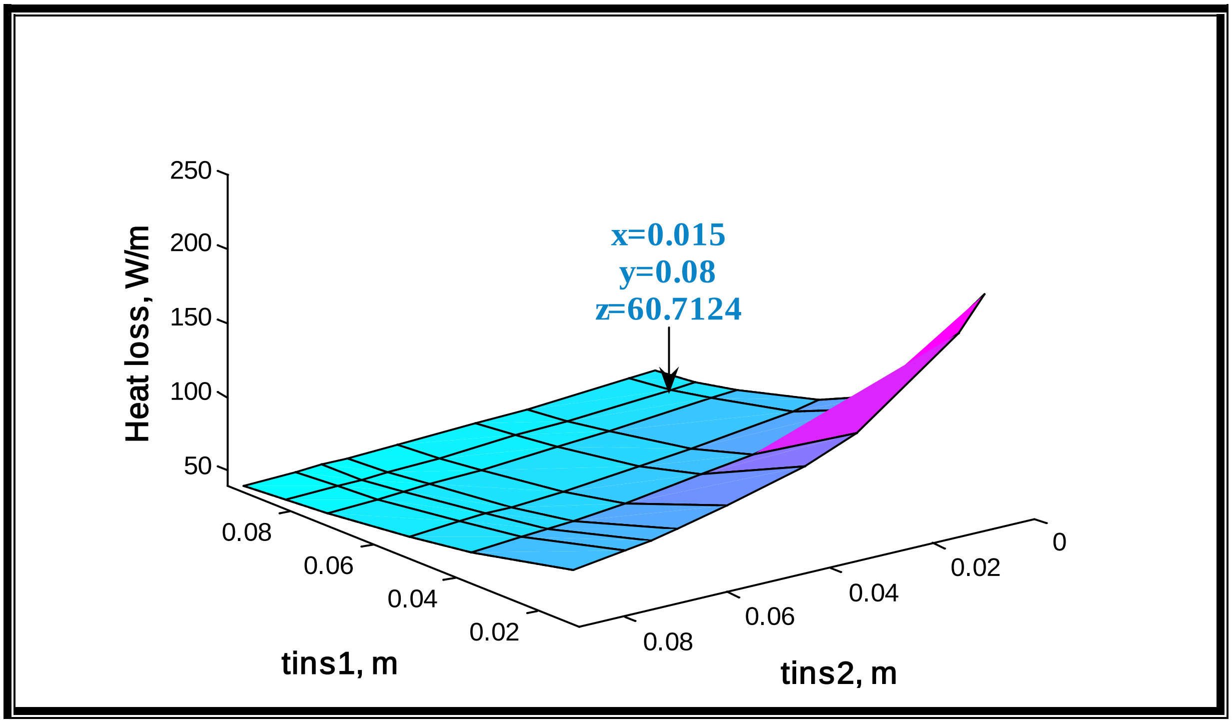


Figure 4-31 The calculation results of heat loss when first layer= Mineral wool at 200 kg/m^3 and second layer=Calcium silicate (case 5 in table 4-5).

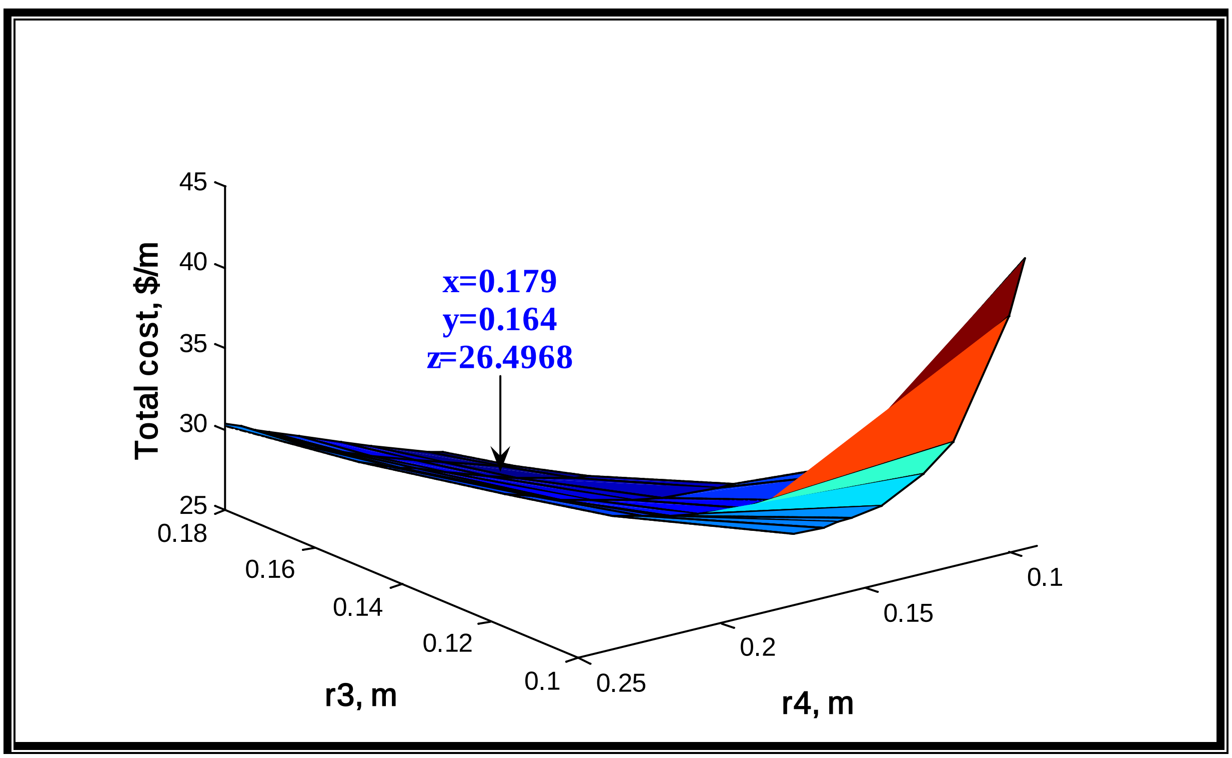


Figure 4-32 The calculation results of total cost when first layer=Mineral wool at 200 kg/m^3 and second layer=Calcium silicate (case 5 in table 4-5).

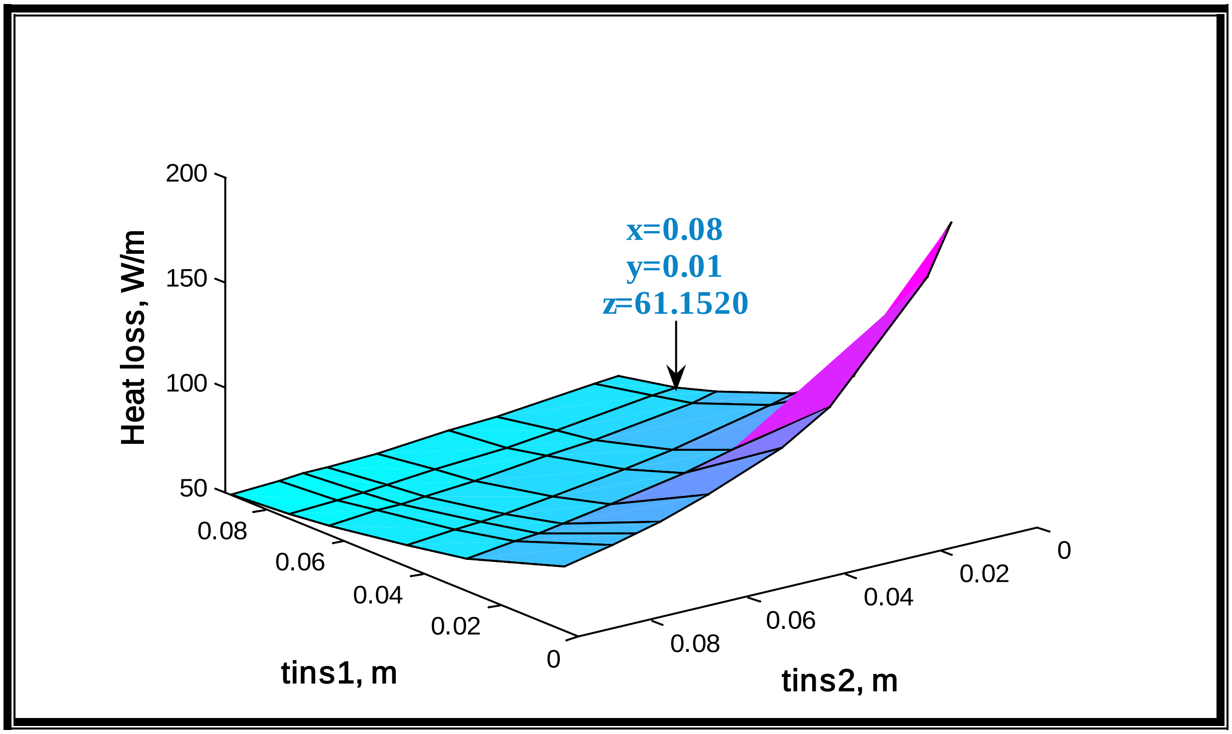


Figure 4-33 The calculation results of heat loss when first layer= Mineral wool at 200 kg/m^3 and second layer=Cellular glass (case 6 in table 4-5).

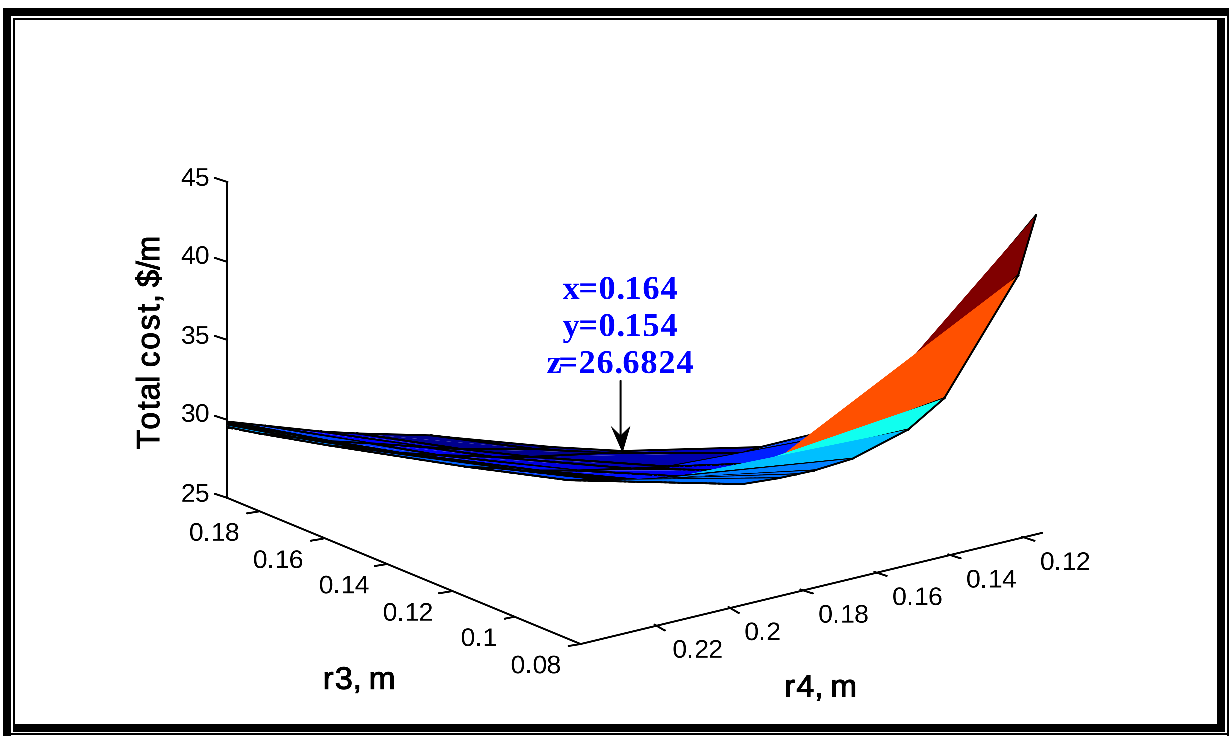


Figure 4-34 The calculation results of total cost when first layer= Mineral wool at 200 kg/m^3 and second layer=Cellular glass (case 6 in table 4-5).

Table 4-5 Optimum composite two-layer insulation in case of third case studied (using steam).

case	**First layer**	**Second layer**	t_{ins1}, **m**	t_{ins2}, **m**	Q_a, **W/m**	T_{4a}, **°C**	C_T, **$/m**
1	Calcium silicate	Cellular glass	0.030	0.060	83.3360	37.3562	31.5918
2	Calcium silicate	Mineral wool (200 kg/m^3)	0.015	0.075	64.6473	34.1892	26.7370
3	Cellular glass	Calcium silicate	0.080	0.015	82.8764	36.9240	32.1730
4	Cellular glass	Mineral wool (200 kg/m^3)	0.010	0.080	63.5374	33.9954	27.0953
5	Mineral wool (200 kg/m^3)	Calcium silicate	0.080	0.015	60.7124	33.2207	26.4968
6	Mineral wool (200 kg/m^3)	Cellular glass	0.080	0.010	61.1520	33.5765	26.68.24

Table 4-6 Optimum single layer insulation of third case studied (using steam).

Single layer	t_{ins}, **m**	Q_a, **W/m**	T_{4a}, **°C**	C_T, **$/m**
Calcium silicate	0.100	84.5922	36.8560	27.7779
Cellular glass	0.090	83.5414	37.3901	28.0374
Mineral wool (200 kg/m^3)	0.085	62.4168	34.0977	21.9582

Chapter five

Conclusions and recommendations

5-1 Conclusions

1. The thermal conductivity generally increases with temperature for the three insulations studied. The results indicate that the lowest thermal conductivity is obtained using the mineral wool insulation at a density equals 200 kg/m^3.
2. The behavior of thermal conductivity with density: at low temperatures the thermal conductivity decreases with density for a particular range and then increases. At high temperatures, the thermal conductivity decreases with increased density.
3. From the cases investigated in this study, it has been demonstrated that neglecting the influence of heat radiation effect, especially at situations of low external ambient air convection coefficients, thin insulation layer and large surface emissivity are likely to produce inaccurate results.
4. The optimum insulation thicknesses for single layer (mineral wool insulation at 200 kg/m^3) of three cases studied (hot water at 98˚C, gas oil at 190˚C and steam at 200˚C) at 20˚C ambient temperature is 0.035, 0.065 and 0.085 m respectively. The heat losses are 20.47, 44.57 and 62.42 W/m respectively.
5. The optimum insulation thicknesses for composite two-layers of six cases studied at 20˚C ambient temperature is 0.03, 0.015, 0.08, 0.01, 0.08, and 0.08 m for first layer and 0.06, 0.075, 0.015, 0.08, 0.15 and 0.01 m for

second layer respectively. The corresponding heat losses are 83.34, 64.65, 82.88, 63.54, 60.71, and 61.15W/m.

6. From the results of composite two layers of (calcium silicate & mineral wool) and (cellular glass & mineral wool) better optimum is obtained than using single layer of calcium silicate or cellular glass. But minimum total cost of double-layer insulation is higher than that of using only mineral wool insulation.

5-2 Recommendations

1. Using insulation materials with better properties, i.e. lower thermal conductivity, not combustible, not absorbing water, and lower cost (Pyrogel®XT).
2. High technical equipment should be used (NIST 1-meter Guarded Hot Plate, and use of heat flow meter ASTM C518 devices)in order to determine the effect of bulk density, moisture content, mean temperature, and barometric pressure, on the thermal conductivity, and using Lambda 2000 to measure the effective thermal conductivity.
3. In multilayer insulation lower cost materials must be used and selected for composite layers insulations depending on the relation of mean temperature with thermal conductivity of insulation.
4. The influence of heat radiation effect may be neglected depending on outermost insulation surface temperature.

References:

1. Kenneth, R. C. and Kathleen, M. P., *Piping handbook,* 7th ed., New York: McGraw-Hill, 2000, pp. 287-349.
2. Eduardo Cao, HEAT TRANSFER IN PROCESS ENGINEERING, McGraw-Hill, New York, 2010.
3. BS 5970:2001, Code of practice for thermal insulation of pipe work and equipment in the temperature range of –100 ºC to +870 ºC, BSI 21 December 2001.
4. Tomas, H.K., Courillon, R.J., John, W.C., Energy management series for industry commerce and institutions, Process Insulation, series 1, 1987
5. ASHRAE, Fundamentals Handbook, 2005.
6. Daryabeigi, K., Heat Transfer Modeling and Validation for Optically Thick Alumina Fibrous Insulation, NASA Langley Research Center, MS 190, Hampton, VA 23681, 2010.
7. Daryabeigi, K., “Analysis and Testing of High Temperature Fibrous Insulation for Reusable Launch Vehicles,” AIAA Paper 99-1044, January 1999.
8. Al-Ajlan, S. A., Measurements of thermal properties of insulation materials by using transient plane source technique, Applied Thermal Engineering 26 (2006) 2184–2191.
9. Song, W. F. and Yu, W. D., Study on radiative heat transfer property of fiber assemblies using FTIR, *J Therm Anal Calorim*, September 2010.
10. Larkin, B. K., and Churchill, S. W., “Heat Transfer by Radiation through Porous Insulations,” *American Institute of Chemical Engineers Journal,* Vol. 4, No. 5, 1959, pp. 467-474.
11. Tong, T. W., and Tien, C. L., “Radiative Heat Transfer in Fibrous

Insulations-Part I: Analytical Study," *Journal of Heat Transfer*, Vol. 105, February 1983, pp. 70-75.

12. Tong, T. W., Yang, Q. S., and Tien, C. L., "Radiative Heat Transfer in Fibrous Insulations- Part II: Experimental Study," *Journal of Heat Transfer*, Vol. 105, February 1983, pp. 76-81.
13. Petrov, V.A., Combined radiation and conduction heat transfer in high temperature fiber thermal insulation, *International Journal of Heat and Mass Transfer*, Vol. 40, Issue 9(1997)2241-2247.
14. Daryabeigi, K., "Analysis and Testing of High Temperature Fibrous Insulation for Reusable Launch Vehicles," AIAA Paper 99-1044, January1999.
15. Tseng, C., and Kuo, K., THERMAL PROPERTIES OF PHENOLIC FOAM INSULATION , *Journal of the Chinese Institute of Engineers*, Vol. 25, No. 6 (2002) pp. 753-758.
16. Wang, H., Dinwiddie, R.B. and Wilkes, K. E., G-Plus Report to Owens Corning Thermal Conductivity Measurements of Fiberglass, Oak Ridge National Laboratory ORNL/TM-2003/98.
17. Karamanos, A.K., Papadopoulos, A.M. and Anastaselos, D. (2004), Heat transfer phenomena in fibrous insulating materials, 2004 IASME / WSEAS International Conference on "HEAT AND MASS TRANSFER", Proceedings of the Conference, CD Rom, 17-19.08.04, Corfu, Greece.
18. Placido, E., Arduini-Schuster, M.C. and Kuhn, J., Thermal properties predictive model for insulating foams, *Infrared Physics & Technology* 46 (2005) 219–231.
19. Abdulagatov, I. M., Emirov, S. N., Abdulagatova, Z. Z. and Askerov, S. Ya., Effect of Pressure and Temperature on the Thermal Conductivity of

Rocks, *J. Chem. Eng 51* (1) 2006, pp 22–33.

20. Manohar, K., Ramlakhan, D., Kochhar,G., and Haldar S. , "Biodegradable Fibrous Thermal Insulation," *J. of the Braz. Soc. of Mech. Sci. & Eng.* No. 1 Vol. XXVIII, January-March 2006.
21. Wei, G., Zhang, X. and Yu, F., Thermal Conductivity of Xonotlite Insulation Material, *Int J Thermophys* 28 (2007) 1718–1729.
22. Mishra, S. C., Krishna, N.A., Gupta, N. and Chaitanya, G.R., Combined conduction and radiation heat transfer with variable thermal conductivity and variable refractive index, *International Journal of Heat and Mass Transfer* 51 (2008) 83–90.
23. Wang, M. and Pan, N. Modeling and prediction of the effective thermal conductivity of random open-cell porous foams, *International Journal of Heat and Mass Transfer* 51 (2008) 1325–1331.
24. Ochs, F., Heidemann, W., Mu¨ller-Steinhagen, H., Effective thermal conductivity of moistened insulation materials as a function of temperature, *International Journal of Heat and Mass Transfer* 51 (2008) 539–552.
25. Veiseh, S., Khodabandeh, N., and Hakkaki-Fard, A., "MATHEMATICAL MODELS FOR THERMAL CONDUCTIVITYDENSITY RELATIONSHIP IN FIBROUS THERMAL INSULATIONS FOR PRACTICAL APPLICATIONS," *ASIAN JOURNAL OF CIVIL ENGINEERING (BUILDING AND HOUSING)* Vol. 10, NO. 2 (2009) 201-214.
26. Pen-Chang Tseng and Hsin-Sen Chu, An experimental study of the heat transfer in PS foam insulation, *Heat Mass Transfer* 45 (2009) 399–406.
27. Shu-yuan Zhao, Bo-ming Zhang, and Xiao-dong He, "Temperature and pressure dependent effective thermal conductivity of fibrous insulation,"

International Journal of Thermal Sciences ,Vol. 48, Issue 2, February 2009, pp. 440-448.

28. Domínguez-Muñoz, F., Anderson, B., Cejudo-López, J. M. and Carrillo-Andrés, A., Uncertainty in the thermal conductivity of insulation materials, *Energy and Buildings*, Vol. 42, Issue 11, November 2010, pp. 2159-2168.
29. Yüksel, N., Avcı, A. and Kılıç, M., The temperature dependence of effective thermal conductivity of the samples of glass wool reinforced with aluminium foil, *International Communications in Heat and Mass Transfer* Vol. 37, Issue 6, July 2010, pp. 675-680.
30. Kamran Daryabeigi, George R. Cunnington, Steve D. Miller, and Jeffry R. Knutson, “Combined Heat Transfer in High-Porosity High- Temperature Fibrous Insulations: Theory and Experimental Validation,” 10th AIAA/ASME Joint Thermophysics and Heat Transfer Conference, June 2010.
31. Daryabeigi, K., “Heat Transfer in High-Temperature Fibrous Insulation,” *Journal of Thermophysics and Heat Transfer*, Vol. 17, No. 1, 2003, pp. 10-20.
32. Nelson, F., P.J. Mago and L.M. Chamra, Energy and economic evaluation of cooling, heating and power systems based on primary energy. Applied Thermal Eng. 29 (2009) 2665-2671.
33. Soponpongpipat, N., Jaruyanon, P. and Nanetoe, S., “The Thermo-Economics Analysis of the Optimum Thickness of Double-Layer Insulation for Air Conditioning Duct,” Energy Research Journal 1 (2) 2010,pp. 146-151.
34. Ahmad, E. H., COST ANALYSIS AND THICKNESS OPTIMIZATION OF THERMAL INSULATION MATERIALS USED IN RESIDENTIAL

BUILDINGS IN SAUDI ARABIA, *The 6th Saudi Engineering Conference,* KFUPM, Dhahran, vol. 1, December 2002, pp.21-32.

35. Kalyon, M. and Sahin, A. Z., APPLICATION OF OPTIMAL CONTROL THEORY IN PIPE INSULATION, *Numerical Heat Transfer*, Part A, 41 (2002) 391– 402.
36. Çomaklı, K. and Yüksel, B. , Optimum insulation thickness of external walls for energy saving, *Applied Thermal Engineering*, Vol. 23, Issue 4, March 2003, pp. 473-479.
37. Al-Khawaja, M.J., Determination and selecting the optimum thickness of insulation for buildings in hot countries by accounting for solar radiation, Applied Thermal Engineering 24 (2004) 2601–2610.
38. Li, Y. F. and Chow, W. K., Optimum insulation-thickness for thermal and freezing protection, *Applied Energy*, Vol. 80, Issue 1, January 2005, pp. 23-33.
39. Öztürk, İ.T., Karabay, H. and Bilgen, E., Thermo-economic optimization of hot water piping systems: A comparison study , *Energy* Vol. 31, Issue 12, September 2006, pp. 2094-2107.
40. Dombayci, O. A., Golcu, M., and Pancar, Y., Optimization of insulation thickness for external walls using different energy-sources, Applied Energy 83 (9) (2006) 921–928.
41. Mahlia, T.M.I., Taufiq, B.N., Ismail and Masjuki, H.H., Correlation between thermal conductivity and the thickness of selected insulation materials for building wall , *Energy and Buildings* Vol. 39, Issue 2, February 2007, pp. 182-187.
42. Yu, J., Yang, C., Tian, L. and Liao, D., A study on optimum insulation thicknesses of external walls in hot summer and cold winter zone of China,

Applied Energy, Vol. 86, Issue 11, November 2009, pp. 2520-2529.

43. Daouas, N., Hassen, Z. and Habib Ben Aissia, Analytical periodic solution for the study of thermal performance and optimum insulation thickness of building walls in Tunisia, *Applied Thermal Engineering* 30 (2010) 319–326.
44. Bahadori, A. and Vuthaluru, H.B., “A simple correlation for estimation of economic thickness of thermal insulation for process piping and equipment,” *Applied Thermal Engineering* 30 (2010) 254–259.
45. Ucar, A. and Balo, F., Determination of the energy savings and the optimum insulation thickness in the four different insulated exterior walls, *Renewable Energy*, Vol. 35, Issue 1, January 2010, pp.88-94.
46. Naouel Daouas, “A study on optimum insulation thickness in walls and energy savings in Tunisian buildings based on analytical calculation of cooling and heating transmission loads,” *Applied Energy* Vol. 88, Issue 1, January 2011, pp.156-164.
47. Kaur, J., Singh, S. P., Sawhney, R. L., and Sodha, M. S., Optimum Layer Distribution of a Building Component, *Int. J. Energy Res*., vol. 15, pp. 11–18, 1991.
48. Kumar, A., Ashutosh, S., and Sodha, M. S., Optimum Distribution of Insulation over Various Components of an Air- Conditioned Building, *Buildg. Environ*., vol. 24, pp. 69–178, 1989.
49. Sofrata, H., and Salmeen, B., Selection of Thermal Insulation Thickness, *Proc. Fourth Saudi Engineering Conf*., King Abdulaziz Univ., Jeddah, vol. 5, pp. 389–394, 1995.
50. Al-Turki, A. M., and Zaki, G. M., Cooling Load Response for Building Walls Comprising Heat Storage and Thermal Insulation Layers, *J. Energy Conv. Mgmt*., vol. 32, no. 3, pp. 235–247, 1991.

51. Lim, J. S., and Bejan, A., Two Fundamental Problems of Refrigerator Thermal Insulation Design, *Heat Transfer Eng*., vol. 15, no. 3, pp. 35–41, 1994.

52. Butch, B., Spreadsheets for Heat Loss Rates and Temperatures, *Chem. Eng*., vol. 102, no. 12, pp. 107–109, 1995.

53. Aziz, A., Critical Thickness of Insulation, *Heat Transfer Eng*., vol. 18, no. 2, pp. 61–91, 1997.

54. Petal, M. R., and Mehta, B. P. S., Optimize Thermal Insulation, *J. Hydrocarbon Processing*, pp. 51–55, October 1993.

55. Turner, W. C., and Malley J. F., *Handbook of Thermal Insulation Design Economics for Pipes and Equipment*, pp. 15–30, McGraw-Hill, New York, 1989.

56. Winiarski, D. W., and Somasundaram, S., Analysis of Cost- Effective Pipe Insulation Requirements, *Energy Eng*., vol. 95, no. 3, pp. 32–52, 1997.

57. Bunday, D. B., *Basic Optimization Methods*, pp. 93–97, Edward Arnold, London, 1984.

58. ZAKI, G. M. and AL-TURKI, A. M., Optimization of Multilayer Thermal Insulation for Pipelines, *Heat Transfer Engineering* 21(2000) 63–70.

59. Iranian Petroleum Standard (IPS), Engineering Standard For ThermalInsulations, first ed., Report IPS-E-TP-700, National Iranian Oil Company, Ahwaz, Iran, January 1996.

60. Gathright, T.R. and Reeve, P.A., EFFECT OF MULTILAYER INSULATION ON RADIATION HEAT TRANSFER AT CRYOGENIC TEMPERATURES, *IEEE TRANSACTIONS ON MAGNETICS*, Vol. 24, No. 2, March 1988.

61. Chorowski, M., Grzegory, P., Parente, C. and Germana Riddone,

Experimental and Mathematical Analysis of Multilayer Insulation below 80 K, 18th International Cryogenic Engineering Conference (ICEC 18), Bombay Mumbai, India, February 2000,21-25.

62. Markus, S., Edgar R.F. Winter, Raymond, V., and Thomas, S., Theoretical studies of high-temperature multilayer thermal insulations using radiation scaling, *Journal of Quantitative Spectroscopy & Radiative Transfer* 84 (2004) 477–491.

63. Augusto, P. A., Castelo-Grande, T., Augusto, P. and Barbosa, D., Optimization of refrigerated shields using multilayer thermal insulation: Cryostats design – analytical solution, *Cryogenics* 46 (2006) 449–457.

64. Li, P. and Cheng, H., Thermal analysis and performance study for multilayer perforated insulation material used in space, Appl. Therm. Eng. 26 (2006) 2020–2026.

65. Ohmori, T., Thermal performance of multilayer insulation around a horizontal cylinder, *Cryogenics* 45 (2006) 725–732.

66. Gyoung-Seok Choi, Jae-Sik Kang, Young-Sun Jeong, Seung-Eon Lee, and Jang-Yeul Sohn, "An experimental study on thermal properties of composite insulation," *6th KSTP Symposium, 6th Symposium of the Korean Society of Thermophysical Properties*, Vol. 455, Issues 1-2, 1 April 2007, pp. 75-79.

67. Chen, J. , and Yu, W., A numerical analysis of heat transfer in an evacuated flexible multilayer insulation material, J Therm Anal Calorim 101(2010) 1183–1188.

68. Johnson, W. L., Demko, J. A., Fesmire, J. E., and Weisend, J. G., ANALYSIS AND TESTING OF MULTILAYER AND AEROGEL INSULATION CONFIGURATIONS, *American Institute of Physics (AIP) Conference Proceedings*, Vol. 1218, Issue 1(2010) 780-787.

69. Johnson, W. L., and Weisend, J. G., OPTIMIZATION OF LAYER DENSITIES FOR MULTILAYERED INSULATION SYSTEMS, *American Institute of Physics (AIP) Conference Proceedings*, vol.1218, Issue 1(2010) 804-811.
70. Bankvall, C., 1974, "Mechanism of Heat Transfer in Permiable Insulation and their Investigation in a Special Guarded Hot Plate," Heat Transmission Measurements in Thermal Insulation, American Society for Testing and Materials, ASTM STP 544, pp. 34-38.
71. Turner, W. C., and Malley J. F., Handbook of Thermal Insulation Design Economics for Pipes and Equipment, McGraw-Hill, New York, 1989.
72. Thermal insulation economics, *Thermal analysis systems company*, American Society of Mechanical Engineers, Nov. 1999 http://findarticles.com/p/articles/mi_qa5325/is_199911/ai_n21446423/?tag=content;col1.
73. Holman J P, Heat transfer, 6th Edition, McGraw-Hill, 1986.
74. 'Standard Practice for Estimate of the Heat Gain or Loss and the Surface Temperatures of Insulated Flat, Cylindrical, and Spherical Systems by Use of Computer Programs', *Annual Book of ASTM Standards*, Vol. 04.06, American Society for Testing and Materials, C 680 – 2003, pp.1-6.
75. Incropera, F. P., and DeWitt, D. P., *Fundamentals of Heat and Mass Transfer*, 3rd ed., John Wiley & Sons, New York, NY, 1990.
76. Churchill, S. W., and Chu, H. H. S., "Correlating Equations for Laminar and Turbulent Free Convection from a Horizontal Cylinder," *Int. J. Heat and Mass Transfer*, Vol. 18, 1975, pp. 1049-1053.
77. Yunus A. Çengel, 'Heat transfer',2^{nd} Edition, July 2002.
78. U.S. Energy Efficiency and Renewable Energy Office of Industrial

Technologies , industrial insulation for Systems Operating Above Ambient Temperature, Washington, D.C. 20585,September, 1995 ORNL/M-4678.

79. (Thermal conductivity coefficient according to the density of rock) http://www.izober.org/download/isil_iletkenlik_eng.pdf.

80. Yen-Hsun Chen, King-Leung Wong, I-Pin Kuo, Chuan-Huang Lin, Jin-Yu Lin ,The Heat Transfer Characteristics of an Insulated Circular Duct Considering and Neglecting the Influence of Heat Radiation, Proceedings of the International Multi Conference of Engineers and Computer Scientists 2009 Vol. II, IMECS 2009, March 18 - 20, 2009, Hong Kong

Appendices

Appendix A

Table A-1 Data of the thermal conductivity for rock wool insulation [79]:

Density, Kg/m^3	Thermal conductivity, W/m. K									
	10˚C	20˚C	30˚C	40˚C	50˚C	60˚C	100˚C	200˚C	300˚C	400˚C
40	0.033	0.036	0.037	0.038	0.040	0.041	0.048	0.065	0.080	0.101
50	0.033	0.036	0.037	0.038	0.040	0.041	0.048	0.065	0.080	0.100
70	0.031	0.035	0.036	0.039	0.041	0.042	0.050	0.067	0.081	0.099
90	0.030	0.034	0.035	0.037	0.039	0.040	0.041	0.063	0.075	0.097
100	0.029	0.033	0.034	0.036	0.038	0.040	0.041	0.057	0.073	0.095
110	0.029	0.033	0.034	0.036	0.038	0.040	0.041	0.060	0.072	0.093
120	0.030	0.034	0.035	0.036	0.038	0.040	0.041	0.055	0.071	0.092
150	0.031	0.035	0.036	0.037	0.038	0.039	0.040	0.053	0.069	0.088
200	0.033	0.036	0.036	0.037	0.038	0.039	0.040	0.052	0.064	0.078

Table A-2 Data of the thermal conductivity as a function of temperature for calcium silicate insulation at 256 Kg/ m^3[1]:

Temperature, K	Thermal conductivity, W/m. K	Temperature, K	Thermal conductivity, W/m. K
255.4	0.0537	560.9	0.0812
283.1	0.0559	588.7	0.0846
310.9	0.0580	616.5	0.0882
338.7	0.0602	644.3	0.0921
366.5	0.0624	672.0	0.0963
394.3	0.0647	699.8	0.1008
422.0	0.0671	727.6	0.1057
449.8	0.0696	755.4	0.1109

Temperature, K	Thermal conductivity, W/m. K	Temperature, K	Thermal conductivity, W/m. K
477.6	0.0722	783.2	0.1165
505.4	0.0750	810.9	0.1224
533.2	0.0780		

Table A-3 Data of the thermal conductivity as a function of temperature for cellular glass insulation at 120 Kg/ m^3[1]:

Temperature, K	Thermal conductivity, W/m. K	Temperature, K	Thermal conductivity, W/m. K
88.70	0.0156	449.8	0.0721
116.5	0.0181	477.6	0.0789
144.3	0.0210	505.4	0.0863
172.0	0.0242	533.2	0.0943
199.8	0.0278	560.9	0.1031
227.6	0.0316	588.7	0.1127
255.4	0.0356	616.5	0.1232
283.1	0.0399	644.3	0.1347
310.9	0.0455	672.0	0.1474
338.7	0.0494	699.8	0.1613
366.5	0.0545	727.6	0.1766
394.3	0.0600	755.4	0.1934
422.0	0.0658		

Properties of water:

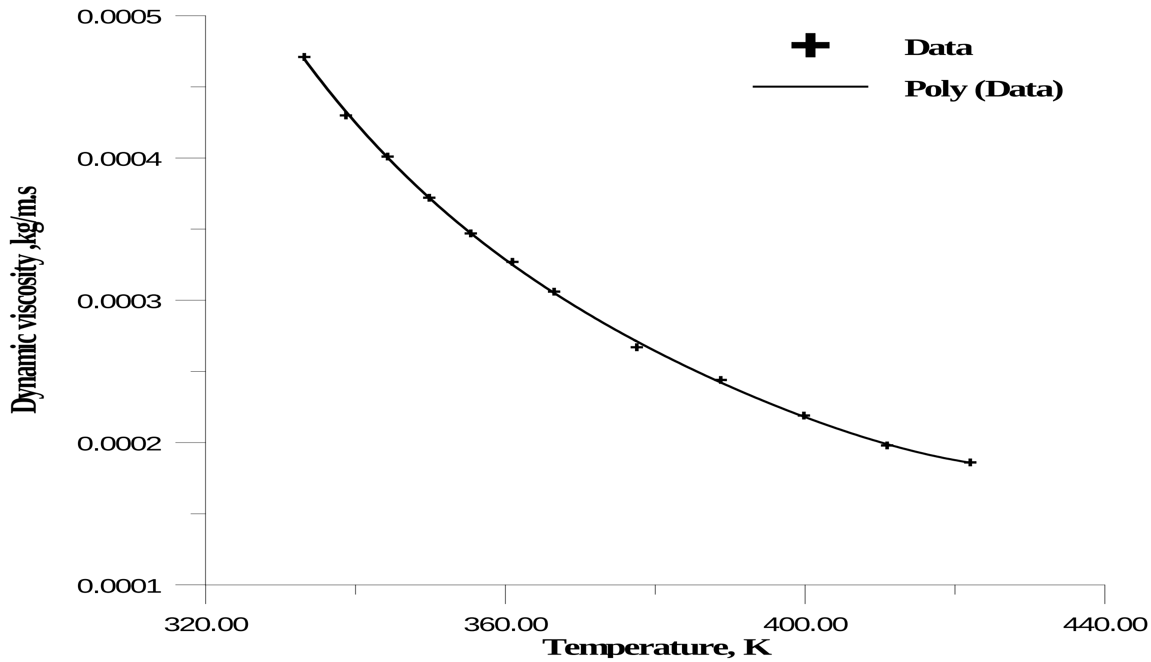


Figure A-1 Comparison of Dynamic viscosity equation and data [73] for water versus temperature.

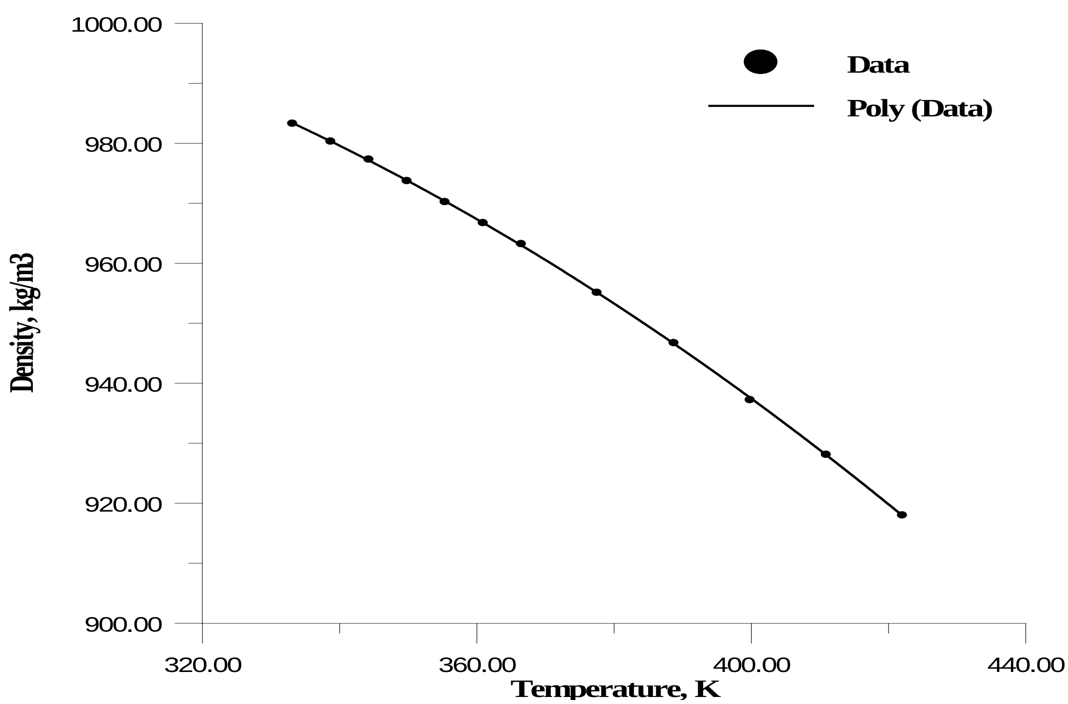


Figure A-2 Comparison of density equation and data [73] for water versus temperature.

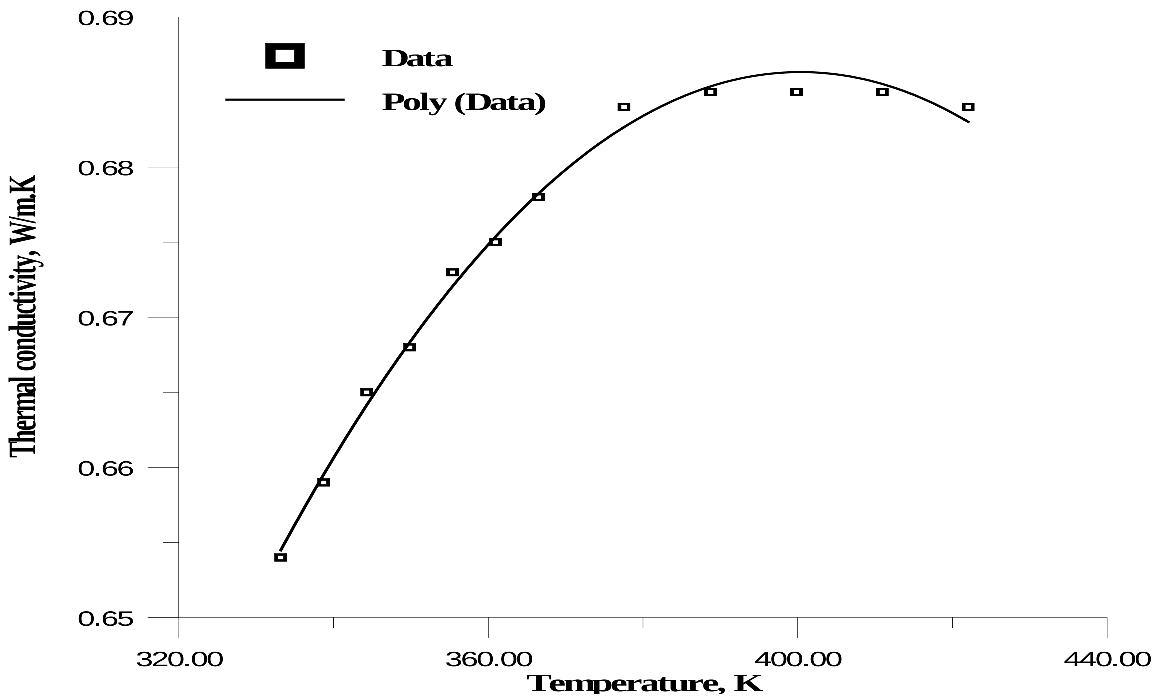


Figure A-3 Comparison of thermal conductivity equation and data [73] for water versus temperature.

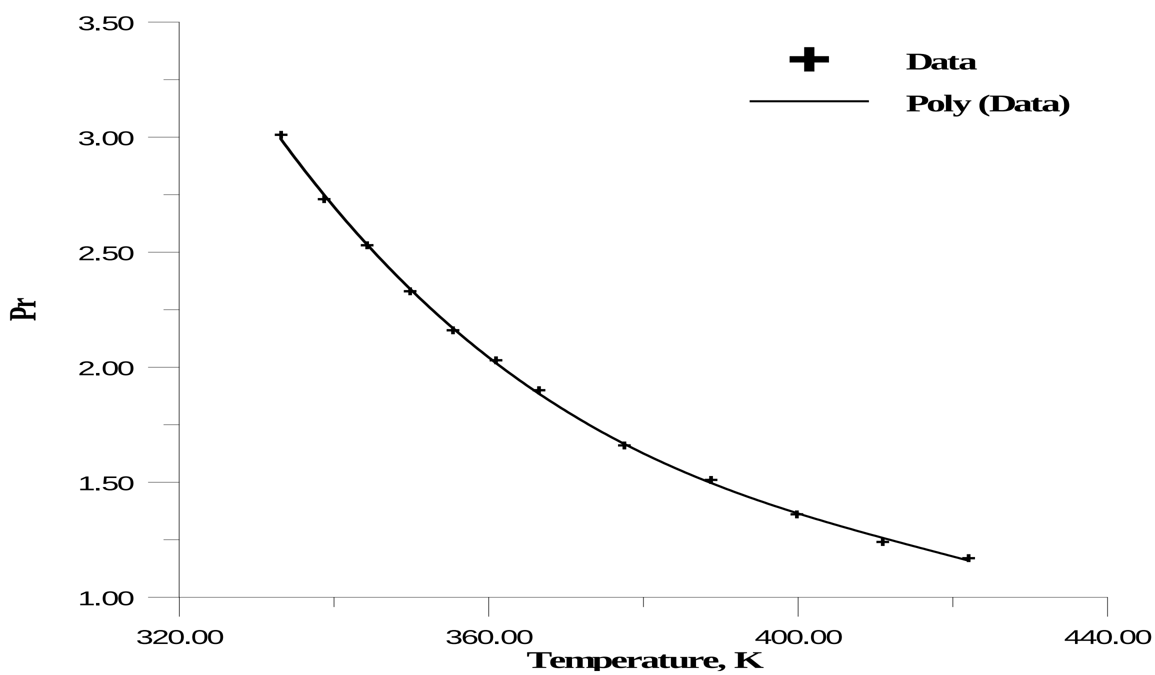


Figure A-4 Comparison of Prandtl number equation and data [73] for water versus temperature.

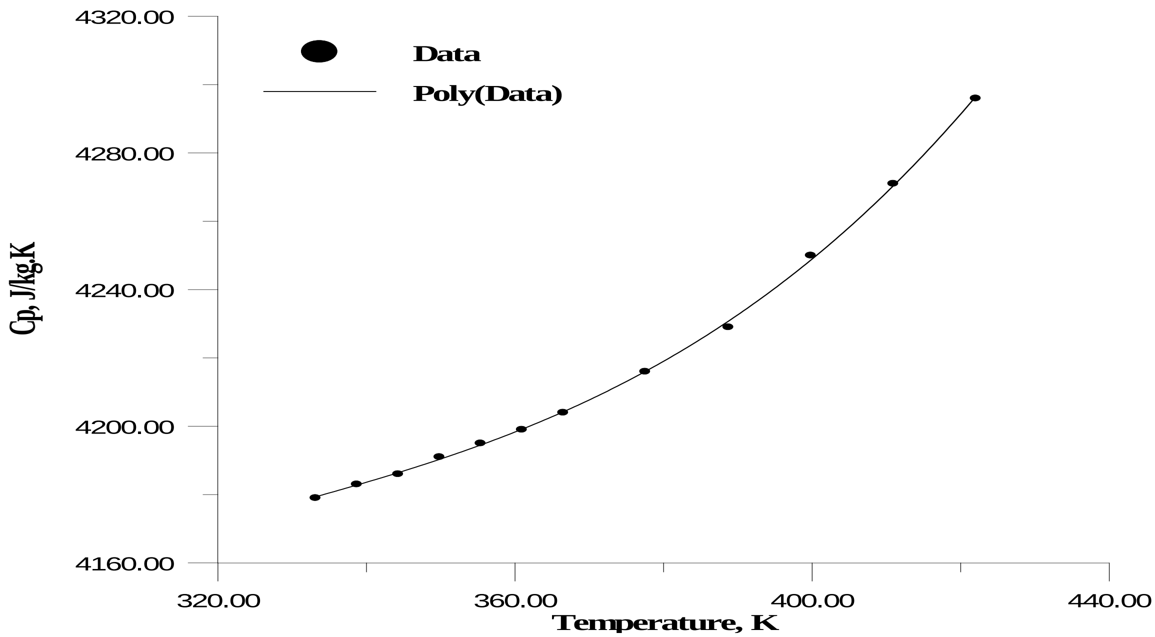


Figure A-5 Comparison of specific capacity equation and data [73] for water versus temperature.

Properties of air:

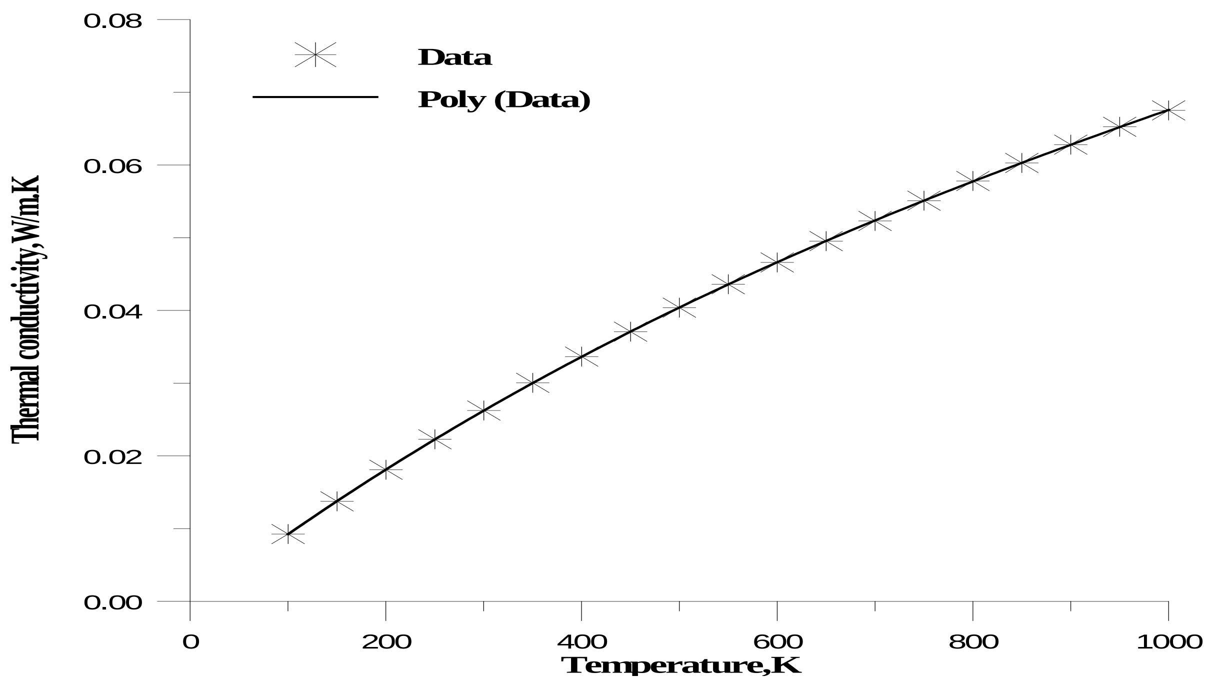


Figure A-6 Comparison of thermal conductivity equation and data [73] for air versus temperature.

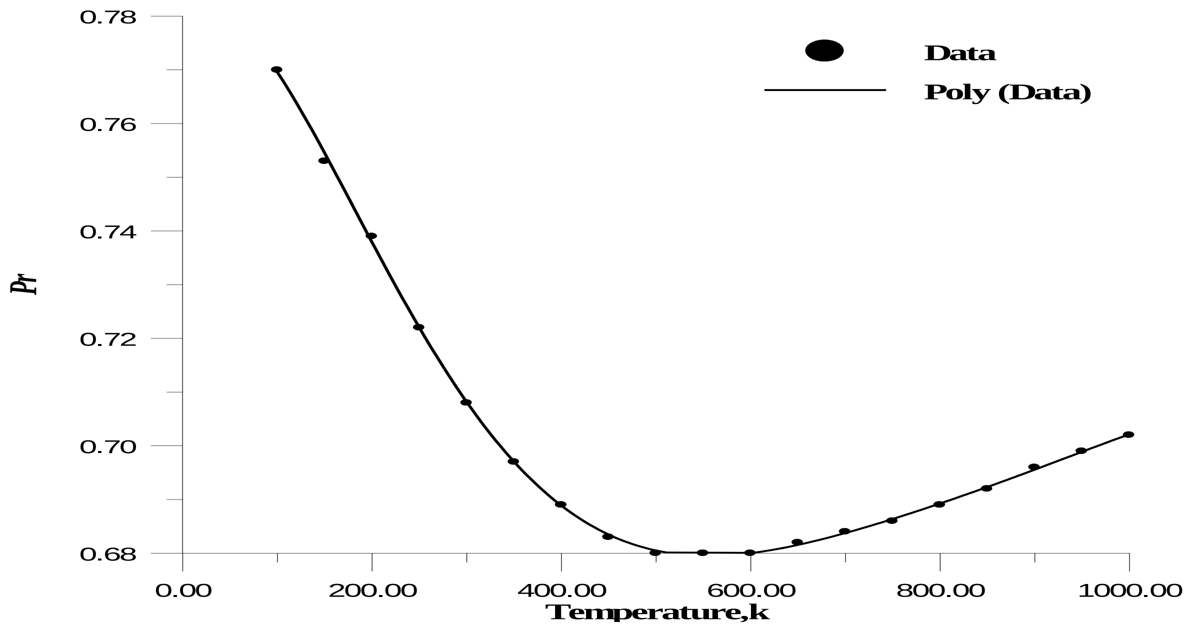


Figure A-7 Comparison of Prandtl number equation and data [73] for air versus temperature.

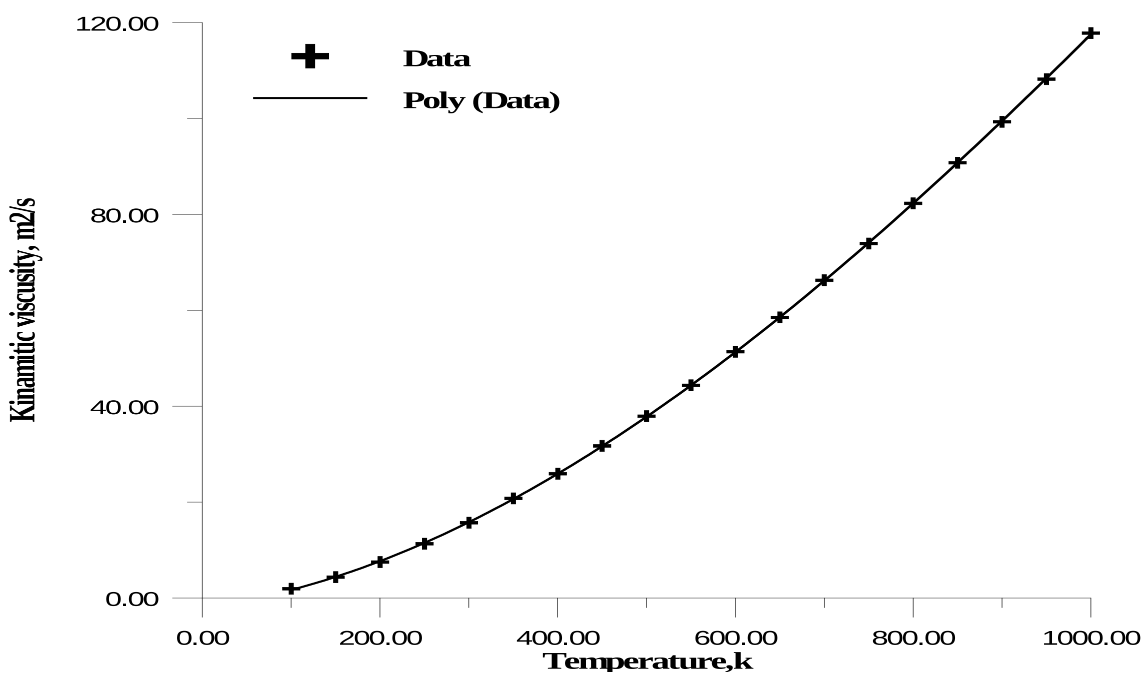


Figure A-8 Comparison of kinematic viscosity equation and data [73] for air versus temperature.

Thermal conductivity of steel pipe:

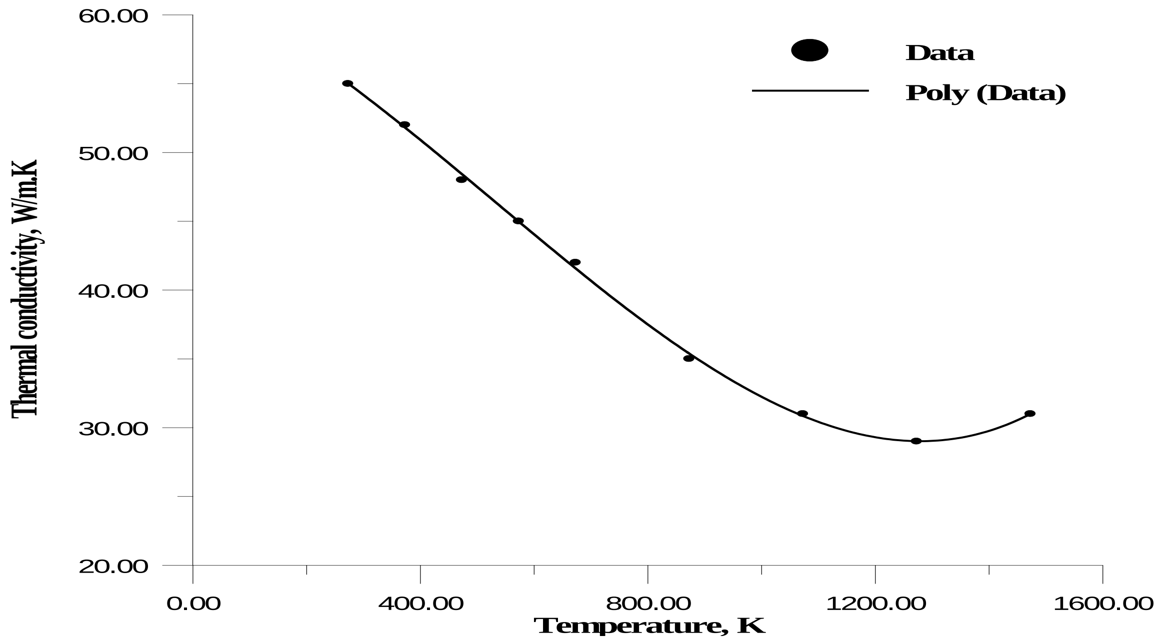


Figure A-9 Comparison of thermal conductivity equation and data [73] for steel pipe versus temperature.

Appendix B

MATLAB Code – For single layer

```
% PIPE CONTAIN HOT WATER
clc
clear
% Tb1=inter water temperature, K
Tb1=98+273.15;
%Ti=bulk water temperature, K
ASSUME Ti
Ti=Tb1;
%T0=air temperature, K
T0=20+273.15;
%ASSUME T1
T1=Ti;
%ASSUME T2
T2=T0;
%ASSUME T3
T3=T0;
%r1=inner pipe radius, m   %NPS=2 inch
r1=0.0525/2;
%r2=outer pipe radius, m   %NPS=2 inch
r2=0.0603/2;
%the water velocity, m/s
u=0.25;
%dx=thickness 0f insulation, m
dx=0.005;
for s=1:31
%r3=radius of pipe with insulation, m
r3=r2+dx;
for n=1:100
%%%%%%Properties of water at Ti, range temperature 60 to 148.9˚C:
%dw= density, kg/m3:
dw=-0.002235*(Ti)^2+0.952*(Ti)+914.323;%correlation coeff.=0.999945
%mw=dynamic viscosity, kg/m. s:
mw=3.395*10^-12*(Ti)^4-5.325*10^-9*(Ti)^3+3.15*10^-6*(Ti)^2-
0.000835*(Ti)+0.08412; %correlation coeff.=0.999629
%kw=thermal conductivity, W/m. K:
kw=-7.05256*10^-6*(Ti)^2+0.005647*(Ti)-0.444139;%correlation coeff.=0.994396
%Prw=Prandtl number:
Prw=-1.70626*10^-6*(Ti)^3+0.00214*(Ti)^2-0.9035*(Ti)+129.573;%correlation
coeff.=0.999491
%Cp:
Cpw=6.8107*10^-5*(Ti)^3-0.06609*(Ti)^2+21.9623*Ti+1679.55;%Correlation
coeff.=0.999568
```

```
%Re=Reynolds number:
Re=dw*u*2*r1/mw;
%Nuw=Nusselt number:
Nuw=0.023*Re^0.8*Prw^0.4;
%hi=heat transfer coefficient inside pipe, w/m2.K
hi=Nuw*kw/(2*r1);
%Ri=R1=thermal resistance of water, K/w
R1=1/(hi*2*pi*r1);
%Tm1=mean temperature of pipe, K
Tm1=(T1+T2)/2;
%kt=thermal conductivity of steel pipe, W/m. K
kt=1.58234*10^-012*Tm1^4+1.5668*10^-008*Tm1^3-2.76413*10^-005*Tm1^2-
0.0194177*Tm1+62.0529;
%R2=thermal resistance of pipe, k/w
R2=log(r2/r1)./(2*pi*kt);
%Tm=mean temperature of insulation, K
Tm=(T2+T3)/2;
%calculation thermal conductivity:
 (1)Temperature dependence: for mineral
%wool mean temperature range 10 to 400 ^oC kins=thermal conductivity of (Mineral
wool) insulation at 40kg/m3, w/m. K
%kins=(5.9239*10^-8)*Tm.^2+(1.13986*10^-4)*Tm-(3.17*10^-3);
%kins=thermal conductivity of (mineral wool) insulation at 70kg/m3, w/m. K
%kins=(-5.2866*10^-8)*Tm.^2+(2.1825*10^-4)*Tm-(2.483*10^-2);
%kins=thermal conductivity of (mineral wool) insulation at 90kg/m3, w/m. K
%kins=(8.56106*10^-8)*Tm.^2+(8.2657*10^-5)*Tm-(1.851*10^-3);
%kins=thermal conductivity of (mineral wool) insulation at 100kg/m3, w/m. K
%kins=(1.4142*10^-7)*Tm.^2+(2.4812*10^-5)*Tm+(1.357*10^-2);
%kins=thermal conductivity of (mineral wool) insulation at 110kg/m3, w/m. K
%kins=(7.4634*10^-8)*Tm.^2+(8.4074*10^-5)*Tm+(1.673*10^-3);
%kins=thermal conductivity of (mineral wool) insulation at 120kg/m3, w/m. K
%kins=(1.5995*10^-7)*Tm.^2-(2.66735*10^-6)*Tm+(2.0818*10^-2);
%kins=thermal conductivity of (mineral wool) insulation at 150kg/m3, w/m. K
%kins=(1.7678*10^-7)*Tm.^2-(3.0263*10^-5)*Tm+(2.8205*10^-2);
%kins=thermal conductivity of (mineral wool) insulation at 200kg/m3, w/m. K
%kins=(1.04688*10^-7)*Tm.^2+(1.19136*10^-5)*Tm+(2.2652*10^-2);
%for calcium silicate mean temperature range -17.75 to 537.75 ^oC
%kins=thermal conductivity of (calcium silicate) insulation at 256kg/m3, w/m. k
%kins= (1.71*10^-10)*Tm.^3-(1.42*10^-7)*Tm.^2+(1.17*10^-4)*Tm+(3.04*10^-2);
%for cellular glass mean temperature range -184.4 to 482.2 ˚C kins=thermal conductivity
of (cellular glass) insulation at 120kg/m3, W/m. k
%kins=(5.82075*10^-13)*Tm.^4-(5.47487*10^-10)*Tm.^3+(3.53567*10^-7)*Tm.^2+(
3.7803*10^-5)*Tm+(9.66184*10^-3);
%(2)depend on temperature and density: Tm= mean temperature range 10 to 400˚C
% D= density range from 40 to 200, kg/m3
D=200;
```

```
%kins=thermal conductivity of (mineral wool) insulation, w/m. K
kins=[-0.02734+(1.7*10^-4)*Tm+(6.10802*10^-8)*Tm.^2]+[(2.896*10^-4)-
(1.04014*10^-6)*Tm+(5.22353*10^-10)*Tm.^2]*D+[0.293269+(5.762*10^-4)*Tm-
(2.20441*10^-6)*Tm.^2]/D;
%R3=resistance of insulation, K/w
R3=log(r3/r2)./(2*pi*kins);
%%%%%%%%%Properties of Air at Tf, range temperature 100 to 1000, K:
%Tf=film temperature, K
Tf=(T3+T0)/2;
%B1= volume coefficient of expansion, K^-1
B1=1/Tf;
%g= acceleration of gravity = 9.81 m/s2
g=9.81;
%v= kinematic viscosity, m2/s
v=(-3.01*10^-8*Tf.^3+1.25*10^-4*Tf.^2+2.45*10^-2*Tf-1.98)*10^-6;
%Gr= Grashof number
Gr=(g*B1*(T3-T0)*(2*r3)^3)/v.^2;
%Pr=Prandtl number
Pr=-1.9*10^-18*Tf.^6+7.69*10^-15*Tf.^5-1.22*10^-11*Tf.^4+9.30*10^-9*Tf.^3-
3.06*10^-6*Tf.^2+1.11*10^-4*Tf+7.81*10^-1;
%ka=thermal conductivity of air, W/m. K
ka=1.3*10^-11*Tf.^3-4.70*10^-8*Tf.^2+1.02*10^-4*Tf-5.31*10^-4;
%Nu=Nusselt number
Nu=[0.6+(0.387*(Gr*Pr).^(1/6))./[1+(0.559/Pr).^(9/16)].^(8/27)].^2;
h0=Nu*ka/(2*r3);
e=0.1;
hrad=(e*5.67*10^-8*(T3^2+T0^2)*(T3+T0));
R4=1/(2*pi*r3*(h0+hrad));
Q=(Ti-T0)/(R1+R2+R3+R4);
qr=2*pi*r3*hrad*(T3-T0);
qc=2*pi*r3*h0*(T3-T0);
m=dw*u*(2*r1)^2*pi/4;
%Q=m*Cpw*(Tb1-Tb2)
Tb2=Tb1-Q/(m*Cpw);
Tb1;
Ti=(Tb1+Tb2)/2;
T3=Q*R4+T0;
T2=Q*R3+T3;
T1=Q*R2+T2;
%Ti=Q*R1+T1;
end
%dTlm=((Tb2-T1)-(Tb1-T1))/log((Tb2-T1)/(Tb1-T1))
Db=Tb1-Tb2;
%cost of energy saving:
CH=(0.0527596*Q+0.00097868)*3.2808;
 %instal cost (material and labor) of insulation (mineral wool):
```

```
 CI1=(19.0551*dx+0.809)*3.2808 ;%NPS=2 inch
 %instal cost (material and labor) of insulation (calcium silicate):
 CI2=(20.36*dx+0.88)*3.2808 ;%NPS=2 inch
 %instal cost (material and labor) of insulation (cellular glass):
 CI3=(22.6884*dx+0.972)*3.2808 ;%NPS=2 inch
%total cost of insulation and energy saving
CT1=CH+CI1;
CT2=CH+CI2;
CT3=CH+CI3;
table = [dx;Db;Q;qc;qr;m;R1;R2;R3;R4;kins;Tm;T3-273.15;T2-273.15;T1-273.15; Ti-
273. 15;Tb2-273.15;Tb1-273.15;h0;CH;CI1;CI2;CI3;CT1;CT2;CT3]
plot(dx,CH,'og','MarkerSize',10),hold on
plot(dx,CI1,'+b','MarkerSize',10),hold on
%plot(dx,CI2,'+b','MarkerSize',10) plot(dx,CI3,'+b','MarkerSize',10)
plot(dx,CT1,'*r','MarkerSize',10)
%plot(dx,CT2,'*r','MarkerSize',10) plot(dx,CT3,'*r','MarkerSize',10)
p=legend('Cost of lost energy','insulation cost curve','Total cost curve');
set(p,'edgecolor','none','FontWeight','bold','FontSize',12)
set(p,'FontName','Times New Roman')
xlabel('Thickness of insulation, m','FontSize',12,'FontWeight','bold');
ylabel('Annual cost, $/m','FontSize',12,'FontWeight','bold');Box off
hold on
dx=dx+0.005;
end
```

Appendix C

MATLAB Code – For multilayer

```
% PIPE CONTAIN STEAM
clc
clear
%T2=outside surface temperature [K]
T2=200+273.15;
T20=T2;
%T0=air temperature [K];
T0=20+273.15;
%r2=outer pipe radius[m]
r2=0.168/2;
%ASSUME T3
 T3=T2;
 T30=T20;
%%%%%%%%%%%%%calcium silicate [density=256 kg/m^3] %%%%%%%%%% %%%%%
%dx1=thickness 0f first insulation[m]
 dx1=0.005;
 for k=1:20
   dx2=0.005;
     for s=1:30
 r3=r2+dx1;
%%%%%%%%%%%%%cellular glass [density=40 kg/m^3] %%%%%%%%%%%%%%%%%
%dx2=thickness of second insulation[m]
 r4=r3+dx2;
 dx=dx1+dx2;
%ASSUME T4
T4=T0;
T40=T0;
for n=1:100
%Tm=mean temperature [K]
Tm1=(T2+T3)/2;
Tm10=(T20+T30)/2;
%D=density [kg/m3]
%D=40;
%kins=thermal conductivity of insulation [w/m. k]
%mineral wool
```

```
%kins1=[-0.02734+(1.7*10^-4)*Tm1+(6.10802*10^-8)*Tm1.^2]+[(2.896*10^-4)-
(1.04014*10^-6)*Tm1+(5.22353*10^-10)*Tm1.^2]*D+[0.293269+(5.762*10^-4)*Tm1-
(2.20441*10^-6)*Tm1.^2]/D;
%calcium silicate:
kins1=(1.71*10^-10)*Tm1.^3-(1.42*10^-7)*Tm1.^2+(1.17*10^-4)*Tm1+(3.04*10^-2);
kins10=(1.71*10^-10)*Tm10.^3-(1.42*10^-7)*Tm10.^2+(1.17*10^-4)*Tm10+(3.04*10^-2);
%cellular glass:
%kins1=(5.82075*10^-13)*Tm1.^4-(5.47487*10^-10)*Tm1.^3+(3.53567*10^
7)*Tm1.^2+(3.7803*10^-5)*Tm1+(9.66184*10^-3);
%kins10=(5.82075*10^-13)*Tm10.^4-(5.47487*10^-10)*Tm10.^3+(3.53567*10^-
7)*Tm10.^2+(3.7803*10^-5)*Tm10+(9.66184*10^-3);
%R3=resistance of insulation [k/w]
%equivalent thickness=r3*log(r3/r2)
R3=r3*log(r3/r2)./(2*pi*r3*kins1);
R30=r3*log(r3/r2)./(2*pi*r3*kins10);
%Tm=mean temperature [K]
Tm2=(T3+T4)/2;
Tm20=(T30+T40)/2;
%D=density [kg/m3]
%D=200;
%kins=thermal conductivity of insulation[w/m.k]
%mineral wool
%kins2=[-0.02734+(1.7*10^-4)*Tm2+(6.10802*10^-8)*Tm2.^2]+[(2.896*10^-4)-
(1.04014*10^-6)*Tm2+(5.22353*10^-10)*Tm2.^2]*D+[0.293269+(5.762*10^-4)*Tm2-
(2.20441*10^-6)*Tm2.^2]/D;
%kins20=[-0.02734+(1.7*10^-4)*Tm20+(6.10802*10^-8)*Tm20.^2]+[(2.896*10^-4)-
(1.04014*10^-6)*Tm20+(5.22353*10^-10)*Tm20.^2]*D+[0.293269+(5.762*10^-4)*Tm20-
(2.20441*10^-6)*Tm20.^2]/D;
%calcium silicate:
%kins2=(1.71*10^-10)*Tm2.^3-(1.42*10^-7)*Tm2.^2+(1.17*10^-4)*Tm2+(3.04*10^-2);
%cellular glass:
kins2=(5.82075*10^-13)*Tm2.^4-(5.47487*10^-10)*Tm2.^3+(3.53567*10^-
7)*Tm2.^2+(3.7803*10^-5)*Tm2+(9.66184*10^-3);
kins20=(5.82075*10^-13)*Tm20.^4-(5.47487*10^-10)*Tm20.^3+(3.53567*10^-
7)*Tm20.^2+(3.7803*10^-5)*Tm20+(9.66184*10^-3);
%R4=resistance of insulation[k/w]
%equivalent thickness=r4*log(r4/r3)
R4=r4*log(r4/r3)./(2*pi*r4*kins2);
R40=r4*log(r4/r3)./(2*pi*r4*kins20);
Tf=(T4+T0)/2;
```

```
Tf0=(T40+T0)/2;
B1=1/Tf;
B10=1/Tf0;
g=9.81;
v=(-3.01*10^-8*Tf.^3+1.25*10^-4*Tf.^2+2.45*10^-2*Tf-1.98)*10^-6;
v0=(-3.01*10^-8*Tf0.^3+1.25*10^-4*Tf0.^2+2.45*10^-2*Tf0-1.98)*10^-6;
Gr=(g*B1*(T4-T0)*(2*r4)^3)/v.^2;
Gr0=(g*B10*(T40-T0)*(2*r4)^3)/v0.^2;
Pr=-1.9*10^-18*Tf.^6+7.69*10^-15*Tf.^5-1.22*10^-11*Tf.^4+9.30*10^-9*Tf.^3-3.06*10^-
6*Tf.^2+1.11*10^-4*Tf+7.81*10^-1;
Pr0=-1.9*10^-18*Tf0.^6+7.69*10^-15*Tf0.^5-1.22*10^-11*Tf0.^4+9.30*10^-9*Tf0.^3-
3.06*10^-6*Tf0.^2+1.11*10^-4*Tf0+7.81*10^-1;
ka=1.3*10^-11*Tf.^3-4.70*10^-8*Tf.^2+1.02*10^-4*Tf-5.31*10^-4;
ka0=1.3*10^-11*Tf0.^3-4.70*10^-8*Tf0.^2+1.02*10^-4*Tf0-5.31*10^-4;
Nu=[0.6+(0.387*(Gr*Pr).^(1/6))./[1+(0.559/Pr).^(9/16)].^(8/27)].^2;
Nu0=[0.6+(0.387*(Gr0*Pr0).^(1/6))./[1+(0.559/Pr0).^(9/16)].^(8/27)].^2;
h0=Nu*ka/(2*r4);
h00=Nu0*ka0/(2*r4);
%e=emissivity (assume low emissivity =0.1)
e=0.1;
hrad=e*5.67*10^-8*(T4^2+T0^2)*(T4+T0);
R5=1/(2*pi*r4*(hrad+h0));
R50=1/(2*pi*r4*(h00));
Q=(T2-T0)/(R3+R4+R5);
Q0=(T20-T0)/(R30+R40+R50);
T4=Q*R5+T0;
T40=Q0*R50+T0;
T3=Q*R4+T4;
T30=Q0*R40+T40;
T2=Q*R3+T3;
T20=Q0*R30+T30;
end
CH=[0.0527596*Q+0.00097868]*3.2808;
%calcium silicate cost single layer:
CI2=[28.1777*dx1+1.185]*3.2808;
%CI2=28.1777*dx2+1.185;
%mineral wool cost single layer:
%CI1=26.8744*dx1+1.11456;
%CI1=[26.8744*dx2+1.11456]*3.2808;
%cellular glass cost single layer:
```

```
%CI3=[31.2036*dx1+1.329]*3.2808;
CI3=[31.2036*dx2+1.329]*3.2808;
%CT21=CH+CI2+CI1;
CT23=CH+CI2+CI3;
%CT13=CH+CI1+CI3;
%Totalcost=costQ+instalcost1+instalcost2;
table= [dx1,dx2,Q,Q0, T3,T4,T40,kins1,kins2,CH,CT21,CT23,CT13]
%plot3(dx2,dx1,CT21,'.k', 'MarkerSize',4.5)
plot3(r4,r3,CT23,'.b', 'MarkerSize',6)
xlabel('r4, m','FontSize',12,'FontWeight','bold');
ylabel('r3, m','FontSize',12,'FontWeight','bold');
zlabel('Total cost, $/m','FontSize',12,'FontWeight','bold');Box off
hold on
dx2=dx2+0.005;
    end
 dx1=dx1+0.005;
 end
```

Appendix D

Table D-1 The results for **first case** study in single layer using **rock mineral wool insulation at 40 kg/m^3.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	T_{b2}, ˚C	T_{b1} ˚C	Annual cost, $/m		
															C_H	C_I	C_T
0.0050	63.3315	56.4484	6.8832	0.0032	0.0004	0.5370	0.6907	0.0455	63.7448	97.7533	97.7802	97.9855	97.9710	98.0000	10.9655	2.9667	13.9322
0.0100	45.8326	40.6430	5.1896	0.0032	0.0004	1.0259	0.6720	0.0444	50.8010	97.8215	97.8409	97.9895	97.9790	98.0000	7.9365	3.2793	11.2159
0.0150	36.8698	32.5052	4.3646	0.0032	0.0004	1.4647	0.6470	0.0439	43.8543	97.8564	97.8720	97.9916	97.9831	98.0000	6.3851	3.5919	9.9770
0.0200	31.3836	27.5091	3.8745	0.0032	0.0004	1.8605	0.6210	0.0435	39.4894	97.8777	97.8911	97.9928	97.9856	98.0000	5.4355	3.9045	9.3400
0.0250	27.6589	24.1099	3.5490	0.0032	0.0004	2.2203	0.5959	0.0433	36.4823	97.8923	97.9040	97.9937	97.9873	98.0000	4.7908	4.2171	9.0079
0.0300	24.9523	21.6358	3.3165	0.0032	0.0004	2.5497	0.5723	0.0431	34.2809	97.9028	97.9134	97.9943	97.9886	98.0000	4.3223	4.5296	8.8520
0.0350	**22.8890**	**19.7470**	**3.1420**	**0.0032**	**0.0004**	**2.8534**	**0.5504**	**0.0430**	**32.5982**	**97.9108**	**97.9206**	**97.9948**	**97.9895**	**98.0000**	**3.9652**	**4.8422**	**8.8074**
0.0400	21.2589	18.2529	3.0060	0.0032	0.0004	3.1350	0.5301	0.0429	31.2695	97.9172	97.9262	97.9951	97.9903	98.0000	3.6830	5.1548	8.8378
0.0450	19.9350	17.0381	2.8969	0.0032	0.0004	3.3975	0.5113	0.0428	30.1936	97.9223	97.9308	97.9954	97.9909	98.0000	3.4538	5.4674	8.9212
0.0500	18.8359	16.0285	2.8074	0.0032	0.0004	3.6432	0.4940	0.0427	29.3044	97.9266	97.9346	97.9957	97.9914	98.0000	3.2636	5.7800	9.0435
0.0550	17.9070	15.1743	2.7326	0.0032	0.0004	3.8741	0.4779	0.0427	28.5572	97.9302	97.9379	97.9959	97.9918	98.0000	3.1028	6.0925	9.1953
0.0600	17.1101	14.4410	2.6692	0.0032	0.0004	4.0919	0.4629	0.0426	27.9205	97.9333	97.9406	97.9961	97.9922	98.0000	2.9649	6.4051	9.3700
0.0650	16.4180	13.8034	2.6146	0.0032	0.0004	4.2980	0.4490	0.0426	27.3716	97.9360	97.9430	97.9962	97.9925	98.0000	2.8451	6.7177	9.5628
0.0700	15.8104	13.2432	2.5672	0.0032	0.0004	4.4936	0.4360	0.0425	26.8934	97.9384	97.9451	97.9964	97.9928	98.0000	2.7399	7.0303	9.7702
0.0750	15.2721	12.7464	2.5256	0.0032	0.0004	4.6796	0.4239	0.0425	26.4731	97.9405	97.9470	97.9965	97.9930	98.0000	2.6467	7.3429	9.9896
0.0800	14.7912	12.3024	2.4888	0.0032	0.0004	4.8570	0.4125	0.0425	26.1008	97.9424	97.9487	97.9966	97.9932	98.0000	2.5635	7.6554	10.2189
0.0850	14.3587	11.9026	2.4561	0.0032	0.0004	5.0266	0.4018	0.0424	25.7688	97.9441	97.9502	97.9967	97.9934	98.0000	2.4886	7.9680	10.4566
0.0900	13.9671	11.5404	2.4267	0.0032	0.0004	5.1890	0.3917	0.0424	25.4709	97.9456	97.9515	97.9968	97.9936	98.0000	2.4208	8.2806	10.7014
0.0950	13.6107	11.2105	2.4002	0.0032	0.0004	5.3447	0.3822	0.0424	25.2021	97.9470	97.9528	97.9969	97.9938	98.0000	2.3591	8.5932	10.9523
0.1000	13.2846	10.9085	2.3761	0.0032	0.0004	5.4943	0.3732	0.0424	24.9583	97.9482	97.9539	97.9970	97.9939	98.0000	2.3027	8.9058	11.2085
0.1050	12.9850	10.6307	2.3542	0.0032	0.0004	5.6383	0.3647	0.0423	24.7362	97.9494	97.9549	97.9970	97.9941	98.0000	2.2508	9.2183	11.4692
0.1100	12.7084	10.3743	2.3342	0.0032	0.0004	5.7771	0.3567	0.0423	24.5331	97.9505	97.9559	97.9971	97.9942	98.0000	2.2030	9.5309	11.7339
0.1150	12.4523	10.1365	2.3158	0.0032	0.0004	5.9109	0.3491	0.0423	24.3465	97.9515	97.9568	97.9972	97.9943	98.0000	2.1586	9.8435	12.0021
0.1200	12.2143	9.9154	2.2988	0.0032	0.0004	6.0403	0.3418	0.0423	24.1747	97.9524	97.9576	97.9972	97.9944	98.0000	2.1174	10.1561	12.2735
0.1250	11.9924	9.7092	2.2832	0.0032	0.0004	6.1654	0.3349	0.0423	24.0159	97.9533	97.9584	97.9973	97.9945	98.0000	2.0790	10.4687	12.5477
0.1300	11.7849	9.5162	2.2686	0.0032	0.0004	6.2865	0.3283	0.0423	23.8686	97.9541	97.9591	97.9973	97.9946	98.0000	2.0431	10.7812	12.8243
0.1350	11.5903	9.3352	2.2551	0.0032	0.0004	6.4039	0.3220	0.0423	23.7317	97.9548	97.9598	97.9973	97.9947	98.0000	2.0094	11.0938	13.1032
0.1400	11.4075	9.1650	2.2425	0.0032	0.0004	6.5178	0.3159	0.0423	23.6042	97.9556	97.9604	97.9974	97.9948	98.0000	1.9778	11.4064	13.3842
0.1450	11.2353	9.0045	2.2307	0.0032	0.0004	6.6283	0.3102	0.0422	23.4850	97.9562	97.9610	97.9974	97.9949	98.0000	1.9480	11.7190	13.6669
0.1500	11.0727	8.8530	2.2197	0.0032	0.0004	6.7358	0.3047	0.0422	23.3735	97.9569	97.9616	97.9975	97.9949	98.0000	1.9198	12.0316	13.9514
0.1550	10.9189	8.7096	2.2093	0.0032	0.0004	6.8403	0.2994	0.0422	23.2688	97.9575	97.9621	97.9975	97.9950	98.0000	1.8932	12.3441	14.2373

Table D-2 The results for **first case** study in single layer using **rock mineral wool insulation at 90 kg/m^3.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	T_{b2}, ˚C	T_{b1} ˚C	Annual cost, $/m		
															C_H	C_I	C_T
0.0050	60.6007	54.0061	6.5945	0.0032	0.0004	0.5864	0.6968	0.0416	62.2277	97.7639	97.7897	97.9861	97.9723	98.0000	10.4928	2.9667	13.4595
0.0100	43.2719	38.3501	4.9219	0.0032	0.0004	1.1189	0.6798	0.0407	49.4158	97.8314	97.8498	97.9901	97.9802	98.0000	7.4933	3.2793	10.7726
0.0150	34.5900	30.4676	4.1225	0.0032	0.0004	1.5958	0.6553	0.0403	42.6665	97.8653	97.8800	97.9921	97.9842	98.0000	5.9905	3.5919	9.5824
0.0200	29.3353	25.6835	3.6518	0.0032	0.0004	2.0256	0.6294	0.0400	38.4646	97.8857	97.8982	97.9933	97.9866	98.0000	5.0810	3.9045	8.9854
0.0250	25.7920	22.4513	3.3407	0.0032	0.0004	2.4160	0.6043	0.0398	35.5856	97.8995	97.9105	97.9941	97.9882	98.0000	4.4676	4.2171	8.6847
0.0300	23.2292	20.1100	3.1192	0.0032	0.0004	2.7734	0.5805	0.0396	33.4857	97.9095	97.9194	97.9947	97.9894	98.0000	4.0240	4.5296	8.5537
0.0350	**21.2820**	**18.3287**	**2.9533**	**0.0032**	**0.0004**	**3.1027**	**0.5584**	**0.0395**	**31.8846**	**97.9171**	**97.9261**	**97.9951**	**97.9903**	**98.0000**	**3.6870**	**4.8422**	**8.5292**
0.0400	19.7475	16.9233	2.8242	0.0032	0.0004	3.4080	0.5379	0.0394	30.6229	97.9231	97.9315	97.9955	97.9910	98.0000	3.4214	5.1548	8.5762
0.0450	18.5037	15.7829	2.7208	0.0032	0.0004	3.6925	0.5190	0.0394	29.6027	97.9279	97.9358	97.9958	97.9915	98.0000	3.2061	5.4674	8.6735
0.0500	17.4728	14.8368	2.6360	0.0032	0.0004	3.9588	0.5014	0.0393	28.7605	97.9319	97.9394	97.9960	97.9920	98.0000	3.0276	5.7800	8.8076
0.0550	16.6026	14.0374	2.5652	0.0032	0.0004	4.2091	0.4851	0.0393	28.0536	97.9353	97.9424	97.9962	97.9924	98.0000	2.8770	6.0925	8.9696
0.0600	15.8571	13.3519	2.5052	0.0032	0.0004	4.4451	0.4699	0.0392	27.4517	97.9382	97.9450	97.9964	97.9927	98.0000	2.7480	6.4051	9.1531
0.0650	15.2101	12.7565	2.4536	0.0032	0.0004	4.6685	0.4558	0.0392	26.9330	97.9407	97.9472	97.9965	97.9930	98.0000	2.6360	6.7177	9.3537
0.0700	14.6426	12.2339	2.4088	0.0032	0.0004	4.8804	0.4426	0.0391	26.4815	97.9430	97.9492	97.9967	97.9933	98.0000	2.5378	7.0303	9.5680
0.0750	14.1401	11.7707	2.3695	0.0032	0.0004	5.0820	0.4303	0.0391	26.0848	97.9449	97.9509	97.9968	97.9935	98.0000	2.4508	7.3429	9.7936
0.0800	13.6916	11.3569	2.3347	0.0032	0.0004	5.2743	0.4188	0.0391	25.7337	97.9467	97.9525	97.9969	97.9937	98.0000	2.3731	7.6554	10.0286
0.0850	13.2884	10.9846	2.3037	0.0032	0.0004	5.4580	0.4079	0.0391	25.4206	97.9482	97.9539	97.9970	97.9939	98.0000	2.3033	7.9680	10.2714
0.0900	12.9235	10.6476	2.2760	0.0032	0.0004	5.6339	0.3977	0.0391	25.1397	97.9497	97.9551	97.9970	97.9941	98.0000	2.2402	8.2806	10.5208
0.0950	12.5916	10.3407	2.2509	0.0032	0.0004	5.8026	0.3881	0.0390	24.8864	97.9509	97.9563	97.9971	97.9942	98.0000	2.1827	8.5932	10.7759
0.1000	12.2880	10.0599	2.2282	0.0032	0.0004	5.9648	0.3790	0.0390	24.6567	97.9521	97.9574	97.9972	97.9944	98.0000	2.1302	8.9058	11.0360
0.1050	12.0092	9.8017	2.2075	0.0032	0.0004	6.1208	0.3703	0.0390	24.4476	97.9532	97.9583	97.9973	97.9945	98.0000	2.0819	9.2183	11.3003
0.1100	11.7520	9.5634	2.1886	0.0032	0.0004	6.2711	0.3622	0.0390	24.2563	97.9542	97.9592	97.9973	97.9946	98.0000	2.0374	9.5309	11.5683
0.1150	11.5138	9.3426	2.1712	0.0032	0.0004	6.4162	0.3544	0.0390	24.0807	97.9551	97.9600	97.9974	97.9947	98.0000	1.9962	9.8435	11.8397
0.1200	11.2925	9.1373	2.1552	0.0032	0.0004	6.5563	0.3470	0.0390	23.9190	97.9560	97.9608	97.9974	97.9948	98.0000	1.9579	10.1561	12.1139
0.1250	11.0862	8.9458	2.1404	0.0032	0.0004	6.6919	0.3400	0.0390	23.7695	97.9568	97.9615	97.9975	97.9949	98.0000	1.9222	10.4687	12.3908
0.1300	10.8934	8.7667	2.1267	0.0032	0.0004	6.8231	0.3333	0.0390	23.6309	97.9576	97.9622	97.9975	97.9950	98.0000	1.8888	10.7812	12.6700
0.1350	10.7126	8.5988	2.1139	0.0032	0.0004	6.9503	0.3269	0.0389	23.5022	97.9583	97.9628	97.9975	97.9951	98.0000	1.8575	11.0938	12.9513
0.1400	10.5428	8.4408	2.1020	0.0032	0.0004	7.0737	0.3208	0.0389	23.3822	97.9589	97.9634	97.9976	97.9952	98.0000	1.8281	11.4064	13.2345
0.1450	10.3829	8.2920	2.0908	0.0032	0.0004	7.1935	0.3149	0.0389	23.2701	97.9596	97.9640	97.9976	97.9953	98.0000	1.8004	11.7190	13.5194
0.1500	10.2319	8.1515	2.0804	0.0032	0.0004	7.3100	0.3093	0.0389	23.1652	97.9601	97.9645	97.9977	97.9953	98.0000	1.7743	12.0316	13.8059
0.1550	10.0892	8.0185	2.0706	0.0032	0.0004	7.4232	0.3040	0.0389	23.0667	97.9607	97.9650	97.9977	97.9954	98.0000	1.7496	12.3441	14.0937

Table D-3 The results for **first case** study in single layer using **rock mineral wool insulation at 200 kg/m^3.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	T_{b2}, ˚C	T_{b1} ˚C	Annual cost, $/m		
															C_H	C_I	C_T
0.0050	58.4662	52.0963	6.3699	0.0032	0.0004	0.6284	0.7018	0.0389	61.0324	97.7722	97.7971	97.9866	97.9733	98.0000	10.1233	2.9667	13.0901
0.0100	41.5760	36.8313	4.7447	0.0032	0.0004	1.1870	0.6852	0.0384	48.4893	97.8380	97.8557	97.9905	97.9810	98.0000	7.1998	3.2793	10.4791
0.0150	33.2200	29.2433	3.9767	0.0032	0.0004	1.6835	0.6606	0.0382	41.9450	97.8706	97.8847	97.9924	97.9848	98.0000	5.7534	3.5919	9.3453
0.0200	28.1822	24.6562	3.5260	0.0032	0.0004	2.1293	0.6345	0.0380	37.8812	97.8902	97.9022	97.9936	97.9871	98.0000	4.8814	3.9045	8.7859
0.0250	24.7898	21.5615	3.2283	0.0032	0.0004	2.5335	0.6091	0.0379	35.0987	97.9034	97.9140	97.9943	97.9887	98.0000	4.2942	4.2171	8.5112
0.0300	22.3371	19.3205	3.0165	0.0032	0.0004	2.9030	0.5851	0.0379	33.0692	97.9130	97.9225	97.9949	97.9898	98.0000	3.8696	4.5296	8.3993
0.0350	**20.4734**	**17.6156**	**2.8578**	**0.0032**	**0.0004**	**3.2432**	**0.5628**	**0.0378**	**31.5214**	**97.9202**	**97.9289**	**97.9953**	**97.9906**	**98.0000**	**3.5470**	**4.8422**	**8.3893**
0.0400	19.0044	16.2702	2.7342	0.0032	0.0004	3.5584	0.5420	0.0378	30.3013	97.9260	97.9340	97.9957	97.9913	98.0000	3.2928	5.1548	8.4476
0.0450	17.8134	15.1782	2.6352	0.0032	0.0004	3.8520	0.5229	0.0377	29.3143	97.9306	97.9382	97.9959	97.9919	98.0000	3.0866	5.4674	8.5540
0.0500	16.8259	14.2719	2.5540	0.0032	0.0004	4.1267	0.5051	0.0377	28.4994	97.9345	97.9416	97.9962	97.9923	98.0000	2.9157	5.7800	8.6956
0.0550	15.9921	13.5059	2.4862	0.0032	0.0004	4.3848	0.4887	0.0377	27.8151	97.9377	97.9445	97.9963	97.9927	98.0000	2.7714	6.0925	8.8639
0.0600	15.2775	12.8489	2.4287	0.0032	0.0004	4.6282	0.4734	0.0377	27.2322	97.9405	97.9470	97.9965	97.9930	98.0000	2.6477	6.4051	9.0528
0.0650	14.6572	12.2780	2.3792	0.0032	0.0004	4.8586	0.4592	0.0376	26.7299	97.9429	97.9491	97.9966	97.9933	98.0000	2.5403	6.7177	9.2580
0.0700	14.1130	11.7767	2.3363	0.0032	0.0004	5.0771	0.4459	0.0376	26.2924	97.9450	97.9510	97.9968	97.9935	98.0000	2.4461	7.0303	9.4764
0.0750	13.6309	11.3324	2.2986	0.0032	0.0004	5.2850	0.4334	0.0376	25.9081	97.9469	97.9527	97.9969	97.9938	98.0000	2.3626	7.3429	9.7055
0.0800	13.2005	10.9353	2.2652	0.0032	0.0004	5.4832	0.4218	0.0376	25.5677	97.9486	97.9542	97.9970	97.9940	98.0000	2.2881	7.6554	9.9436
0.0850	12.8135	10.5780	2.2355	0.0032	0.0004	5.6726	0.4108	0.0376	25.2642	97.9501	97.9555	97.9971	97.9941	98.0000	2.2211	7.9680	10.1892
0.0900	12.4632	10.2544	2.2088	0.0032	0.0004	5.8540	0.4005	0.0376	24.9920	97.9514	97.9567	97.9971	97.9943	98.0000	2.1605	8.2806	10.4411
0.0950	12.1445	9.9597	2.1848	0.0032	0.0004	6. 0279	0.3908	0.0376	24.7463	97.9527	97.9579	97.9972	97.9944	98.0000	2.1053	8.5932	10.6985
0.1000	11.8530	9.6900	2.1630	0.0032	0.0004	6.1951	0.3816	0.0376	24.5235	97.9538	97.9589	97.9973	97.9946	98.0000	2.0549	8.9058	10.9606
0.1050	11.5851	9.4420	2.1431	0.0032	0.0004	6.3560	0.3729	0.0376	24.3206	97.9549	97.9598	97.9974	97.9947	98.0000	2.0085	9.2183	11.2269
0.1100	11.3379	9.2130	2.1249	0.0032	0.0004	6.5110	0.3647	0.0376	24.1351	97.9558	97.9607	97.9974	97.9948	98.0000	1.9657	9.5309	11.4967
0.1150	11.1090	9.0008	2.1082	0.0032	0.0004	6.6605	0.3569	0.0376	23.9647	97.9567	97.9614	97.9975	97.9949	98.0000	1.9261	9.8435	11.7696
0.1200	10.8963	8.8035	2.0929	0.0032	0.0004	6.8050	0.3495	0.0375	23.8078	97.9576	97.9622	97.9975	97.9950	98.0000	1.8893	10.1561	12.0454
0.1250	10.6981	8.6194	2.0786	0.0032	0.0004	6.9448	0.3424	0.0375	23.6627	97.9583	97.9629	97.9976	97.9951	98.0000	1.8550	10.4687	12.3236
0.1300	10.5127	8.4473	2.0654	0.0032	0.0004	7.0801	0.3356	0.0375	23.5282	97.9590	97.9635	97.9976	97.9952	98.0000	1.8229	10.7812	12.6041
0.1350	10.3389	8.2858	2.0531	0.0032	0.0004	7.2113	0.3292	0.0375	23.4032	97.9597	97.9641	97.9976	97.9953	98.0000	1.7928	11.0938	12.8866
0.1400	10.1756	8.1339	2.0417	0.0032	0.0004	7.3385	0.3230	0.0375	23.2868	97.9604	97.9647	97.9977	97.9953	98.0000	1.7645	11.4064	13.1709
0.1450	10.0218	7.9908	2.0310	0.0032	0.0004	7.4620	0.3171	0.0375	23.1780	97.9610	97.9652	97.9977	97.9954	98.0000	1.7379	11.7190	13.4569
0.1500	9.8766	7.8556	2.0210	0.0032	0.0004	7.5821	0.3115	0.0375	23.0761	97.9615	97.9657	97.9977	97.9955	98.0000	1.7128	12.0316	13.7443
0.1550	9.7393	7.7277	2.0115	0.0032	0.0004	7.6989	0.3060	0.0375	22.9805	97.9621	97.9662	97.9978	97.9955	98.0000	1.6890	12.3441	14.0332

Table D-4 The results for **first case** study in single layer using **calcium silicate insulation.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	T_{b2}, ˚C	T_{b1} ˚C	Annual cost, $/m		
															C_H	C_I	C_T
0.0050	72.8484	64.9482	7.9002	0.0032	0.0004	0.3951	0.6717	0.0618	68.9339	97.7162	97.7472	97.9833	97.9667	98.0000	12.6128	3.2211	15.8339
0.0100	55.9812	49.7258	6.2554	0.0032	0.0004	0.7437	0.6457	0.0613	56.1467	97.7819	97.8057	97.9872	97.9744	98.0000	9.6932	3.5551	13.2483
0.0150	46.5702	41.1778	5.3924	0.0032	0.0004	1.0536	0.6174	0.0610	48.7541	97.8186	97.8384	97.9893	97.9787	98.0000	8.0642	3.8891	11.9533
0.0200	40.5029	35.6446	4.8583	0.0032	0.0004	1.3318	0.5901	0.0608	43.9000	97.8422	97.8594	97.9907	97.9815	98.0000	7.0140	4.2230	11.2371
0.0250	36.2358	31.7417	4.4942	0.0032	0.0004	1.5841	0.5645	0.0607	40.4566	97.8588	97.8742	97.9917	97.9834	98.0000	6.2754	4.5570	10.8324
0.0300	33.0543	28.8248	4.2295	0.0032	0.0004	1.8149	0.5410	0.0606	37.8826	97.8712	97.8853	97.9924	97.9849	98.0000	5.7247	4.8910	10.6157
0.0350	30.5805	26.5524	4.0281	0.0032	0.0004	2.0273	0.5194	0.0605	35.8841	97.8809	97.8939	97.9930	97.9860	98.0000	5.2965	5.2250	10.5215
0.0400	**28.5952**	**24.7256**	**3.8696**	**0.0032**	**0.0004**	**2.2242**	**0.4996**	**0.0604**	**34.2866**	**97.8886**	**97.9008**	**97.9935**	**97.9869**	**98.0000**	**4.9529**	**5.5590**	**10.5118**
0.0450	26.9621	23.2206	3.7414	0.0032	0.0004	2.4076	0.4814	0.0604	32.9803	97.8950	97.9064	97.9938	97.9877	98.0000	4.6702	5.8930	10.5631
0.0500	25.5918	21.9561	3.6357	0.0032	0.0004	2.5793	0.4647	0.0603	31.8920	97.9003	97.9112	97.9941	97.9883	98.0000	4.4330	6.2270	10.6599
0.0550	24.4231	20.8763	3.5468	0.0032	0.0004	2.7406	0.4492	0.0603	30.9714	97.9049	97.9152	97.9944	97.9888	98.0000	4.2307	6.5609	10.7916
0.0600	23.4129	19.9417	3.4712	0.0032	0.0004	2.8927	0.4349	0.0603	30.1825	97.9088	97.9187	97.9946	97.9893	98.0000	4.0558	6.8949	10.9508
0.0650	22.5294	19.1236	3.4059	0.0032	0.0004	3.0366	0.4216	0.0602	29.4989	97.9122	97.9218	97.9948	97.9897	98.0000	3.9029	7.2289	11.1318
0.0700	21.7492	18.4003	3.3490	0.0032	0.0004	3.1732	0.4093	0.0602	28.9010	97.9153	97.9245	97.9950	97.9901	98.0000	3.7679	7.5629	11.3308
0.0750	21.0543	17.7554	3.2989	0.0032	0.0004	3.3031	0.3977	0.0602	28.3735	97.9180	97.9269	97.9952	97.9904	98.0000	3.6476	7.8969	11.5445
0.0800	20.4306	17.1761	3.2545	0.0032	0.0004	3.4270	0.3869	0.0602	27.9047	97.9204	97.9291	97.9953	97.9907	98.0000	3.5396	8.2309	11.7705
0.0850	19.8672	16.6523	3.2149	0.0032	0.0004	3.5454	0.3768	0.0602	27.4854	97.9226	97.9311	97.9955	97.9909	98.0000	3.4421	8.5649	12.0070
0.0900	19.3553	16.1760	3.1794	0.0032	0.0004	3.6588	0.3672	0.0601	27.1082	97.9246	97.9328	97.9956	97.9911	98.0000	3.3535	8.8988	12.2523
0.0950	18.8877	15.7405	3.1472	0.0032	0.0004	3.7675	0.3583	0.0601	26.7670	97.9264	97.9344	97.9957	97.9914	98.0000	3.2726	9.2328	12.5054
0.1000	18.4585	15.3405	3.1181	0.0032	0.0004	3.8720	0.3498	0.0601	26.4569	97.9281	97.9359	97.9958	97.9916	98.0000	3.1983	9.5668	12.7651
0.1050	18.0630	14.9715	3.0915	0.0032	0.0004	3.9725	0.3418	0.0601	26.1739	97.9296	97.9373	97.9959	97.9917	98.0000	3.1298	9.9008	13.0306
0.1100	17.6970	14.6299	3.0671	0.0032	0.0004	4.0694	0.3342	0.0601	25.9146	97.9311	97.9386	97.9960	97.9919	98.0000	3.0664	10.2348	13.3012
0.1150	17.3572	14.3124	3.0448	0.0032	0.0004	4.1629	0.3270	0.0601	25.6761	97.9324	97.9398	97.9960	97.9921	98.0000	3.0076	10.5688	13.5764
0.1200	17.0406	14.0165	3.0241	0.0032	0.0004	4.2532	0.3202	0.0601	25.4560	97.9336	97.9409	97.9961	97.9922	98.0000	2.9528	10.9028	13.8556
0.1250	16.7448	13.7398	3.0050	0.0032	0.0004	4.3406	0.3137	0.0601	25.2523	97.9348	97.9419	97.9962	97.9923	98.0000	2.9016	11.2367	14.1384
0.1300	16.4677	13.4804	2.9873	0.0032	0.0004	4.4252	0.3075	0.0601	25.0632	97.9358	97.9428	97.9962	97.9925	98.0000	2.8537	11.5707	14.4244
0.1350	16.2074	13.2365	2.9709	0.0032	0.0004	4.5072	0.3015	0.0601	24.8873	97.9369	97.9438	97.9963	97.9926	98.0000	2.8086	11.9047	14.7133
0.1400	15.9623	13.0068	2.9555	0.0032	0.0004	4.5867	0.2959	0.0600	24.7231	97.9378	97.9446	97.9963	97.9927	98.0000	2.7662	12.2387	15.0049
0.1450	15.7311	12.7900	2.9411	0.0032	0.0004	4.6640	0.2905	0.0600	24.5695	97.9387	97.9454	97.9964	97.9928	98.0000	2.7262	12.5727	15.2988
0.1500	15.5125	12.5848	2.9277	0.0032	0.0004	4.7390	0.2853	0.0600	24.4256	97.9396	97.9462	97.9965	97.9929	98.0000	2.6883	12.9067	15.5950
0.1550	15.3054	12.3903	2.9151	0.0032	0.0004	4.8120	0.2803	0.0600	24.2905	97.9404	97.9469	97.9965	97.9930	98.0000	2.6525	13.2407	15.8931

Table D-5 The results for **first case** study in single layer using **cellular glass insulation.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	T_{b2}, ˚C	T_{b1} ˚C	Annual cost, $/m		
															C_H	C_I	C_T
0.0050	67.7456	60.3930	7.3526	0.0032	0.0004	0.4660	0.6815	0.0524	66.1697	97.7361	97.7649	97.9845	97.9690	98.0000	11.7296	3.5611	15.2907
0.0100	50.1923	44.5459	5.6463	0.0032	0.0004	0.8902	0.6599	0.0512	53.1240	97.8045	97.8258	97.9885	97.9770	98.0000	8.6912	3.9333	12.6245
0.0150	40.8599	36.0724	4.7875	0.0032	0.0004	1.2712	0.6338	0.0506	45.8982	97.8408	97.8582	97.9907	97.9813	98.0000	7.0758	4.3055	11.3813
0.0200	35.0311	30.7619	4.2692	0.0032	0.0004	1.6152	0.6075	0.0501	41.2814	97.8635	97.8784	97.9920	97.9840	98.0000	6.0669	4.6777	10.7445
0.0250	31.0228	27.1012	3.9217	0.0032	0.0004	1.9280	0.5824	0.0498	38.0676	97.8791	97.8923	97.9929	97.9858	98.0000	5.3731	5.0498	10.4229
0.0300	28.0843	24.4123	3.6720	0.0032	0.0004	2.2145	0.5590	0.0496	35.6982	97.8906	97.9025	97.9936	97.9872	98.0000	4.8644	5.4220	10.2864
0.0350	**25.8292**	**22.3456**	**3.4837**	**0.0032**	**0.0004**	**2.4787**	**0.5373**	**0.0495**	**33.8777**	**97.8994**	**97.9104**	**97.9941**	**97.9882**	**98.0000**	**4.4741**	**5.7942**	**10.2683**
0.0400	24.0386	20.7021	3.3365	0.0032	0.0004	2.7236	0.5173	0.0493	32.4347	97.9064	97.9166	97.9945	97.9890	98.0000	4.1641	6.1664	10.3305
0.0450	22.5783	19.3602	3.2181	0.0032	0.0004	2.9519	0.4988	0.0492	31.2625	97.9120	97.9216	97.9948	97.9897	98.0000	3.9114	6.5386	10.4499
0.0500	21.3621	18.2412	3.1209	0.0032	0.0004	3.1657	0.4818	0.0492	30.2914	97.9168	97.9259	97.9951	97.9902	98.0000	3.7009	6.9107	10.6116
0.0550	20.3313	17.2918	3.0394	0.0032	0.0004	3.3666	0.4660	0.0491	29.4738	97.9208	97.9294	97.9954	97.9907	98.0000	3.5224	7.2829	10.8053
0.0600	19.4450	16.4747	2.9703	0.0032	0.0004	3.5561	0.4513	0.0490	28.7759	97.9243	97.9325	97.9956	97.9911	98.0000	3.3690	7.6551	11.0241
0.0650	18.6736	15.7628	2.9108	0.0032	0.0004	3.7354	0.4377	0.0490	28.1732	97.9273	97.9352	97.9957	97.9915	98.0000	3.2355	8.0273	11.2628
0.0700	17.9952	15.1361	2.8591	0.0032	0.0004	3.9056	0.4250	0.0489	27.6475	97.9299	97.9375	97.9959	97.9918	98.0000	3.1181	8.3995	11.5175
0.0750	17.3932	14.5795	2.8136	0.0032	0.0004	4.0675	0.4131	0.0489	27.1851	97.9322	97.9396	97.9960	97.9920	98.0000	3.0139	8.7716	11.7855
0.0800	16.8547	14.0812	2.7734	0.0032	0.0004	4.2219	0.4020	0.0488	26.7751	97.9343	97.9415	97.9961	97.9923	98.0000	2.9206	9.1438	12.0645
0.0850	16.3697	13.6321	2.7376	0.0032	0.0004	4.3695	0.3915	0.0488	26.4091	97.9362	97.9432	97.9963	97.9925	98.0000	2.8367	9.5160	12.3527
0.0900	15.9302	13.2248	2.7054	0.0032	0.0004	4.5108	0.3817	0.0488	26.0804	97.9379	97.9447	97.9964	97.9927	98.0000	2.7606	9.8882	12.6488
0.0950	15.5298	12.8533	2.6764	0.0032	0.0004	4.6463	0.3724	0.0488	25.7836	97.9395	97.9461	97.9964	97.9929	98.0000	2.6913	10.2604	12.9517
0.1000	15.1631	12.5130	2.6501	0.0032	0.0004	4.7765	0.3637	0.0487	25.5143	97.9409	97.9474	97.9965	97.9931	98.0000	2.6278	10.6325	13.2604
0.1050	14.8258	12.1997	2.6261	0.0032	0.0004	4.9018	0.3554	0.0487	25.2689	97.9422	97.9485	97.9966	97.9932	98.0000	2.5695	11.0047	13.5742
0.1100	14.5143	11.9101	2.6042	0.0032	0.0004	5.0226	0.3475	0.0487	25.0443	97.9435	97.9496	97.9967	97.9934	98.0000	2.5156	11.3769	13.8925
0.1150	14.2257	11.6416	2.5841	0.0032	0.0004	5.1391	0.3401	0.0487	24.8379	97.9446	97.9506	97.9967	97.9935	98.0000	2.4656	11.7491	14.2147
0.1200	13.9572	11.3917	2.5655	0.0032	0.0004	5.2516	0.3330	0.0487	24.6477	97.9456	97.9516	97.9968	97.9936	98.0000	2.4191	12.1213	14.5404
0.1250	13.7067	11.1584	2.5483	0.0032	0.0004	5.3605	0.3263	0.0486	24.4719	97.9466	97.9524	97.9969	97.9937	98.0000	2.3758	12.4935	14.8692
0.1300	13.4724	10.9400	2.5324	0.0032	0.0004	5.4659	0.3198	0.0486	24.3088	97.9475	97.9532	97.9969	97.9938	98.0000	2.3352	12.8656	15.2008
0.1350	13.2526	10.7350	2.5176	0.0032	0.0004	5.5681	0.3137	0.0486	24.1571	97.9484	97.9540	97.9970	97.9939	98.0000	2.2972	13.2378	15.5350
0.1400	13.0459	10.5421	2.5038	0.0032	0.0004	5.6672	0.3078	0.0486	24.0157	97.9492	97.9547	97.9970	97.9940	98.0000	2.2614	13.6100	15.8714
0.1450	12.8511	10.3602	2.4909	0.0032	0.0004	5.7634	0.3022	0.0486	23.8836	97.9499	97.9554	97.9971	97.9941	98.0000	2.2277	13.9822	16.2098
0.1500	12.6672	10.1884	2.4788	0.0032	0.0004	5.8569	0.2968	0.0486	23.7599	97.9507	97.9560	97.9971	97.9942	98.0000	2.1958	14.3544	16.5502
0.1550	12.4931	10.0257	2.4675	0.0032	0.0004	5.9479	0.2917	0.0486	23.6438	97.9513	97.9566	97.9971	97.9943	98.0000	2.1657	14.7265	16.8922

Table D-6 The results for **second case** study in single layer using **rock mineral wool insulation at 40 kg/m^3.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m		
															C_H	C_I	C_T
0.0050	230.2722	202.6069	27.6653	0.0204	0.0003	0.2978	0.4198	0.0570	116.6585	185.2349	185.2954	190.0000	6.7463	0.9212	39.8619	3.4459	43.3077
0.0100	165.1967	146.0624	19.1343	0.0204	0.0003	0.5892	0.4192	0.0548	89.2529	186.5815	186.6249	190.0000	6.1649	0.8076	28.5977	3.7979	32.3957
0.0150	131.1395	115.9788	15.1607	0.0204	0.0003	0.8630	0.4127	0.0536	74.1192	187.2863	187.3207	190.0000	5.7371	0.7500	22.7026	4.1500	26.8526
0.0200	110.1938	97.3141	12.8797	0.0204	0.0003	1.1184	0.4036	0.0529	64.4797	187.7197	187.7487	190.0000	5.4027	0.7151	19.0771	4.5021	23.5791
0.0250	95.9845	84.5821	11.4025	0.0204	0.0003	1.3568	0.3936	0.0523	57.7816	188.0137	188.0390	190.0000	5.1303	0.6916	16.6175	4.8541	21.4717
0.0300	85.6904	75.3222	10.3681	0.0204	0.0003	1.5799	0.3833	0.0520	52.8473	188.2268	188.2493	190.0000	4.9021	0.6748	14.8357	5.2062	20.0419
0.0350	77.8732	68.2698	9.6033	0.0204	0.0003	1.7892	0.3731	0.0517	49.0567	188.3885	188.4090	190.0000	4.7066	0.6621	13.4826	5.5583	19.0408
0.0400	71.7234	62.7088	9.0146	0.0204	0.0003	1.9863	0.3632	0.0514	46.0512	188.5158	188.5346	190.0000	4.5365	0.6521	12.4181	5.9103	18.3284
0.0450	66.7505	58.2033	8.5473	0.0204	0.0003	2.1724	0.3537	0.0512	43.6086	188.6187	188.6362	190.0000	4.3865	0.6442	11.5573	6.2624	17.8197
0.0500	62.6400	54.4728	8.1671	0.0204	0.0003	2.3487	0.3446	0.0511	41.5837	188.7038	188.7202	190.0000	4.2528	0.6376	10.8458	6.6145	17.4603
0.0550	59.1805	51.3287	7.8518	0.0204	0.0003	2.5160	0.3359	0.0509	39.8776	188.7754	188.7909	190.0000	4.1325	0.6321	10.2470	6.9665	17.2135
0.0600	56.2251	48.6393	7.5858	0.0204	0.0003	2.6753	0.3276	0.0508	38.4202	188.8365	188.8513	190.0000	4.0235	0.6275	9.7354	7.3186	17.0540
0.0650	53.6682	46.3097	7.3585	0.0204	0.0003	2.8272	0.3198	0.0507	37.1609	188.8894	188.9035	190.0000	3.9241	0.6235	9.2928	7.6707	16.9635
0.0700	**51.4319**	**44.2701**	**7.1618**	**0.0204**	**0.0003**	**2.9724**	**0.3123**	**0.0506**	**36.0618**	**188.9357**	**188.9492**	**190.0000**	**3.8328**	**0.6201**	**8.9058**	**8.0227**	**16.9285**
0.0750	49.4577	42.4676	6.9900	0.0204	0.0003	3.1114	0.3052	0.0506	35.0942	188.9765	188.9895	190.0000	3.7487	0.6170	8.5640	8.3748	16.9388
0.0800	47.7005	40.8618	6.8386	0.0204	0.0003	3.2448	0.2984	0.0505	34.2358	189.0129	189.0254	190.0000	3.6708	0.6143	8.2599	8.7269	16.9867
0.0850	46.1251	39.4209	6.7042	0.0204	0.0003	3.3729	0.2920	0.0504	33.4691	189.0455	189.0576	190.0000	3.5984	0.6120	7.9872	9.0789	17.0661
0.0900	44.7036	38.1196	6.5839	0.0204	0.0003	3.4962	0.2859	0.0504	32.7803	189.0749	189.0867	190.0000	3.5308	0.6098	7.7411	9.4310	17.1721
0.0950	43.4137	36.9379	6.4758	0.0204	0.0003	3.6151	0.2800	0.0503	32.1579	189.1016	189.1130	190.0000	3.4675	0.6079	7.5178	9.7831	17.3009
0.1000	42.2371	35.8591	6.3779	0.0204	0.0003	3.7297	0.2745	0.0503	31.5930	189.1260	189.1371	190.0000	3.4081	0.6062	7.3142	10.1351	17.4493
0.1050	41.1589	34.8699	6.2890	0.0204	0.0003	3.8405	0.2691	0.0503	31.0779	189.1483	189.1591	190.0000	3.3521	0.6046	7.1276	10.4872	17.6148
0.1100	40.1667	33.9589	6.2078	0.0204	0.0003	3.9476	0.2641	0.0502	30.6062	189.1688	189.1794	190.0000	3.2993	0.6031	6.9558	10.8393	17.7951
0.1150	39.2501	33.1167	6.1334	0.0204	0.0003	4.0513	0.2592	0.0502	30.1729	189.1878	189.1981	190.0000	3.2494	0.6018	6.7972	11.1913	17.9885
0.1200	38.4004	32.3355	6.0649	0.0204	0.0003	4.1518	0.2545	0.0501	29.7733	189.2054	189.2155	190.0000	3.2020	0.6006	6.6501	11.5434	18.1935
0.1250	37.6101	31.6085	6.0017	0.0204	0.0003	4.2493	0.2500	0.0501	29.4037	189.2217	189.2316	190.0000	3.1571	0.5994	6.5133	11.8955	18.4088
0.1300	36.8730	30.9299	5.9431	0.0204	0.0003	4.3440	0.2457	0.0501	29.0608	189.2370	189.2467	190.0000	3.1143	0.5984	6.3857	12.2476	18.6332
0.1350	36.1835	30.2948	5.8887	0.0204	0.0003	4.4360	0.2416	0.0501	28.7419	189.2512	189.2607	190.0000	3.0735	0.5974	6.2663	12.5996	18.8660
0.1400	35.5369	29.6989	5.8381	0.0204	0.0003	4.5254	0.2376	0.0500	28.4446	189.2646	189.2740	190.0000	3.0346	0.5965	6.1544	12.9517	19.1061
0.1450	34.9292	29.1384	5.7908	0.0204	0.0003	4.6125	0.2338	0.0500	28.1667	189.2772	189.2864	190.0000	2.9974	0.5957	6.0492	13.3038	19.3530
0.1500	34.3567	28.6101	5.7465	0.0204	0.0003	4.6973	0.2301	0.0500	27.9064	189.2890	189.2981	190.0000	2.9618	0.5949	5.9501	13.6558	19.6060
0.1550	33.8162	28.1112	5.7050	0.0204	0.0003	4.7799	0.2266	0.0500	27.6620	189.3002	189.3091	190.0000	2.9277	0.5942	5.8566	14.0079	19.8645

Table D-7 The results for **second case** study in single layer using **rock mineral wool insulation at 200 kg/m^3.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m		
															C_H	C_I	C_T
0.0050	204.4380	180.3063	24.1317	0.0204	0.0003	0.3809	0.4300	0.0445	107.9020	185.7695	185.8232	190.0000	6.6019	0.8836	35.3901	3.4459	38.8360
0.0100	141.9523	125.6382	16.3141	0.0204	0.0003	0.7446	0.4323	0.0434	81.3627	187.0625	187.0998	190.0000	5.9847	0.7771	24.5743	3.7979	28.3722
0.0150	111.1602	98.3018	12.8584	0.0204	0.0003	1.0819	0.4268	0.0428	67.4398	187.6997	187.7289	190.0000	5.5474	0.7256	19.2443	4.1500	23.3943
0.0200	92.7456	81.8323	10.9133	0.0204	0.0003	1.3942	0.4181	0.0424	58.7740	188.0808	188.1051	190.0000	5.2117	0.6950	16.0569	4.5021	20.5590
0.0250	80.4474	70.7825	9.6649	0.0204	0.0003	1.6844	0.4081	0.0422	52.8272	188.3353	188.3564	190.0000	4.9413	0.6747	13.9281	4.8541	18.7823
0.0300	71.6244	62.8298	8.7946	0.0204	0.0003	1.9552	0.3976	0.0420	48.4795	188.5178	188.5367	190.0000	4.7162	0.6601	12.4009	5.2062	17.6071
0.0350	64.9684	56.8158	8.1526	0.0204	0.0003	2.2088	0.3872	0.0418	45.1560	188.6556	188.6727	190.0000	4.5243	0.6492	11.2488	5.5583	16.8071
0.0400	59.7567	52.0976	7.6590	0.0204	0.0003	2.4471	0.3770	0.0417	42.5299	188.7634	188.7791	190.0000	4.3579	0.6407	10.3467	5.9103	16.2571
0.0450	55.5570	48.2896	7.2674	0.0204	0.0003	2.6720	0.3672	0.0417	40.4010	188.8503	188.8649	190.0000	4.2115	0.6338	9.6198	6.2624	15.8822
0.0500	52.0948	45.1458	6.9489	0.0204	0.0003	2.8848	0.3578	0.0416	38.6395	188.9220	188.9357	190.0000	4.0813	0.6282	9.0205	6.6145	15.6350
0.0550	49.1871	42.5024	6.6847	0.0204	0.0003	3.0867	0.3488	0.0415	37.1574	188.9821	188.9951	190.0000	3.9644	0.6235	8.5172	6.9665	15.4837
0.0600	46.7073	40.2454	6.4618	0.0204	0.0003	3.2787	0.3403	0.0415	35.8929	189.0335	189.0457	190.0000	3.8585	0.6195	8.0879	7.3186	15.4066
0.0650	**44.5648**	**38.2935**	**6.2713**	**0.0204**	**0.0003**	**3.4618**	**0.3321**	**0.0414**	**34.8013**	**189.0778**	**189.0895**	**190.0000**	**3.7621**	**0.6161**	**7.7171**	**7.6707**	**15.3878**
0.0700	42.6931	36.5868	6.1064	0.0204	0.0003	3.6368	0.3244	0.0414	33.8492	189.1165	189.1278	190.0000	3.6737	0.6131	7.3931	8.0227	15.4159
0.0750	41.0424	35.0801	5.9623	0.0204	0.0003	3.8043	0.3170	0.0414	33.0116	189.1507	189.1615	190.0000	3.5922	0.6105	7.1074	8.3748	15.4822
0.0800	39.5743	33.7391	5.8352	0.0204	0.0003	3.9650	0.3100	0.0413	32.2689	189.1811	189.1915	190.0000	3.5168	0.6082	6.8533	8.7269	15.5802
0.0850	38.2591	32.5367	5.7223	0.0204	0.0003	4.1193	0.3033	0.0413	31.6059	189.2083	189.2183	190.0000	3.4468	0.6062	6.6256	9.0789	15.7046
0.0900	37.0731	31.4517	5.6214	0.0204	0.0003	4.2679	0.2970	0.0413	31.0103	189.2328	189.2426	190.0000	3.3814	0.6044	6.4203	9.4310	15.8513
0.0950	35.9974	30.4669	5.5305	0.0204	0.0003	4.4109	0.2909	0.0413	30.4725	189.2551	189.2645	190.0000	3.3203	0.6027	6.2341	9.7831	16.0172
0.1000	35.0167	29.5685	5.4483	0.0204	0.0003	4.5490	0.2851	0.0412	29.9845	189.2754	189.2846	190.0000	3.2629	0.6012	6.0644	10.1351	16.1995
0.1050	34.1185	28.7450	5.3735	0.0204	0.0003	4.6823	0.2796	0.0412	29.5396	189.2940	189.3029	190.0000	3.2089	0.5999	5.9089	10.4872	16.3961
0.1100	33.2921	27.9869	5.3052	0.0204	0.0003	4.8113	0.2743	0.0412	29.1323	189.3111	189.3198	190.0000	3.1580	0.5986	5.7659	10.8393	16.6051
0.1150	32.5290	27.2865	5.2426	0.0204	0.0003	4.9362	0.2692	0.0412	28.7582	189.3269	189.3354	190.0000	3.1098	0.5975	5.6338	11.1913	16.8251
0.1200	31.8218	26.6369	5.1849	0.0204	0.0003	5.0572	0.2644	0.0412	28.4133	189.3415	189.3499	190.0000	3.0641	0.5964	5.5114	11.5434	17.0548
0.1250	31.1643	26.0326	5.1317	0.0204	0.0003	5.1745	0.2597	0.0412	28.0943	189.3551	189.3633	190.0000	3.0208	0.5955	5.3976	11.8955	17.2930
0.1300	30.5511	25.4688	5.0823	0.0204	0.0003	5.2885	0.2553	0.0411	27.7985	189.3678	189.3758	190.0000	2.9795	0.5946	5.2914	12.2476	17.5390
0.1350	29.9777	24.9412	5.0365	0.0204	0.0003	5.3992	0.2510	0.0411	27.5233	189.3797	189.3875	190.0000	2.9402	0.5937	5.1922	12.5996	17.7918
0.1400	29.4401	24.4463	4.9938	0.0204	0.0003	5.5069	0.2468	0.0411	27.2668	189.3908	189.3985	190.0000	2.9027	0.5930	5.0991	12.9517	18.0508
0.1450	28.9349	23.9810	4.9539	0.0204	0.0003	5.6117	0.2429	0.0411	27.0271	189.4012	189.4088	190.0000	2.8669	0.5922	5.0117	13.3038	18.3154
0.1500	28.4590	23.5424	4.9166	0.0204	0.0003	5.7138	0.2390	0.0411	26.8026	189.4111	189.4186	190.0000	2.8326	0.5916	4.9293	13.6558	18.5851
0.1550	28.0099	23.1283	4.8816	0.0204	0.0003	5.8132	0.2353	0.0411	26.5919	189.4204	189.4277	190.0000	2.7997	0.5909	4.8515	14.0079	18.8594

Table D-8 The results for **second case** study in single layer using **calcium silicate insulation.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m C_H	C_I	C_T
0.0050	248.3961	218.1584	30.2377	0.0204	0.0003	0.2503	0.4134	0.0678	122.6806	184.8598	184.9251	190.0000	6.8381	0.9478	42.9990	3.5712	46.5703
0.0100	185.8904	164.1589	21.7315	0.0204	0.0003	0.4845	0.4093	0.0667	96.0896	186.1533	186.2021	190.0000	6.3061	0.8348	32.1797	3.9667	36.1463
0.0150	151.4538	133.8903	17.5635	0.0204	0.0003	0.7010	0.4008	0.0660	80.7021	186.8659	186.9057	190.0000	5.9049	0.7746	26.2189	4.3621	30.5810
0.0200	129.5139	114.4162	15.0978	0.0204	0.0003	0.9013	0.3906	0.0656	70.5869	187.3199	187.3539	190.0000	5.5853	0.7370	22.4213	4.7575	27.1788
0.0250	114.2456	100.7762	13.4694	0.0204	0.0003	1.0875	0.3799	0.0653	63.3994	187.6359	187.6659	190.0000	5.3214	0.7112	19.7784	5.1530	24.9314
0.0300	102.9683	90.6547	12.3136	0.0204	0.0003	1.2611	0.3692	0.0651	58.0159	187.8692	187.8963	190.0000	5.0978	0.6924	17.8264	5.5484	23.3748
0.0350	94.2735	82.8232	11.4504	0.0204	0.0003	1.4238	0.3588	0.0649	53.8268	188.0492	188.0739	190.0000	4.9048	0.6781	16.3214	5.9438	22.2652
0.0400	87.3490	76.5681	10.7809	0.0204	0.0003	1.5767	0.3488	0.0648	50.4713	188.1924	188.2154	190.0000	4.7356	0.6668	15.1228	6.3393	21.4620
0.0450	81.6926	71.4463	10.2463	0.0204	0.0003	1.7209	0.3393	0.0647	47.7216	188.3095	188.3310	190.0000	4.5857	0.6576	14.1437	6.7347	20.8784
0.0500	76.9769	67.1675	9.8094	0.0204	0.0003	1.8575	0.3303	0.0646	45.4264	188.4071	188.4273	190.0000	4.4514	0.6501	13.3274	7.1301	20.4575
0.0550	72.9790	63.5336	9.4455	0.0204	0.0003	1.9870	0.3218	0.0645	43.4812	188.4898	188.5090	190.0000	4.3301	0.6438	12.6354	7.5255	20.1610
0.0600	69.5421	60.4045	9.1376	0.0204	0.0003	2.1102	0.3136	0.0644	41.8115	188.5609	188.5792	190.0000	4.2198	0.6384	12.0405	7.9210	19.9615
0.0650	66.5521	57.6783	8.8738	0.0204	0.0003	2.2277	0.3060	0.0644	40.3624	188.6228	188.6403	190.0000	4.1190	0.6337	11.5230	8.3164	19.8394
0.0700	63.9242	55.2792	8.6450	0.0204	0.0003	2.3400	0.2987	0.0643	39.0930	188.6772	188.6940	190.0000	4.0262	0.6296	11.0681	8.7118	19.7799
0.0750	**61.5941**	**53.1493**	**8.4448**	**0.0204**	**0.0003**	**2.4475**	**0.2918**	**0.0643**	**37.9717**	**188.7254**	**188.7416**	**190.0000**	**3.9404**	**0.6261**	**10.6648**	**9.1073**	**19.7720**
0.0800	59.5121	51.2440	8.2680	0.0204	0.0003	2.5506	0.2852	0.0642	36.9741	188.7685	188.7841	190.0000	3.8608	0.6229	10.3044	9.5027	19.8071
0.0850	57.6388	49.5280	8.1108	0.0204	0.0003	2.6497	0.2790	0.0642	36.0807	188.8073	188.8224	190.0000	3.7867	0.6201	9.9801	9.8981	19.8783
0.0900	55.9432	47.9732	7.9700	0.0204	0.0003	2.7450	0.2731	0.0642	35.2762	188.8423	188.8570	190.0000	3.7174	0.6176	9.6866	10.2936	19.9802
0.0950	54.4000	46.5568	7.8433	0.0204	0.0003	2.8369	0.2674	0.0641	34.5478	188.8743	188.8886	190.0000	3.6525	0.6153	9.4195	10.6890	20.1085
0.1000	52.9887	45.2602	7.7285	0.0204	0.0003	2.9255	0.2620	0.0641	33.8853	188.9035	188.9174	190.0000	3.5914	0.6133	9.1752	11.0844	20.2596
0.1050	51.6921	44.0681	7.6241	0.0204	0.0003	3.0111	0.2569	0.0641	33.2801	188.9303	188.9439	190.0000	3.5338	0.6114	8.9508	11.4798	20.4306
0.1100	50.4963	42.9677	7.5286	0.0204	0.0003	3.0939	0.2520	0.0641	32.7252	188.9551	188.9683	190.0000	3.4794	0.6097	8.7438	11.8753	20.6191
0.1150	49.3893	41.9481	7.4411	0.0204	0.0003	3.1740	0.2473	0.0641	32.2145	188.9780	188.9909	190.0000	3.4279	0.6081	8.5522	12.2707	20.8229
0.1200	48.3610	41.0005	7.3606	0.0204	0.0003	3.2517	0.2428	0.0640	31.7430	188.9992	189.0120	190.0000	3.3791	0.6066	8.3742	12.6661	21.0403
0.1250	47.4030	40.1168	7.2861	0.0204	0.0003	3.3271	0.2385	0.0640	31.3063	189.0191	189.0315	190.0000	3.3326	0.6053	8.2084	13.0616	21.2699
0.1300	46.5078	39.2906	7.2172	0.0204	0.0003	3.4002	0.2344	0.0640	30.9008	189.0376	189.0498	190.0000	3.2884	0.6040	8.0534	13.4570	21.5104
0.1350	45.6691	38.5160	7.1531	0.0204	0.0003	3.4713	0.2304	0.0640	30.5231	189.0549	189.0669	190.0000	3.2462	0.6029	7.9082	13.8524	21.7607
0.1400	44.8815	37.7881	7.0934	0.0204	0.0003	3.5405	0.2266	0.0640	30.1706	189.0712	189.0830	190.0000	3.2059	0.6018	7.7719	14.2479	22.0198
0.1450	44.1401	37.1024	7.0376	0.0204	0.0003	3.6077	0.2229	0.0640	29.8409	189.0866	189.0982	190.0000	3.1673	0.6008	7.6436	14.6433	22.2869
0.1500	43.4407	36.4552	6.9855	0.0204	0.0003	3.6733	0.2194	0.0639	29.5317	189.1011	189.1125	190.0000	3.1304	0.5998	7.5225	15.0387	22.5612
0.1550	42.7797	35.8432	6.9365	0.0204	0.0003	3.7371	0.2160	0.0639	29.2413	189.1147	189.1260	190.0000	3.0950	0.5990	7.4081	15.4341	22.8422

Table D-9 The results for **second case** study in single layer using **cellular glass insulation.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_1, K/ W	R_2, K/ W	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_3, ˚C	T_2, ˚C	T_1, ˚C	T_i, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m C_H	C_I	C_T
0.0050	247.1029	217.0513	30.0516	0.0204	0.0003	0.2535	0.4138	0.0669	122.2540	184.8866	184.9515	190.0000	6.8318	0.9459	42.7752	4.1618	46.9369
0.0100	181.6415	160.4501	21.1914	0.0204	0.0003	0.5040	0.4112	0.0641	94.6990	186.2412	186.2889	190.0000	6.2784	0.8292	31.4442	4.5703	36.0145
0.0150	145.9074	129.0064	16.9009	0.0204	0.0003	0.7406	0.4038	0.0625	78.9235	186.9807	187.0190	190.0000	5.8613	0.7679	25.2589	4.9788	30.2377
0.0200	123.4499	109.0534	14.3965	0.0204	0.0003	0.9620	0.3944	0.0615	68.6913	187.4454	187.4778	190.0000	5.5308	0.7301	21.3716	5.3874	26.7590
0.0250	108.0149	95.2542	12.7606	0.0204	0.0003	1.1689	0.3843	0.0608	61.5048	187.7648	187.7932	190.0000	5.2594	0.7046	18.6999	5.7959	24.4958
0.0300	96.7347	85.1252	11.6095	0.0204	0.0003	1.3627	0.3739	0.0602	56.1734	187.9982	188.0236	190.0000	5.0307	0.6861	16.7474	6.2044	22.9518
0.0350	88.1151	77.3594	10.7557	0.0204	0.0003	1.5448	0.3638	0.0598	52.0574	188.1766	188.1997	190.0000	4.8340	0.6721	15.2554	6.6130	21.8683
0.0400	81.3022	71.2050	10.0972	0.0204	0.0003	1.7163	0.3540	0.0595	48.7820	188.3176	188.3389	190.0000	4.6624	0.6612	14.0761	7.0215	21.0976
0.0450	75.7729	66.1992	9.5738	0.0204	0.0003	1.8782	0.3446	0.0593	46.1125	188.4320	188.4519	190.0000	4.5107	0.6523	13.1190	7.4300	20.5491
0.0500	71.1890	62.0414	9.1476	0.0204	0.0003	2.0317	0.3357	0.0590	43.8947	188.5269	188.5456	190.0000	4.3752	0.6451	12.3256	7.8386	20.1641
0.0550	67.3218	58.5279	8.7938	0.0204	0.0003	2.1774	0.3271	0.0589	42.0226	188.6069	188.6246	190.0000	4.2531	0.6390	11.6562	8.2471	19.9033
0.0600	64.0114	55.5160	8.4954	0.0204	0.0003	2.3161	0.3190	0.0587	40.4212	188.6754	188.6922	190.0000	4.1424	0.6339	11.0832	8.6556	19.7388
0.0650	61.1424	52.9023	8.2401	0.0204	0.0003	2.4484	0.3113	0.0586	39.0356	188.7348	188.7508	190.0000	4.0412	0.6295	10.5866	9.0641	19.6507
0.0700	**58.6295**	**50.6102**	**8.0193**	**0.0204**	**0.0003**	**2.5748**	**0.3040**	**0.0585**	**37.8250**	**188.7868**	**188.8022**	**190.0000**	**3.9483**	**0.6256**	**10.1516**	**9.4727**	**19.6243**
0.0750	56.4082	48.5819	7.8264	0.0204	0.0003	2.6960	0.2971	0.0584	36.7582	188.8327	188.8475	190.0000	3.8626	0.6223	9.7671	9.8812	19.6483
0.0800	54.4289	46.7725	7.6563	0.0204	0.0003	2.8122	0.2905	0.0583	35.8111	188.8737	188.8880	190.0000	3.7831	0.6193	9.4245	10.2897	19.7142
0.0850	52.6525	45.1472	7.5053	0.0204	0.0003	2.9238	0.2842	0.0582	34.9646	188.9104	188.9243	190.0000	3.7092	0.6166	9.1170	10.6983	19.8153
0.0900	51.0483	43.6781	7.3702	0.0204	0.0003	3.0312	0.2782	0.0581	34.2036	188.9436	188.9570	190.0000	3.6402	0.6142	8.8393	11.1068	19.9462
0.0950	49.5913	42.3426	7.2487	0.0204	0.0003	3.1348	0.2725	0.0580	33.5156	188.9738	188.9868	190.0000	3.5755	0.6121	8.5872	11.5153	20.1025
0.1000	48.2614	41.1226	7.1388	0.0204	0.0003	3.2347	0.2671	0.0580	32.8908	189.0013	189.0140	190.0000	3.5148	0.6102	8.3570	11.9239	20.2808
0.1050	47.0420	40.0030	7.0390	0.0204	0.0003	3.3312	0.2619	0.0579	32.3209	189.0265	189.0389	190.0000	3.4576	0.6084	8.1459	12.3324	20.4783
0.1100	45.9190	38.9713	6.9478	0.0204	0.0003	3.4245	0.2569	0.0579	31.7988	189.0498	189.0618	190.0000	3.4036	0.6068	7.9515	12.7409	20.6924
0.1150	44.8811	38.0169	6.8642	0.0204	0.0003	3.5149	0.2522	0.0578	31.3189	189.0713	189.0830	190.0000	3.3525	0.6053	7.7719	13.1495	20.9213
0.1200	43.9185	37.1312	6.7873	0.0204	0.0003	3.6025	0.2476	0.0578	30.8763	189.0912	189.1027	190.0000	3.3040	0.6040	7.6052	13.5580	21.1632
0.1250	43.0228	36.3064	6.7163	0.0204	0.0003	3.6874	0.2433	0.0578	30.4667	189.1097	189.1210	190.0000	3.2580	0.6027	7.4502	13.9665	21.4167
0.1300	42.1869	35.5363	6.6506	0.0204	0.0003	3.7699	0.2391	0.0577	30.0867	189.1270	189.1381	190.0000	3.2142	0.6015	7.3055	14.3751	21.6806
0.1350	41.4047	34.8151	6.5896	0.0204	0.0003	3.8500	0.2351	0.0577	29.7331	189.1432	189.1541	190.0000	3.1724	0.6005	7.1701	14.7836	21.9537
0.1400	40.6709	34.1382	6.5327	0.0204	0.0003	3.9280	0.2312	0.0577	29.4034	189.1584	189.1691	190.0000	3.1326	0.5994	7.0431	15.1921	22.2352
0.1450	39.9810	33.5013	6.4797	0.0204	0.0003	4.0038	0.2275	0.0576	29.0951	189.1727	189.1832	190.0000	3.0944	0.5985	6.9237	15.6007	22.5243
0.1500	39.3308	32.9008	6.4300	0.0204	0.0003	4.0777	0.2239	0.0576	28.8063	189.1861	189.1964	190.0000	3.0579	0.5976	6.8111	16.0092	22.8203
0.1550	38.7169	32.3334	6.3835	0.0204	0.0003	4.1497	0.2205	0.0576	28.5351	189.1988	189.2090	190.0000	3.0229	0.5968	6.7049	16.4177	23.1226

Table D-10 The results for **third case** study in single layer using **rock mineral wool insulation at 40 kg/m^3.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_m, ˚C	T_0, ˚C	T_3, ˚C	T_2, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m		
													C_H	C_I	C_T
0.0050	456.0858	396.4584	59.6274	0.1554	0.2393	0.0592	164.5695	20.0000	129.1389	200.0000	6.4960	0.9770	78.9488	4.0975	83.0463
0.0100	319.8028	280.2339	39.5689	0.3155	0.2473	0.0567	149.5483	20.0000	99.0966	200.0000	5.9987	0.8470	55.3591	4.5383	59.8974
0.0150	248.9227	218.5958	30.3269	0.4724	0.2507	0.0554	141.2079	20.0000	82.4159	200.0000	5.6303	0.7811	43.0902	4.9792	48.0694
0.0200	205.5759	180.5225	25.0534	0.6240	0.2516	0.0545	135.8645	20.0000	71.7290	200.0000	5.3405	0.7412	35.5871	5.4200	41.0071
0.0250	176.3258	154.6758	21.6500	0.7698	0.2510	0.0539	132.1289	20.0000	64.2577	200.0000	5.1030	0.7143	30.5241	5.8609	36.3850
0.0300	155.2423	135.9707	19.2716	0.9101	0.2494	0.0534	129.3600	20.0000	58.7199	200.0000	4.9026	0.6949	26.8747	6.3017	33.1764
0.0350	139.3102	121.7953	17.5148	1.0449	0.2472	0.0531	127.2200	20.0000	54.4399	200.0000	4.7298	0.6802	24.1169	6.7426	30.8595
0.0400	126.8359	110.6727	16.1632	1.1745	0.2446	0.0528	125.5132	20.0000	51.0265	200.0000	4.5783	0.6686	21.9577	7.1834	29.1411
0.0450	116.7954	101.7050	15.0904	1.2994	0.2418	0.0525	124.1185	20.0000	48.2370	200.0000	4.4438	0.6593	20.2198	7.6243	27.8440
0.0500	108.5333	94.3157	14.2176	1.4197	0.2387	0.0524	122.9561	20.0000	45.9123	200.0000	4.3231	0.6517	18.7896	8.0651	26.8548
0.0550	101.6103	88.1170	13.4933	1.5358	0.2356	0.0522	121.9719	20.0000	43.9438	200.0000	4.2138	0.6453	17.5913	8.5060	26.0973
0.0600	95.7213	82.8390	12.8823	1.6480	0.2325	0.0521	121.1273	20.0000	42.2545	200.0000	4.1141	0.6398	16.5720	8.9468	25.5188
0.0650	90.6474	78.2878	12.3596	1.7564	0.2293	0.0519	120.3942	20.0000	40.7884	200.0000	4.0226	0.6351	15.6937	9.3877	25.0814
0.0700	86.2278	74.3206	11.9073	1.8613	0.2262	0.0518	119.7518	20.0000	39.5035	200.0000	3.9382	0.6310	14.9287	9.8285	24.7572
0.0750	82.3415	70.8297	11.5118	1.9629	0.2231	0.0517	119.1840	20.0000	38.3680	200.0000	3.8599	0.6273	14.2560	10.2694	24.5254
0.0800	78.8956	67.7326	11.1630	2.0615	0.2200	0.0517	118.6785	20.0000	37.3570	200.0000	3.7870	0.6241	13.6595	10.7102	24.3697
0.0850	75.8178	64.9648	10.8530	2.1571	0.2170	0.0516	118.2255	20.0000	36.4510	200.0000	3.7189	0.6213	13.1268	11.1511	24.2778
0.0900	**73.0508**	**62.4752**	**10.5756**	**2.2500**	**0.2140**	**0.0515**	**117.8172**	**20.0000**	**35.6344**	**200.0000**	**3.6551**	**0.6187**	**12.6478**	**11.5919**	**24.2397**
0.0950	70.5488	60.2230	10.3258	2.3403	0.2111	0.0515	117.4472	20.0000	34.8945	200.0000	3.5950	0.6164	12.2148	12.0328	24.2475
0.1000	68.2745	58.1748	10.0996	2.4281	0.2083	0.0514	117.1105	20.0000	34.2209	200.0000	3.5384	0.6143	11.8211	12.4736	24.2947
0.1050	66.1974	56.3035	9.8939	2.5136	0.2055	0.0513	116.8026	20.0000	33.6051	200.0000	3.4849	0.6124	11.4616	12.9144	24.3760
0.1100	64.2922	54.5863	9.7059	2.5969	0.2028	0.0513	116.5200	20.0000	33.0400	200.0000	3.4342	0.6106	11.1318	13.3553	24.4871
0.1150	62.5379	53.0045	9.5334	2.6781	0.2002	0.0513	116.2597	20.0000	32.5194	200.0000	3.3861	0.6090	10.8281	13.7961	24.6243
0.1200	60.9167	51.5421	9.3745	2.7572	0.1976	0.0512	116.0192	20.0000	32.0384	200.0000	3.3403	0.6075	10.5475	14.2370	24.7845
0.1250	59.4134	50.1857	9.2277	2.8345	0.1951	0.0512	115.7963	20.0000	31.5926	200.0000	3.2966	0.6062	10.2873	14.6778	24.9651
0.1300	58.0154	48.9238	9.0916	2.9099	0.1927	0.0511	115.5892	20.0000	31.1783	200.0000	3.2550	0.6049	10.0453	15.1187	25.1640
0.1350	56.7116	47.7465	8.9651	2.9837	0.1903	0.0511	115.3961	20.0000	30.7922	200.0000	3.2152	0.6037	9.8196	15.5595	25.3792
0.1400	55.4923	46.6452	8.8471	3.0557	0.1880	0.0511	115.2158	20.0000	30.4316	200.0000	3.1771	0.6026	9.6086	16.0004	25.6090
0.1450	54.3495	45.6126	8.7369	3.1262	0.1857	0.0511	115.0470	20.0000	30.0940	200.0000	3.1405	0.6016	9.4108	16.4412	25.8520
0.1500	53.2758	44.6421	8.6337	3.1951	0.1835	0.0510	114.8887	20.0000	29.7773	200.0000	3.1055	0.6006	9.2249	16.8821	26.1070
0.1550	52.2649	43.7282	8.5367	3.2626	0.1814	0.0510	114.7398	20.0000	29.4797	200.0000	3.0718	0.5997	9.0499	17.3229	26.3729

Table D-11 The results for **third case** study in single layer using **rock mineral wool insulation at 200 kg/m^3.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_m, ˚C	T_0, ˚C	T_3, ˚C	T_2, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m		
													C_H	C_I	C_T
0.0050	403.1122	351.5917	51.5205	0.2011	0.2454	0.0458	159.4697	20.0000	118.9394	200.0000	6.3548	0.9312	69.7794	4.0975	73.8769
0.0100	273.1292	239.7632	33.3660	0.4036	0.2555	0.0444	144.8874	20.0000	89.7748	200.0000	5.8180	0.8097	47.2802	4.5383	51.8185
0.0150	209.5690	184.1315	25.4375	0.5991	0.2598	0.0436	137.2209	20.0000	74.4419	200.0000	5.4373	0.7511	36.2783	4.9792	41.2575
0.0200	171.7943	150.7942	21.0001	0.7866	0.2611	0.0432	132.4295	20.0000	64.8590	200.0000	5.1442	0.7164	29.7397	5.4200	35.1598
0.0250	146.7057	128.5461	18.1596	0.9662	0.2607	0.0429	129.1246	20.0000	58.2491	200.0000	4.9072	0.6932	25.3970	5.8609	31.2579
0.0300	128.7992	112.6173	16.1820	1.1383	0.2592	0.0427	126.6948	20.0000	53.3897	200.0000	4.7088	0.6766	22.2975	6.3017	28.5993
0.0350	115.3570	100.6335	14.7236	1.3033	0.2571	0.0425	124.8270	20.0000	49.6540	200.0000	4.5387	0.6641	19.9708	6.7426	26.7134
0.0400	104.8814	91.2792	13.6022	1.4618	0.2544	0.0424	123.3429	20.0000	46.6858	200.0000	4.3903	0.6542	18.1575	7.1834	25.3409
0.0450	96.4787	83.7666	12.7121	1.6142	0.2515	0.0423	122.1333	20.0000	44.2666	200.0000	4.2589	0.6463	16.7031	7.6243	24.3273
0.0500	89.5824	77.5947	11.9877	1.7609	0.2484	0.0422	121.1273	20.0000	42.2546	200.0000	4.1412	0.6398	15.5094	8.0651	23.5745
0.0550	83.8158	72.4294	11.3864	1.9023	0.2452	0.0421	120.2768	20.0000	40.5536	200.0000	4.0349	0.6343	14.5112	8.5060	23.0172
0.0600	78.9185	68.0398	10.8787	2.0389	0.2420	0.0421	119.5478	20.0000	39.0956	200.0000	3.9381	0.6297	13.6635	8.9468	22.6103
0.0650	74.7048	64.2605	10.4443	2.1708	0.2387	0.0420	118.9157	20.0000	37.8315	200.0000	3.8494	0.6256	12.9341	9.3877	22.3218
0.0700	71.0385	60.9705	10.0681	2.2984	0.2354	0.0420	118.3622	20.0000	36.7245	200.0000	3.7676	0.6221	12.2995	9.8285	22.1280
0.0750	67.8176	58.0786	9.7389	2.4220	0.2322	0.0419	117.8734	20.0000	35.7468	200.0000	3.6919	0.6191	11.7420	10.2694	22.0114
0.0800	64.9639	55.5154	9.4485	2.5418	0.2290	0.0419	117.4385	20.0000	34.8769	200.0000	3.6214	0.6163	11.2481	10.7102	21.9583
0.0850	**62.4168**	**53.2266**	**9.1902**	**2.6580**	**0.2259**	**0.0419**	**117.0489**	**20.0000**	**34.0977**	**200.0000**	**3.5556**	**0.6139**	**10.8072**	**11.1511**	**21.9582**
0.0900	60.1283	51.1694	8.9589	2.7708	0.2228	0.0418	116.6978	20.0000	33.3956	200.0000	3.4940	0.6117	10.4110	11.5919	22.0029
0.0950	58.0600	49.3094	8.7506	2.8805	0.2198	0.0418	116.3799	20.0000	32.7598	200.0000	3.4360	0.6098	10.0530	12.0328	22.0858
0.1000	56.1807	47.6189	8.5618	2.9871	0.2168	0.0418	116.0905	20.0000	32.1811	200.0000	3.3814	0.6080	9.7277	12.4736	22.2013
0.1050	54.4651	46.0751	8.3901	3.0909	0.2139	0.0418	115.8261	20.0000	31.6522	200.0000	3.3298	0.6063	9.4308	12.9144	22.3452
0.1100	52.8921	44.6591	8.2330	3.1920	0.2111	0.0417	115.5834	20.0000	31.1668	200.0000	3.2809	0.6048	9.1585	13.3553	22.5138
0.1150	51.4440	43.3552	8.0888	3.2906	0.2084	0.0417	115.3600	20.0000	30.7200	200.0000	3.2346	0.6035	8.9078	13.7961	22.7040
0.1200	50.1062	42.1502	7.9560	3.3867	0.2057	0.0417	115.1535	20.0000	30.3071	200.0000	3.1905	0.6022	8.6763	14.2370	22.9133
0.1250	48.8661	41.0329	7.8332	3.4804	0.2031	0.0417	114.9623	20.0000	29.9245	200.0000	3.1484	0.6010	8.4616	14.6778	23.1395
0.1300	47.7130	39.9937	7.7193	3.5720	0.2006	0.0417	114.7845	20.0000	29.5690	200.0000	3.1084	0.6000	8.2620	15.1187	23.3807
0.1350	46.6378	39.0244	7.6133	3.6615	0.1981	0.0417	114.6189	20.0000	29.2378	200.0000	3.0701	0.5989	8.0759	15.5595	23.6355
0.1400	45.6326	38.1180	7.5146	3.7489	0.1957	0.0416	114.4642	20.0000	28.9284	200.0000	3.0334	0.5980	7.9019	16.0004	23.9023
0.1450	44.6905	37.2683	7.4222	3.8344	0.1933	0.0416	114.3194	20.0000	28.6388	200.0000	2.9983	0.5971	7.7389	16.4412	24.1801
0.1500	43.8056	36.4699	7.3356	3.9181	0.1910	0.0416	114.1836	20.0000	28.3672	200.0000	2.9645	0.5963	7.5857	16.8821	24.4678
0.1550	42.9725	35.7182	7.2544	4.0000	0.1888	0.0416	114.0560	20.0000	28.1120	200.0000	2.9321	0.5955	7.4415	17.3229	24.7644

Table D-12 The results for **third case** study in single layer using **calcium silicate insulation.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_m, ˚C	T_0, ˚C	T_3, ˚C	T_2, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m		
													C_H	C_I	C_T
0.0050	487.2845	422.6869	64.5977	0.1333	0.2361	0.0690	167.5139	20.0000	135.0277	200.0000	6.5712	1.0043	84.3491	4.3500	88.6991
0.0100	355.4324	310.9411	44.4912	0.2644	0.2420	0.0677	153.0124	20.0000	106.0248	200.0000	6.1199	0.8757	61.5263	4.8122	66.3385
0.0150	283.6393	248.8599	34.7794	0.3905	0.2441	0.0670	144.6188	20.0000	89.2377	200.0000	5.7783	0.8075	49.0994	5.2744	54.3738
0.0200	238.2855	209.2099	29.0756	0.5113	0.2441	0.0665	139.0814	20.0000	78.1628	200.0000	5.5046	0.7650	41.2489	5.7367	46.9856
0.0250	206.9441	181.6203	25.3238	0.6270	0.2428	0.0661	135.1268	20.0000	70.2536	200.0000	5.2770	0.7358	35.8239	6.1989	42.0228
0.0300	183.9394	161.2725	22.6669	0.7378	0.2408	0.0659	132.1479	20.0000	64.2959	200.0000	5.0829	0.7144	31.8420	6.6611	38.5031
0.0350	166.3046	145.6196	20.6850	0.8440	0.2383	0.0657	129.8164	20.0000	59.6329	200.0000	4.9140	0.6980	28.7895	7.1233	35.9128
0.0400	152.3359	133.1871	19.1488	0.9461	0.2355	0.0655	127.9381	20.0000	55.8761	200.0000	4.7649	0.6851	26.3716	7.5856	33.9572
0.0450	140.9838	123.0615	17.9222	1.0442	0.2325	0.0654	126.3901	20.0000	52.7803	200.0000	4.6317	0.6745	24.4066	8.0478	32.4544
0.0500	131.5661	114.6464	16.9197	1.1387	0.2294	0.0653	125.0912	20.0000	50.1823	200.0000	4.5115	0.6658	22.7765	8.5100	31.2865
0.0550	123.6197	107.5353	16.0844	1.2298	0.2263	0.0652	123.9846	20.0000	47.9693	200.0000	4.4023	0.6585	21.4010	8.9722	30.3733
0.0600	116.8192	101.4418	15.3775	1.3178	0.2231	0.0651	123.0302	20.0000	46.0605	200.0000	4.3022	0.6522	20.2239	9.4345	29.6584
0.0650	110.9290	96.1579	14.7711	1.4027	0.2199	0.0650	122.1982	20.0000	44.3964	200.0000	4.2101	0.6467	19.2043	9.8967	29.1010
0.0700	105.7743	91.5292	14.2451	1.4849	0.2168	0.0650	121.4662	20.0000	42.9324	200.0000	4.1249	0.6420	18.3121	10.3589	28.6710
0.0750	101.2225	87.4381	13.7843	1.5645	0.2137	0.0649	120.8170	20.0000	41.6340	200.0000	4.0456	0.6378	17.5242	10.8212	28.3453
0.0800	97.1714	83.7941	13.3772	1.6417	0.2107	0.0649	120.2372	20.0000	40.4745	200.0000	3.9717	0.6341	16.8230	11.2834	28.1063
0.0850	93.5407	80.5259	13.0148	1.7166	0.2077	0.0648	119.7163	20.0000	39.4325	200.0000	3.9025	0.6307	16.1945	11.7456	27.9401
0.0900	90.2667	77.5766	12.6901	1.7892	0.2048	0.0648	119.2455	20.0000	38.4910	200.0000	3.8374	0.6277	15.6278	12.2078	27.8356
0.0950	87.2978	74.9005	12.3973	1.8599	0.2020	0.0647	118.8180	20.0000	37.6359	200.0000	3.7762	0.6250	15.1139	12.6701	27.7840
0.1000	**84.5922**	**72.4602**	**12.1320**	**1.9286**	**0.1993**	**0.0647**	**118.4280**	**20.0000**	**36.8560**	**200.0000**	**3.7183**	**0.6226**	**14.6456**	**13.1323**	**27.7779**
0.1050	82.1153	70.2249	11.8904	1.9955	0.1966	0.0647	118.0708	20.0000	36.1415	200.0000	3.6636	0.6203	14.2168	13.5945	27.8114
0.1100	79.8384	68.1690	11.6694	2.0606	0.1940	0.0647	117.7424	20.0000	35.4847	200.0000	3.6116	0.6182	13.8227	14.0567	27.8795
0.1150	77.7375	66.2710	11.4665	2.1241	0.1914	0.0646	117.4394	20.0000	34.8788	200.0000	3.5622	0.6164	13.4591	14.5190	27.9780
0.1200	75.7923	64.5128	11.2795	2.1860	0.1889	0.0646	117.1590	20.0000	34.3180	200.0000	3.5152	0.6146	13.1224	14.9812	28.1036
0.1250	73.9855	62.8790	11.1065	2.2464	0.1865	0.0646	116.8988	20.0000	33.7976	200.0000	3.4704	0.6130	12.8096	15.4434	28.2531
0.1300	72.3024	61.3562	10.9461	2.3054	0.1841	0.0646	116.6567	20.0000	33.3133	200.0000	3.4275	0.6115	12.5183	15.9056	28.4239
0.1350	70.7301	59.9332	10.7969	2.3630	0.1818	0.0645	116.4307	20.0000	32.8615	200.0000	3.3865	0.6101	12.2462	16.3679	28.6140
0.1400	69.2578	58.6001	10.6577	2.4194	0.1796	0.0645	116.2195	20.0000	32.4390	200.0000	3.3472	0.6088	11.9913	16.8301	28.8214
0.1450	67.8758	57.3482	10.5276	2.4745	0.1774	0.0645	116.0215	20.0000	32.0430	200.0000	3.3095	0.6075	11.7521	17.2923	29.0444
0.1500	66.5758	56.1701	10.4057	2.5284	0.1753	0.0645	115.8356	20.0000	31.6712	200.0000	3.2734	0.6064	11.5271	17.7546	29.2816
0.1550	65.3503	55.0591	10.2912	2.5811	0.1732	0.0645	115.6607	20.0000	31.3214	200.0000	3.2385	0.6053	11.3149	18.2168	29.5317

Table D-13 The results for **third case** study in single layer using **cellular glass insulation.**

t_{ins}, m	Q_a, W/m	q_c W/m	q_r W/m	R_3, K/ W	R_4, K/ W	k_{ins}, W/m.K	T_m, ˚C	T_0, ˚C	T_3, ˚C	T_2, ˚C	h_0, W/$m^2.K$	h_r W/$m^2.K$	Annual cost, $/m		
													C_H	C_I	C_T
0.0050	490.5784	425.4474	65.1310	0.1312	0.2357	0.0701	167.8223	20.0000	135.6447	200.0000	6.5789	1.0071	84.9192	4.8720	89.7913
0.0100	352.5018	308.4217	44.0801	0.2682	0.2424	0.0667	152.7303	20.0000	105.4605	200.0000	6.1104	0.8733	61.0190	5.3839	66.4030
0.0150	277.5905	243.5967	33.9938	0.4032	0.2452	0.0648	144.0310	20.0000	88.0619	200.0000	5.7538	0.8029	48.0524	5.8958	53.9481
0.0200	230.7976	202.6518	28.1458	0.5342	0.2457	0.0636	138.3534	20.0000	76.7068	200.0000	5.4689	0.7596	39.9528	6.4076	46.3605
0.0250	198.8223	174.4802	24.3421	0.6605	0.2449	0.0628	134.3410	20.0000	68.6820	200.0000	5.2332	0.7301	34.4181	6.9195	41.3376
0.0300	175.5825	153.9097	21.6728	0.7820	0.2431	0.0622	131.3460	20.0000	62.6919	200.0000	5.0331	0.7087	30.3954	7.4314	37.8268
0.0350	157.9173	138.2210	19.6964	0.8990	0.2409	0.0617	129.0198	20.0000	58.0395	200.0000	4.8597	0.6925	27.3377	7.9432	35.2809
0.0400	144.0255	125.8519	18.1736	1.0115	0.2383	0.0613	127.1578	20.0000	54.3156	200.0000	4.7072	0.6797	24.9331	8.4551	33.3882
0.0450	132.8060	115.8421	16.9638	1.1200	0.2354	0.0610	125.6320	20.0000	51.2639	200.0000	4.5714	0.6694	22.9911	8.9670	31.9580
0.0500	123.5485	107.5694	15.9791	1.2245	0.2324	0.0607	124.3576	20.0000	48.7152	200.0000	4.4493	0.6609	21.3887	9.4788	30.8675
0.0550	115.7744	100.6127	15.1617	1.3254	0.2294	0.0605	123.2766	20.0000	46.5531	200.0000	4.3385	0.6538	20.0430	9.9907	30.0337
0.0600	109.1491	94.6772	14.4719	1.4229	0.2263	0.0603	122.3475	20.0000	44.6950	200.0000	4.2373	0.6477	18.8962	10.5025	29.3988
0.0650	103.4320	89.5501	13.8819	1.5171	0.2231	0.0601	121.5402	20.0000	43.0804	200.0000	4.1444	0.6425	17.9066	11.0144	28.9211
0.0700	98.4455	85.0742	13.3713	1.6084	0.2201	0.0600	120.8319	20.0000	41.6638	200.0000	4.0585	0.6379	17.0435	11.5263	28.5698
0.0750	94.0554	81.1305	12.9249	1.6968	0.2170	0.0599	120.2054	20.0000	40.4108	200.0000	3.9787	0.6339	16.2836	12.0381	28.3218
0.0800	90.1589	77.6277	12.5312	1.7825	0.2140	0.0597	119.6472	20.0000	39.2944	200.0000	3.9045	0.6303	15.6092	12.5500	28.1592
0.0850	86.6755	74.4942	12.1813	1.8657	0.2111	0.0596	119.1466	20.0000	38.2932	200.0000	3.8350	0.6271	15.0062	13.0619	28.0681
0.0900	**83.5414**	**71.6731**	**11.8683**	**1.9465**	**0.2082**	**0.0595**	**118.6951**	**20.0000**	**37.3901**	**200.0000**	**3.7698**	**0.6242**	**14.4637**	**13.5737**	**28.0374**
0.0950	80.7053	69.1189	11.5865	2.0250	0.2053	0.0595	118.2858	20.0000	36.5715	200.0000	3.7085	0.6217	13.9728	14.0856	28.0584
0.1000	78.1257	66.7944	11.3313	2.1014	0.2026	0.0594	117.9130	20.0000	35.8259	200.0000	3.6507	0.6193	13.5263	14.5975	28.1237
0.1050	75.7684	64.6691	11.0993	2.1758	0.1999	0.0593	117.5720	20.0000	35.1440	200.0000	3.5960	0.6172	13.1182	15.1093	28.2276
0.1100	73.6050	62.7177	10.8873	2.2482	0.1972	0.0593	117.2589	20.0000	34.5178	200.0000	3.5441	0.6152	12.7438	15.6212	28.3650
0.1150	71.6119	60.9191	10.6927	2.3189	0.1947	0.0592	116.9705	20.0000	33.9409	200.0000	3.4949	0.6134	12.3988	16.1331	28.5318
0.1200	69.7692	59.2555	10.5136	2.3878	0.1922	0.0591	116.7038	20.0000	33.4077	200.0000	3.4480	0.6118	12.0798	16.6449	28.7247
0.1250	68.0599	57.7118	10.3481	2.4550	0.1897	0.0591	116.4566	20.0000	32.9133	200.0000	3.4033	0.6102	11.7839	17.1568	28.9407
0.1300	66.4696	56.2749	10.1947	2.5206	0.1874	0.0590	116.2268	20.0000	32.4536	200.0000	3.3607	0.6088	11.5087	17.6686	29.1773
0.1350	64.9859	54.9337	10.0521	2.5848	0.1850	0.0590	116.0126	20.0000	32.0252	200.0000	3.3199	0.6075	11.2519	18.1805	29.4324
0.1400	63.5980	53.6788	9.9192	2.6475	0.1828	0.0590	115.8125	20.0000	31.6250	200.0000	3.2808	0.6063	11.0116	18.6924	29.7040
0.1450	62.2966	52.5016	9.7950	2.7088	0.1806	0.0589	115.6251	20.0000	31.2502	200.0000	3.2434	0.6051	10.7864	19.2042	29.9906
0.1500	61.0736	51.3949	9.6787	2.7688	0.1784	0.0589	115.4493	20.0000	30.8985	200.0000	3.2074	0.6040	10.5747	19.7161	30.2908
0.1550	59.9219	50.3524	9.5695	2.8275	0.1764	0.0589	115.2840	20.0000	30.5679	200.0000	3.1729	0.6030	10.3753	20.2280	30.6033

شكر وتقدير

الحمد لله الذي انعم علينا بتمام الصحة و فضلنا بنعمة العقل وزيننا بتاج العلم والصلاة والسلام على اشرف الخلق محمد وعلى آله الطيبين الطاهرين.

اقف عاجزةً على التعبير عن مدى الشكر والامتنان الذي اود ان ابديه الى مشرفي الفاضل الاستاذ الدكتور قاسم جبار سليمان لما بذله من جهد وعناء لاخراج هذا النتاج العلمي بهذه الحله الغراء.

واتقدم بالشكر الجزيل الى استاذي الدكتور باسم عبيد حسن رئيساً لقسم الهندسة الكيمياوية وجميع الكادر فيه لما بذلوه في مساعدتي لانجاز هذا العمل.

كما واتقدم بالشكر والاجلال الى والديّ الكريمين اللذان حفوني بالدموع والدعاء كما واشكر اخواني واختي اللذين لطالما وقفوا الى جانبي.

وشكري الجزيل لجميع زملائي الذين لم يقصرو في مساعدتي.

سمر سعدي حسين

طبقتي العزل المركبة (calcium silicate & mineral wool) و (cellular glass & mineral wool) افضل من استعمال طبقة العزل الواحدة من calcium silicate او cellular glass. لكن اوطئ تكلفة سنوية عند استعمال طبقتي العزل المركبة تكون اعلى من استعمال طبقة واحدة من الصوف الصخري.

الخلاصة

يسعى البحث الحالي الى الحصول على بيانات قياسية لتحليل رياضي لإنتقال الحرارة في انابيب تحمل موائع ساخنة ويشمل التحليل الحالات التي تستخدم فيها طبقة واحدة من المواد العازلة حراريا أو طبقات متعددة من عوازل حرارية مختلفة الأنواع. وقد استخدمت طريقة **quasi-Newton** في برنامج **STATISTICA** لايجاد معادلة التوصيل الحرارية الدالة على عوامل تؤثر على كفاءة العزل (درجة حرارة والكثافة) بإستعمال البيانات في الأدب لثلاثة من أنواع العزل الحراري (mineral wool, calcium silicate, and cellular glass) . إنّ توزيع درجة الحرارة هو مكتسب لكلّ طبقة عن طريق التجربة والخطا باستعمال البرنامج الخطي **MATLAB** . وتم ملاحظة درجة حرارة السطح الخارجي للعزل الحراري والتي روعي ألا تتعدى حد الأمان المسموح به في مصافي تكرير البترول "60 م°". طُبق التحليل على ثلاث حالات لمواد مختلفة (Hot water, Petroleum products in refineries, and Steam) عند درجات حرارة متباينة ولكل من هذه الحالات تم حساب توزيع درجة الحرارة وكذلك معدل إنتقال الحرارة المفقودة اذا كانت خطوط الأنابيب غير معزولة حرارياً بطبقة أو طبقتين من العزل الحراري.

قدم تحليل اقتصادي باعتماد دالة تكاليف سنوية والتي قسمت الى تكاليف العزل الحراري وكذلك تكاليف الطاقة المفقودة. والهدف من التحليل الإقتصادي هو الحصول على أقل قيمة للتكاليف السنوية مع مراعاة الشرط الوحيد وهو عدم زيادة درجة الحرارة على السطح الخارجي عن أقصى درجة مسموح بها. وقد استخدم البرنامج التفاعلي **MATLAB** للحصول على التكاليف السنوية للتشكيلات الثلاث المتباينة للعوازل، إختيار السمك القصوي للعزل الوحيد ومتعدّد الطبقات مستند على التحليل الإقتصادي.

نُلاحظ من النتائج توفر كلفة هامّة بالعزل القصوي مقارنةً إلى الأبعاد الهندسية الممارسة. البيانات المفصّلة لهذه الحالات قُدِمت وبُحِثت بالتفصيل. حيث اتضح ان التوصيلية الحرارية عموماً تزداد مع درجة الحرارة للعوازل الثلاث المدروسة . والنتائج تشير ان اوطئ توصيلية حرارية حصلت عند استعمال الصوف الصخري ذو كثافة $200\ kg/m^3$. ومن نتائج طبقتي العزل المركبة نستنتج انه استعمال

افضل سمك طبقة وطبقتين من عازل انبوبي لمنع تسرب الحرارة تحت ظروف انتقال الحرارة بالحمل الطبيعي

رسالة

مقدمة الى كلية الهندسة في جامعة النهرين

وهي جزء من متطلبات نيل درجة ماجستير علوم

في الهندسة الكيمياوية

من قبل

سمر سعدي حسين

بكالوريوس علوم في الهندسة الكيمياوية 2008

ربيع الاول 1432

اذار 2011