PREDICTION OF PRESSURE DROP IN PACKED BED FOR WATER AND AIR SYSTEMS

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by

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Certification

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Abstract

Semi-empirical equations for fluid flow through packed bed have been proposed, depending on statistical fitting of experimental data. Two types of fluids have been used (water and air) separately (single phase flow). Several size distributions of sphere packing materials have been used in the packed bed, and each had been studied separately.

Different parameters affecting the pressure drop of fluid flow through packed bed have been studied. These parameters are fluid velocity, bed porosity, bed diameter, pore diameter, tortuosity and packing height.

A certain semi-empirical equations for fluid flow through packed bed have been proposed for a certain size and type of packing system called singular equation (mono, binary, ternary, quaternary, quinary and multi-sized spherical particle system). There were ten singular equations have been written, five of them for water flow and five for air flow through packed beds.

A general semi-empirical equation has been proposed that can be used for all types of packing systems is:

A. for air flow through packed bed.

$$\frac{\Delta P}{L} = -5.47872\frac{\tau(\tau-1)^2}{\varepsilon^2}\frac{\mu u}{d_{pore}^2} + 2.7267\frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)}\frac{\rho u^2}{d_{pore}}$$

The average percentages errors were found 13.57423%.

B. for water flow through packed bed.

$$\frac{\Delta P}{L} = 108.3983\frac{\tau(\tau-1)^2}{\varepsilon^2}\frac{\mu u}{d_{pore}^2} + 2.1188\frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)}\frac{\rho u^2}{d_{pore}}$$

The average percentages errors were found 12.9576%.

The empirical formulas of pore diameter had been proposed for all the equations used in the calculations. The calculation results of these formulas have been compared with experimental results taken from documented literature data; the result equation of pore diameter for water and air is:

$$d_{pore} = 1.9612 \ d_p \ \varepsilon^{1.61605}$$

The average percentages errors for air flow were found to be 0.3897%, and for water flow were found to be 0.328%.

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Notations

Symbols		Notations
D_b	=	Diameter of the bed (m).
D_e	=	Effective diffusion coefficient (m^2/s).
D_0	=	Diffusion coefficient in the bulk medium (m^2/s).
d_{eff}	=	Effective particles diameter (m).
d_{pi}	=	Diameter of particles i in mixture (m).
d_p	=	Diameter of the particle (m).
d_{pore}	=	Equivalent diameter of the pore channels (m).
d_t	=	Diameter of tube (m).
f_w	=	Correction factor.
K	=	Kozeny's coefficient.
K_C	=	Kozeny's constant.
K_1	=	Representation of packing and fluid characteristics at laminar flow.
K_2	=	Representation of packing and fluid characteristics at turbulent flow
k	=	Permeability coefficient for the bed (m^2).
L	=	Height of packing in the bed (m).
L_e	=	Average length of porous medium (m).
l	=	Thickness of the bed (m).
Δp	=	Pressure drop through packed bed, Pa ($kg/m.s^2$).
R	=	Reduce of horizontal pipe (m).
Δr	=	An annulus thickness of element (m).
S	=	Specific surface area of the particles (m^2/m^3).
S_B	=	Specific surface area of the bed (m^2/m^3).
S_c	=	Surface of the container per unit volume of the bed (m^{-1}).

u	=	Superficial velocity (m/s).
u_1	=	Average velocity through the pore channels (m/s).
V_{total}	=	Total void of the bed (m^3).
V_{void}	=	Volume of the void in a bed (m^3).
x_i	=	The weight fraction of particle i.

Greek Symbols

ε	=	Porosity of the bed.
μ	=	Fluid viscosity (kg/m.s).
Φ_s	=	Sphericity.
ρ	=	Density of the fluid (kg/m^3).
τ	=	Tortuosity factor.
λ	=	orientation factor
α	=	Angle which the normal solid-liquid interface makes with the stream direction.

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Chapter One

Introduction

Fluid flow through packed bed has many important applications in chemical and other process engineering fields it is commonly used in industry to contact two fluid phases, or a fluid and solid phase. This process is used for catalytic reactions, combustion, gas absorption, gas-liquid adsorption, distillation, drying, separation [1], filter bed, waste water treatment and the flow of crude oil in petroleum reservoir [2].

Packed bed usually consists of a cylindrical column containing a large number of solid particles in contact with one another or as a network of tortuous passages formed by the spaces between the particles [3]. The packing material may be glass marbles, ceramics, plastics, pea gravel, or mixtures of materials [4]. It should have a large void volume to allow flow of fluid without excessive pressure drop and it should be chemically inert to fluids being processed [5].

In the packed column the fluid is distributed as uniformly as possible from the bottom, passes through the packed material and exits at the top of the column. There are inlet and outlet pores for the fluid and two pressure nodes above and below the packing that measure the pressure drop across the column [6]. Because of the nature of packed material, a packed column can operate using strongly corrosive fluids [7].

The surface area of the packing material increasing in efficiency of packed bed. Because increase in surface area provided by these particulate materials provides a bigger reaction area for the desired operation [1]

An understanding of pressure drop across a packed column as a function of flow rate is important when determining the energy requirements to pump a fluid for any given bed [8], in order to decrease costs of the system, but also to maintain the optimal operating conditions and to maximize the product [4].

The aim of this work is to:

I. Propose a semi-empirical equation of pressure drop over the packed bed as a function of different parameters; including fluid velocity, height of packing, pore diameter, tortuosity, bed porosity, and density and viscosity of fluid. This equation proposed for spherical particles for different size distribution of the particles (mono, binary, ternary, quaternary and quinary) and two systems of fluid, water and air flowing through a packed bed.

II. Propose an empirical equation for the tortuosity of packed bed as function of the bed porosity.

III. Propose an empirical equation for pore diameter of packed bed as a function of porosity and particle diameter.

Chapter Two

Literature Survey

2.1 Introductions

The study of fluid flow through the packed bed is an important issue. Chemical engineering operations commonly involve the use of packed and fluidized beds. These are devices in which a large surface area for contact between a liquid and a gas (absorption, distillation) or a solid and a gas or liquid (adsorption, catalysis) is obtained for achieving rapid mass and heat transfer [9].

For flow of one fluid, the fluid flow from the bottom, passes through the packed material and exits at the top of the column. With two fluids, liquid enters from the top of the column and flows downward, wetting the packing material. A gas enters at the bottom, and flows upward, contacting the liquid in a countercurrent fashion, initiating mass and energy transfer between the fluids [10]

The flow of fluid through porous medium composed either of irregular shaped material or of packing of regular geometrical form has attracted considerable attention from many investigators [11].

Blake-Kozeny in 1927 [12] derived an equation that correlated the pressure drop to low fluid flow rates and it is generally good for void fractions less than 0.5 and is valid only in the laminar region given by $(\rho u d_p/(1-\varepsilon)\,\mu < 10)$.

Burke-Plummer in 1928 [12] derived an expression for change in pressure at turbulent flow resulting from kinetic energy loss and it is only valid in the turbulent region given by $(\rho u d_p/(1-\varepsilon)\ \mu > 1000$.

Ergun in 1952 [13] studied the pressure drop for incompressible packed beds composed of uniform spherical particles. Despite this, the Ergun model has been used in situations where the particle shape was non-spherical and/or the particle size distribution was non-uniform. Ergun equation can provide the pressure drop along the length of the packed bed given a fluid velocity. Ergun found that the pressure drop over the packing length was dependent on the rate of fluid flow, the viscosity and density of the fluid, the closeness and orientation of the packing, and the size, shape and surface of the packing material. The Ergun equation was designed for fluid flow until the fluidization point.

Leva in 1959[12] predicted of pressure drop versus flow rate based on the study of single incompressible fluids through an incompressible bed of granular salts.

Harkonen in 1987 [13], Lindqvist in 1994, Lammi in 1996, Wang and Gullichsen in 1999 [10] and Lee and Bennington in 2004 [14] measured the average void fraction and flow resistance through packed columns. And found that the pressure drop of liquid through a packed bed depends on a number of factors, including the particle species and the type and size distribution of the particles.

Gibson and Ashby in 1988, Duplessis in 1994 and Richardson in 2000 [15] studies the influence of several structural parameters such as porosity, tortuosity, surface area and pore diameter, on predicting the pressure drop through porous medium

2.2 Packed Bed

The flow of fluid through bed composed of stationary granular particles is a frequent occurrence in the chemical industry and therefore expressions are needed to predict pressure drop across beds due to the resistance caused by the presence of the particles [16].

Packed systems in industry may be divided into the following classes:

1. Fixed beds
    a. Solid- gas system.
    b. Solid- liquid system.
2. Moving beds.
3. Solid- liquid- gas system.

Typical example of solid-gas fixed-bed systems are the catalytic reactors which were used in the Fischer-Tropsch synthesis, retorting of oil Shale, roasting of ores, combustion of coal and coke in fuel beds, and blast furnace operations.

The most important solid-liquid fixed-bed applications are water filtration, flow of oil through sand strata, coal washing, and leaching [17].

Moving beds are employed in the FCC (fluidized catalytic cracking) process and others.

The Solid-liquid-gas system comprises fractionating towers, absorbers, scrubbers, and many other kinds of chemical engineering equipment [17].

2.3 Flows in Porous Media

Flow in porous media has received much attention in recent years because of its important role in a large variety of engineering and technical applications, such as filtration units, packed beds, and certain types of chemical reactors.

Dullien 1992 [18] Porous materials are encountered everywhere in everyday life, in technology and in nature. A material can be defined as a porous medium if the material has the following properties.

1- The material must contain relatively small spaces, called pores or voids, imbedded in the solid. The pores usually contain some fluid, such as air, water, etc., or a mixture of different fluids [18].

2- The fluids should be able to penetrate through one face of the material and emerge on the other side [18].

Building materials, such as porcelain, or various plastics, thin-walled metal rings of steel or aluminum, glass, stone, bricks, concrete, soil, or other material are examples of porous materials. All properties of porous media are influenced by the pore structure. Pore structure parameters represent average behavior of a sample containing many pores and the some important pore structure parameters are the porosity, the tortuosity and the permeability. The porosity and the tortuosity are the characteristics of a porous medium; the permeability is the mass transfer property of the porous media [18].

Analysis of pore structure and pore radius distribution is necessary in order to construct on effective model for a porous medium [19].

The models of a porous medium consist of lattice networks of cylindrical channels with distribution of radii and also of randomly packed rotund particles examples spheres as described by **Iczkowski**, **Mason and Haynes** [20].

The significant properties of porous media are:

1. Porosity which is a measure of the pore space and hence of the fluid capacity of the medium [21].
2. Tortuosity which is a measure of fluid path through bed compared with actual depth of bed [22].
3. Permeability which is a measure of ease with which fluids may traverse the medium under the influence of a driving pressure [21].

2.3.1 Porosity of the Bed

The porosity (ε) is defined as the ratio of the void volume to the total volume of the bed (the volume fraction occupied by the fluid phase). Other names given the porosity include the void fraction, fractional voidage, or simply voidage [23], i.e.:

$$porosity\,(\varepsilon) = \frac{V_{void}}{V_{total}}$$

Depending on the type of the porous medium, the porosity may vary from near zero to almost unity**. Kaviany in 1995** suggested that the normal range of average void fraction was from 0.36 to 0.43 [24]. Measurement of porosity is made by using several techniques, such as imbibitions, mercury injection and gas injection methods to give effective porosity value [18].

The porosity is the most important property of a porous medium and it affects most of the physical properties of the medium. For a homogeneous porous medium, the porosity may be a constant. But in general, the porosity is space dependent. Each void in the porous medium is connected to more than one other pore (through pore or interconnected), connected only to one other pore (blind pore or dead end), or not connected to any other pore (closed pore or isolated) and fluid flows through the interconnected pores [18, 25]. Figure 2.1 shows the three possible kinds of pores.

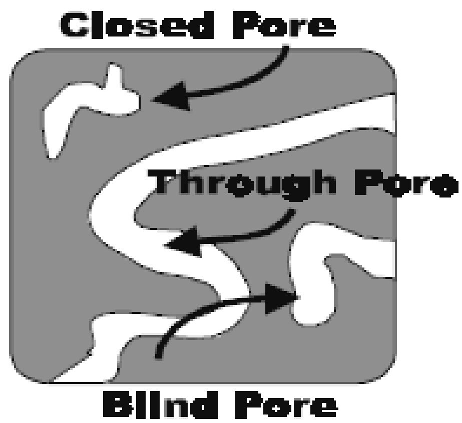

Figure 2.1 Three possible kinds of pores [25]

Many investigators described the fractional void volume of a bed of solid particles and found that the packing porosity depends upon the particle size and size distribution, the particle shape and surface roughness, the method of packing, and the size of the container relative to the particle diameter [5]; Such as **Fuller and Thompson in 1987** [26] studied the influence of distribution of the particle size upon the density of granular material. **Moallemi in 1989, Summers in 1994 and Ismail in 2000** studied the local voidage for the mixtures of sphere packing (mono, binary and ternary) and found that the local voidage variations in the axial, radial and angular directions [27].

The classification of pores according to size has been under discussion for many years, but in the past the terms micro pore {pore of internal width smaller than 2 mm}.Macro pore {pore of internal width greater than 50 mm}, has been applied in different ways by physical and chemists and some other scientists in an attempt to clarify this situation .Mesopores {pore of Internal width between 2mm and 50 mm} are especially important in the Context of adsorption [28].

2.3.2 Tortuosity Factor

Tortuosity is the ratio of the average pore length (L_e) to the porous medium thickness (l) [18].Figure 2.2 shows the effect of tortuosity in a porous medium between the average length and the bed thickness. It's classically defined as follows [29]:

$$\tau = \frac{L_e}{l} \qquad \dots (2.1)$$

where:

L_e= average length of porous medium.

l = the bed thickness.

The ratio L_e / l being the tortuosity factor and it is usually represented by τ as described by **Bear in 1972 and Dullien in 1979** [31].

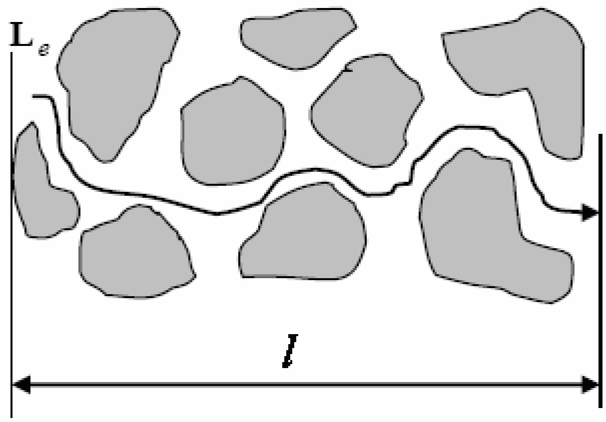

Figure 2.2 The effect of tortuosity in a porous medium showing the average length and the bed thickness [30]

Tortuosity is not a physical constant and depends first of all on other porous media characteristics, like porosity, pore diameter, channel shape, etc. In general, in granular packing or beds the value of tortuosity lies in the range 1.1-1.7 [31].

It is difficult to determine tortuosity experimentally and in general, tortuosity is calculated by using the porosity and the effective diffusion coefficient or from the Kozeny coefficient [18].

Several empirical correlations, which suggested a relationship between tortuosity and porosity, such as **Maxwell in 1873, Weissberg in 1963, Comiti and Renaud in 1989 and Boudreau in 1996** [31], are shown below:

$\tau = 1.5 - 0.5\varepsilon$ **(Maxwell, 1873)** … (2.2)

$\tau = 1 - 0.5\ln(\varepsilon)$ **(Weissberg, 1963)** … (2.3)

$\tau = 1 - 0.41\ln(\varepsilon)$ **(Comiti and Renaud, 1989)** … (2.4)

$$\tau = \sqrt{1 - \ln(\varepsilon^2)} \qquad \textbf{(Boudreau, 1996)} \qquad \ldots (2.5)$$

Archie in 1942 suggested most frequently relationship between tortuosity and porosity for a mixed bed of particles dependent on the methods applied for packing preparation [32], as:

$$\tau = \frac{1}{\varepsilon^n} \qquad \ldots (2.6)$$

Where n is a numerical value, and depend on the properties of the packing bed. The value of n lies in the range from 0.4 for loose packing to 0.5 for dense packing [32]. Equations all satisfy the condition $\tau = 1$ for $\varepsilon = 1$, and this consistent with the physical situation observed [31].

Also tortuosity can be calculated from the effective diffusion coefficient D_e, which characterizes mass transfer in porous media, it is written as [29]:

$$\tau = \frac{D_\circ}{D_e}\varepsilon \qquad \ldots (2.7)$$

The tortuosity may be expressed as a function of **kozeny's** coefficient K as [29]:

$$\tau = \sqrt{\frac{K}{K_c}} \qquad \ldots (2.8)$$

where:

K_c is the kozeny's constant.

K is the kozeny's coefficient.

Sen in 1981 and Yun in 2005 [31] showed that for an isotropic medium with spherical particles the tortuosity of porous and granular media decreases with increasing bed voidage.

2.3.3 Permeability of the Bed

The permeability, *k*, is the measure of the flow conductance of the porous medium and it is defined by the **Darcy's** law and can be written by using the porosity [18] as:

$$k = \left(\frac{\varepsilon}{\tau}\right)^2 \frac{\varepsilon\, d_{eff.}^2}{36(1-\varepsilon)^2 K_c} \qquad \dots (2.9)$$

The **kozeny's** constant, $K_{c,}$ is dependent of the porosity for packing [18]. Figure 2.3 shows the variation of **Kozeny's** constant with porosity for different shaped particles [16].

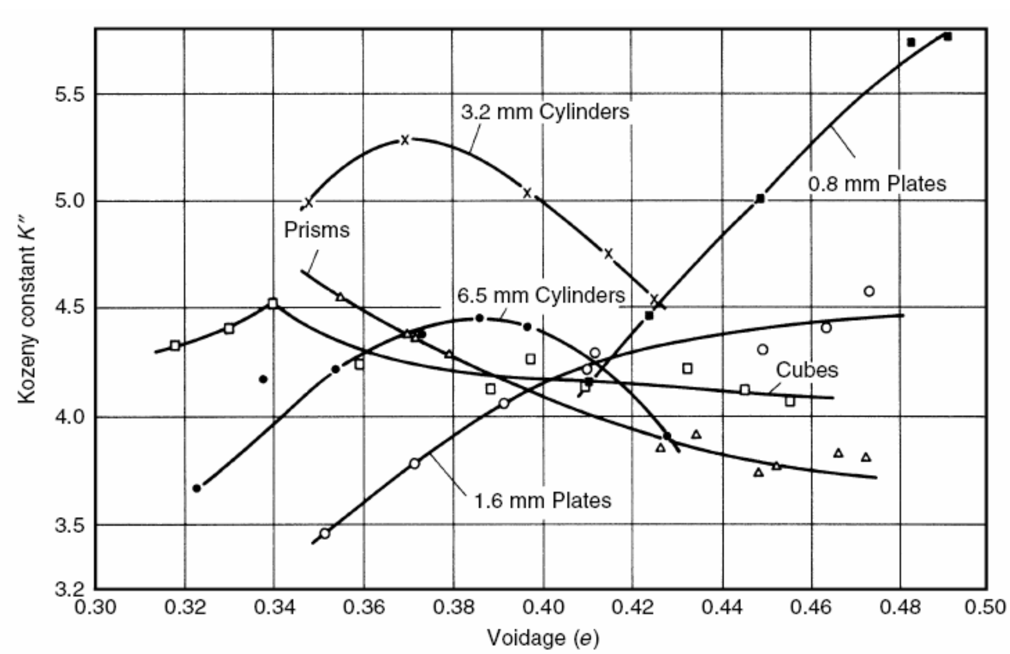

Figure 2.3 Variation of **Kozeny's** constant with porosity for different shaped particles [16].

The permeability is independent of the nature of the fluid but it depends on the geometry of the porous medium. The size of pore space and interconnectivity of the spaces help determine permeability [18]. Generally, the permeability of metal foam increases as the cell size increases for fixed porosity. **Paek, Kang, Kim and Hyun in 2000** [33] found that for different flow velocity, pressure drop were minimum at the same solid fraction (1-ε). This indicates that the pressure drop depends on the solid fraction.

2.4 Factors Affecting Pressure Drop through Packed Bed

The flow of single phase through a packed bed is extensively used for many chemical engineering applications, particularly for the design of fixed catalytic beds. Therefore expressions are needed to predict pressure drop across beds [34]. There are several factors affecting the pressure drop, some of them related to the physical properties of fluid such as viscosity and density, and others consists the rate of fluid flow, closeness and orientation of packing, size, shape and surface roughness of particles [4].

2.4.1 Rate of Fluid Flow

The flow rate of fluid is an important factor affecting the pressure drop through packed bed. When there is no flow through the packed bed, the net gravitational force acts downward. When fluid flows upwards, friction forces act upward and counter balance the net gravitational force [35]. For a high enough fluid velocity, the friction force is large enough to lift the particles. With increasing fluid velocity, the pressure drop through the bed rises until it reaches the weight of the packing per unit area. At this point, the bed starts to expand until it reaches the point of fluidization [7].

In a fixed bed, the particles are in direct contact with each other, supporting each other's weight. In an expanded bed, the particles are not in direct contact and are supported by the drag force of the fluid [36]. Figure 2.4 shows pressure drop verses velocity.

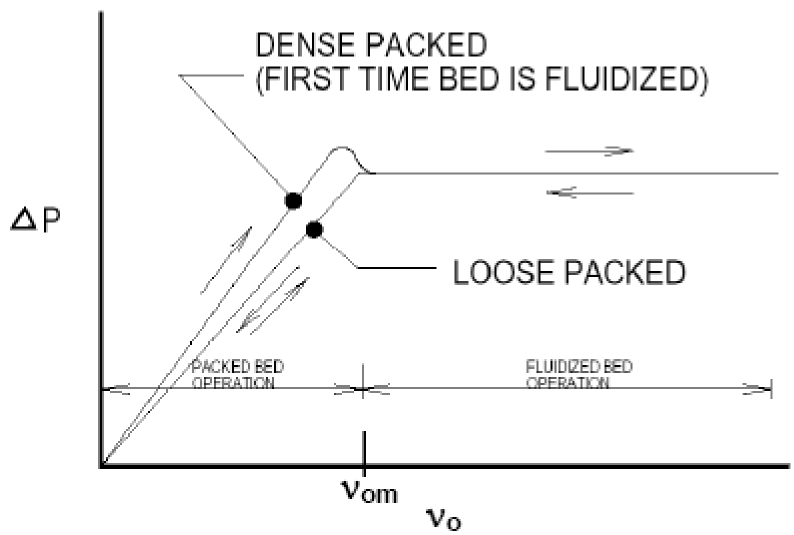

Figure 2.4 Pressure drop versus velocity. At ν_{om}, the bed fluidizes [36].

Different crystal structures (dense or loose packing) have different void fractions. The dense packing being when the arrangement of particles; face centered cubic and hexagonal close packed. The ideal void fraction for these forms of packing is 0.26 and 0.31 respectively [36]. Figure 2.5 and 2.6 shows arrangement of sphere particles in the bed.

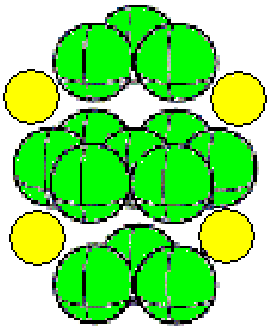

Figure 2.5 Hexagonal close packing [36]

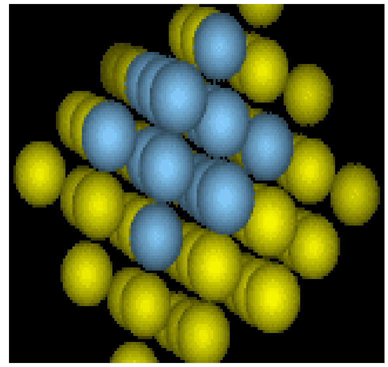

Figure 2.6 Face centered cubic [37]

Carman and Kozeny [4] found that for viscous flow, the change in pressure was proportional to $(1-\varepsilon)^2/\varepsilon^3$, and derived an expression for pressure under viscous flow as:

$$\frac{\Delta p}{L} = \frac{150\,(1-\varepsilon)^2\, u\, \mu}{\varepsilon^3\, \phi_s^2\, d_p^2} \qquad \ldots (2.10)$$

At the same time, **Burke and Plummer** [4] discovered that the change in pressure at turbulent flow, resulting from kinematic energy loss, is proportional to $(1-\varepsilon)/\varepsilon^3$, and derived an expression for change in pressure at turbulent flow as:

$$\frac{\Delta p}{L} = \frac{1.75(1-\varepsilon)\rho u^2}{\varepsilon^3 \phi_s d_p} \qquad \ldots (2.11)$$

2.4.2 Closeness and Orientation of Packing

It is apparent that the orientation of the particles composing the bed with respect to the direction of flow has a significant effect which depends on the shape and the arrangement of the particles. **Coulson and Gupta in 1938** reported that for the same porosity, the resistance to flow changes with the arrangement of the particles relative to each other [38].

Sullivan and Hertel in 1940 [38] suggested a factor for the orientation and the equation for flow through a packed bed including this factor is:

$$\lambda = (\sin^2 \alpha) \qquad \dots (2.12)$$

where:
λ is the orientation factor
α is the angle which the normal to the solid-liquid interface makes with the stream direction.

They presented data for three specific packing; sphere, cylinders perpendicular to the direction of flow and cylinders parallel to the direction of flow. In the case of the spheres, the orientation factor equals 2/3, for cylinders perpendicular to the direction of flow, the factor equals 1/2 and for cylinders parallel to the direction of flow the factor equals one [38].

2.4.3 The Size of the Particles

To define regular particles such as cubes, cylinders or spheres, the length, width, thickness or diameter are usually used. However it becomes difficult when the particles are irregular [38].

The particle size and vessel size are interrelated in their influence upon porosity. The presence of the container wall interrupts the pattern of particle-to-particle contacts and hence makes for a large fraction voids at the wall [5], Figure 2.7 shows the fluctuation of porosity in a bed of spheres and cylinders [39].

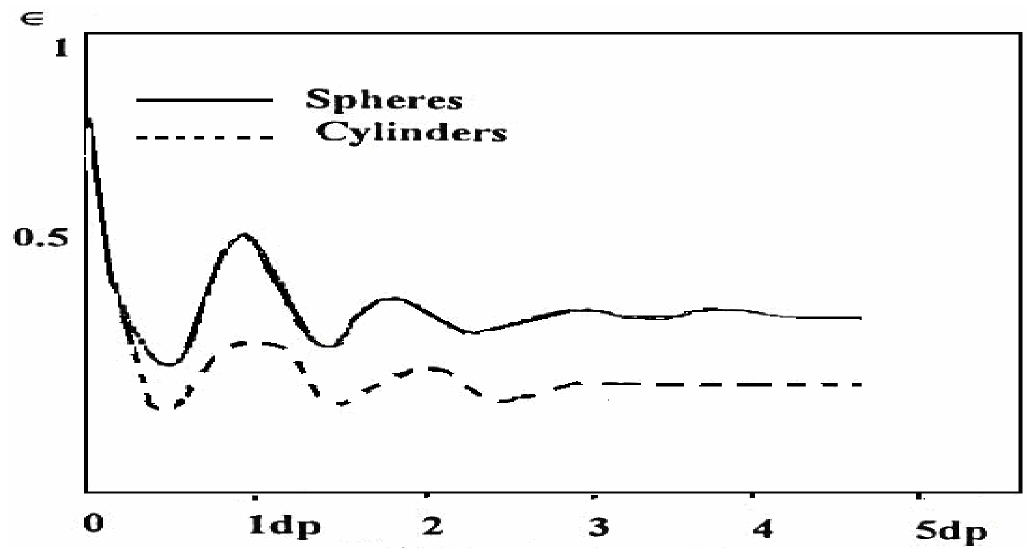

Figure 2.7 The Fluctuation of porosity in a bed of spheres and cylinders [39]

To decrease wall effects, the particle diameter should be small in comparison with the column diameter in which the packing is contained [40]. **Furnas in 1931** [41] studied the wall effect and found that when the ratio of the diameter of the container (D_b) to that of the particle (d_p) is greater than 10:1, the wall effect can be neglected [39].

Carman in 1937 [46] and **Coulson** in 1949 [49] made no correction for the change in porosity near the wall. They used the mean porosity and for low rates added half the area of the walls to the surface area of the particles. A wall affects correction factor *fw*, for the velocity though the packed bed has been determined experimentally by **Coulson** [16] as:

$$f_w = \left(1 + \frac{1}{2}\frac{S_c}{S}\right)^2 \qquad \text{… (2.13)}$$

where:

S_c= is the surface area of the container per unit volume of bed.

S= is the specific surface area of the particles.

2.4.4 The Shape of the Particles.

Particle shape is a more important variable in porosity determination than is surface roughness, though both of the variables act in the same way. The lower the particle sphericity the more open is the bed. Particles settle cross each other and packed with pointed ends against each other, preventing a close packing. Figure 2.8 shows the sphericity as a function of porosity for random-packed beds of uniformly sized particles [5].

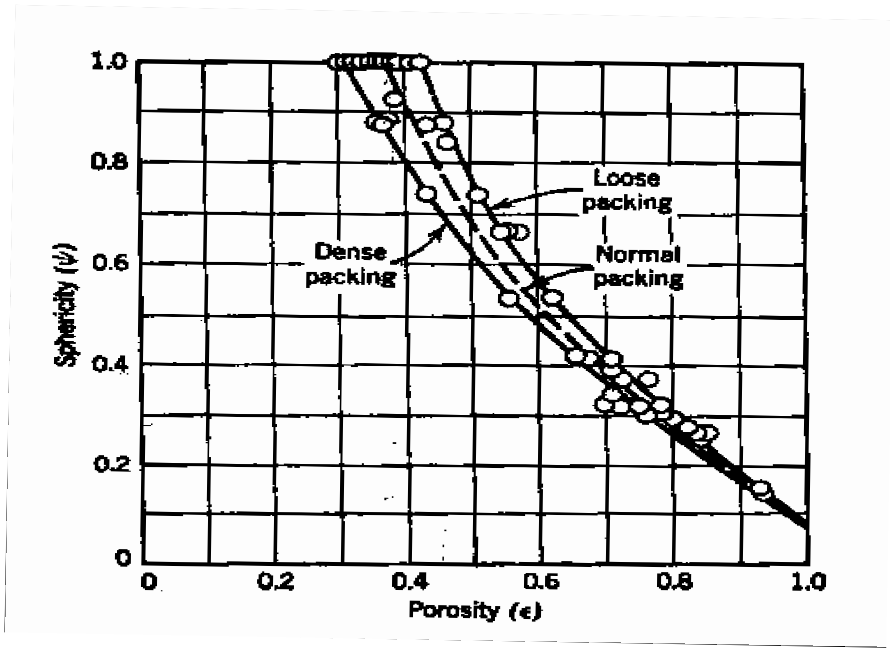

Figure 2.8 Sphericity as a function of porosity for random-packed beds of uniformly sized particles [5].

Particle shape is the ratio of the particle property to the property a sphere having a diameter equal to the measured particle dimension. The most commonly used of the shape factors is the sphericity ($\O_s$). Sphericity is defined as the ratio of the surface area of this sphere to the surface area of particle [13], as shown below:

$$\phi_S = \frac{S_{sphere}}{S_{particle}} = \frac{\pi d_p^2}{S_{particle}} \qquad \dots (2.14)$$

where:

d_p is the diameter of a sphere of the same volume as the particle

Therefore the value of sphericity for spheres particle equal one and for other shapes is less than one [38].Table 2.1 shows the shape factor of different shape materials [43].

Table 2.1 Sphericity of different shape materials [43]

Material	Sphericity	Material	Sphericity
Sphere, cubes, short cylinders ($L=d_p$) Rashing rings ($L=d_p$)	1.0	Ottawa sand	0.95
		Rounded sand	0.83
		Coal dust	0.73
Rashing rings $L=d_{po}$*, $d_{pi} = 0.5d_{po}$ * $L=d_{po}$, $d_{pi}= 0.75d_{po}$	0.58 0.33	Flint sand	0.65
		Crushed glass	0.65
Berl saddles	0.3	Mica flakes	0.28
* d_{po}, d_{pi} is the outside and inside diameter of rashing rings.			

2.4.5 The surface roughness of the particles

Surface roughness has two major effects; it increases the resistance of the bed to fluid flow and increases the porosity of the bed [38].

The effects of surface roughness upon the pressure drop have indicated a marked effect in the turbulence range. Roughness dose not affect friction in the laminar region [8].

Roughness has an effect on the effective path and the kinematic velocity of the fluid; both terms of friction factor are subject to change. The effective path is the actual path that the fluid travels [8].

For slow flow rates, the effective path for rough and smooth particles should be approximating the same because channeling effect does not differ with the degree of roughness. For high flow rates, the rough surface has more friction therefore there is a substantial difference in the pressure drops [8].

2.5 Specific Surface Area

The general surface of a bed of particles can often be characterized by the specific area of the bed (S_B) and the fractional voidage of the bed (ε). S_B is the surface area presented to the fluid per unit volume of bed when the particles are packed in bed. Its units are $(\text{length})^{-1}$.

S is the specific surface area of the particles and is the surface area of a particle divided by its volume. Its units are again $(\text{length})^{-1}$. Therefore for spherical particle [16, 44]:

$$S = \frac{Surface\,area}{volume} = \frac{\pi d_p^2}{\pi\left(d_p^3/6\right)} = \frac{6}{d_p} \qquad \text{… (2.15)}$$

It can be seen that S and S_B are not equal due to the voidage occurring when the particles are packed in to a bed. If contact points occur between particles so that only a very small fraction of surface area is lost by overlapping, then [16, 44]:

$$S_B = S\,(1-\varepsilon) \qquad \text{… (2.16)}$$

For a given shape of particle S increases as the particle size is reduced [45].

When mixtures of sizes are studied the value of S for sphere of mixed sizes is given by [46]:

$$S = 6(1-\varepsilon)\sum_{i=1}^{n}\frac{x_i}{d_{pi}} \quad \dots (2.17)$$

where:

x_i is the fractional weight of spherical particle.

d_p is the diameter of spherical particle.

For beds consisting of a mixture of different particle diameters, the effective particle diameter (dp_{eff}) can be used instead of d_p as [13]:

$$dp_{eff} = \frac{1}{\sum_{i=1}^{n}\frac{x_i}{d_{pi}}} \quad \dots (2.18)$$

2.6 Prediction of Voidage Distribution

The characteristics of the flow through packed bed are important in filter design and understanding of the relationship between void fraction and the flow distribution is essential. Figure 2.9 shows a typical radial voidage distribution (in a bed of 98 mm in diameter packed with 4 mm spherical beads). It has been shown that, for flow through a fixed bed of uniform particles that there is a maximum velocity approximately one particle diameter from the outer wall of the bed, which decrease sharply toward the wall and more gradually away from it. The fraction of the bed influenced by this velocity profile depends on the ratio of particle size to bed diameter [47].

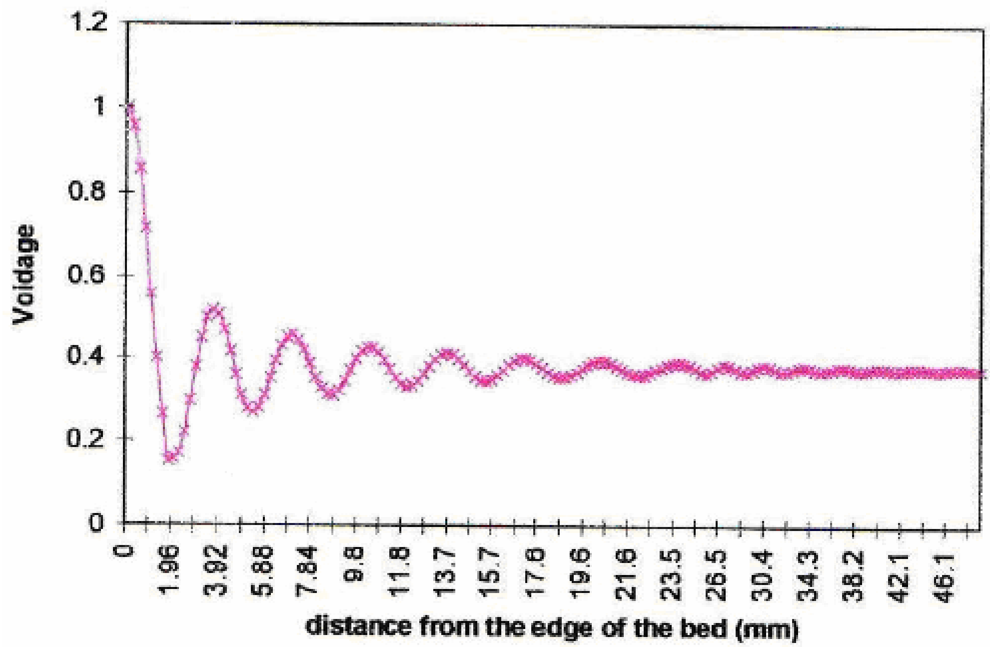

Figure 2.9 Typical radial voidage distributions [47]

The voidage is found to be a minimum about half a particle diameter from the wall of the bed and then follow a dumped oscillatory function until it reaches a constant value about 5 particle diameters from the wall, where the packing is random [47].

Large randomly packed beds of uniform spheres tend to pack with an average void fraction of 39% since locally; the voidage varies from point to point. Near the wall of the containing vessel the void fraction will be larger than near the center of the bed. Immediately adjacent to the wall the void fraction approach unity and in the center of the bed a minimum voidage observed [48].

2.7 The Relation between depth of bed and pressure drop

From the readings of the manometers, Colson found that the differences in pressure over varying depths of the packing were obtained directly. Some results for beds of spherical particles (d_p= 5/32″ and 5/16″), plates (1/16″) and cylinders (1/8″) are shown graphically in Figure 2.10. The experimental points are seen to lie on straight lines indicating a linear relation between pressure drop (Δp) and depth of bed (L) [49].

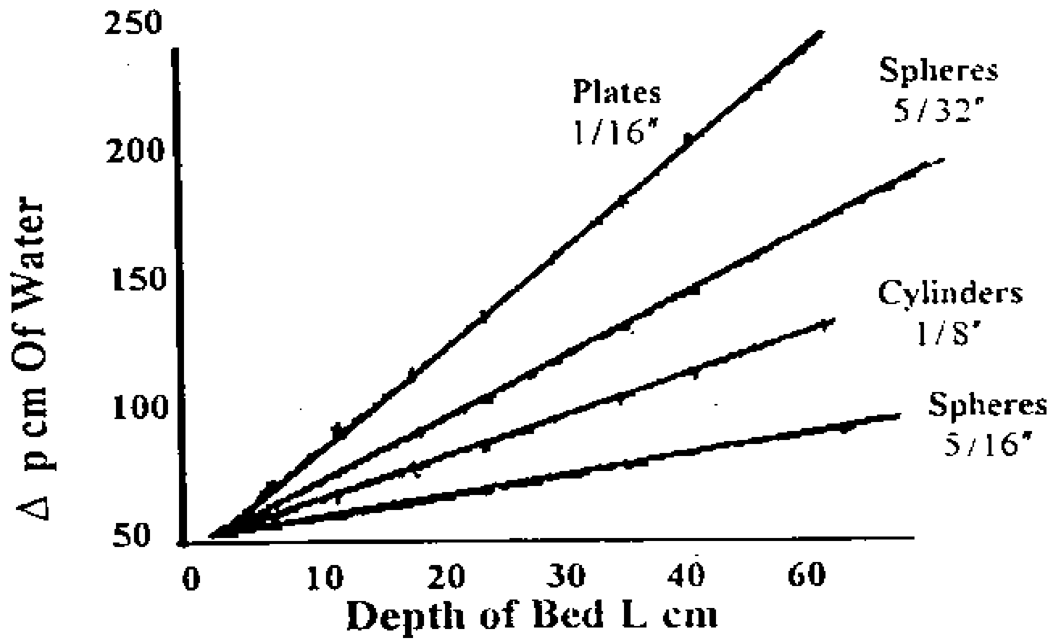

Figure 2.10 Relation between depth of bed and pressure drop [49]

Chapter Three

Theoretical Aspects

3.1 Introduction

This chapter deals with proposed semi-empirical equations for modeling fluid flow through packed bed. The most important parameter in the equations is the pressure drop. The semi-empirical equations proposed can be divided into several types according to the packing system, mono size packing system, binary size packing system, ternary size packing system, quaternary size packing system and quinary size packing system.

A semi-empirical equation was proposed for each type of packing referred to a singular equation. An equation that can be used for all types of packing systems is called general equation. The shape of the singular and general equations are similar, the difference between them is in the constants used in the terms of these equations.

The second factor affecting the fluid flow through packed bed is the pore diameter. The pore diameter is included in the pressure drop equation. An empirical formula was proposed to evaluate the pore diameter for each type of packing using experimental data.

The third factor affecting on the pressure drop of fluid flow through packed bed is the tortuosity factor. The simplest expression was proposed to evaluate the tortuosity for each type of packing using experimental data.

3.2 Fluid Flow through Randomly Packed Columns

Many attempts have been made to obtain general expressions for pressure drop and mean velocity for flow through packing in terms of voidage and specific surface, as these quantities are often known or can be measured. Alternatively, measurement of the pressure drop, velocity, and voidage provide a convenient way of measuring the surface area of some particulate materials [16].

Considering a horizontal pipe of radius R and length L with an annulus element of thickness Δr as shown in figure 2.3 below [50]:

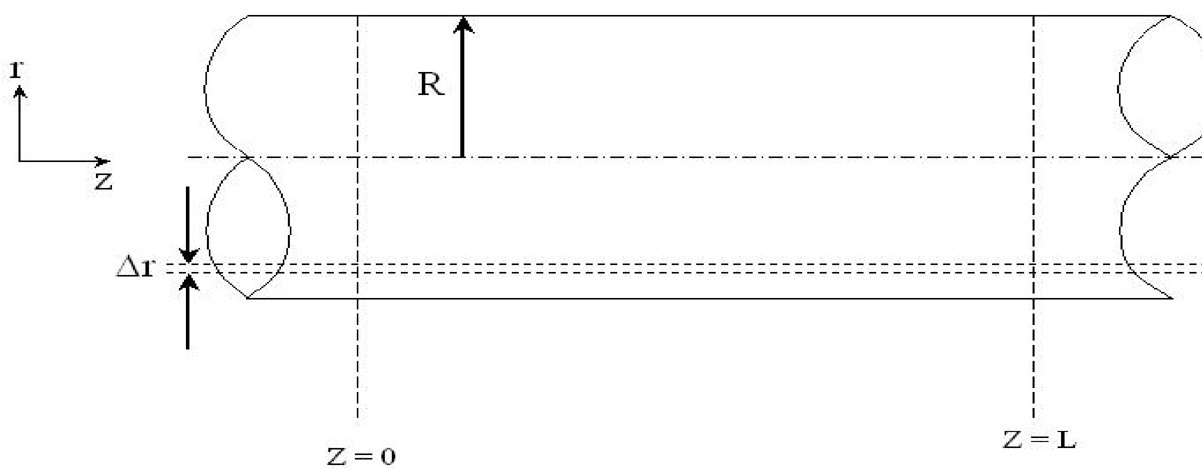

Figure3.1 Schematic diagrams for a pipe

The momentum balance on the increment Δr is as follows [50]:

Rate of momentum in- Rate of momentum in+ Sum of forces acting on system= accumulation … (3.1)

1. Rate of momentum in across cylindrical surface = $(2\pi\ r\ L)\ \tau_{rz}|_r$
2. Rate of momentum out across cylindrical surface = $(2\pi\ r\ L)\ \tau_{rz}|_{r+\Delta r}$
3. Rate of momentum in across annular surface at z = 0 is

   $(2\pi\ r\ \Delta r\ u_z)(\rho\ u_z)|_{z=0}$

4. Rate of momentum out across annular surface at z = L is

   $(2\pi\ r\ \Delta r\ u_z)(\rho\ u_z)|_{z=L}$

5. Pressure force acting on system = $(2\pi\ r\ \Delta r)(P_0 - P_L)$

   Where P_0 and P_L is the fluid pressure at z=0 and at z=L, respectively.

For horizontal pipe the gravitational force is neglected.

Substitution of the above five terms into the general momentum balance equation (3.1) the resulting equation will be as follows [50]:

$$(2\pi\, rL)\left(\tau_{rz}\big|_r - \tau_{rz}\big|_{r+\Delta r}\right) + (2\pi\, r\Delta r u_z)\,(\rho u_z)\big|_{z=0} - (2\pi\, r\Delta r u_z)\,(\rho u_z)\big|_{z=L} + (2\pi\, r\Delta r)\,(P_0 - P_L) = 0 \quad \dots (3.2)$$

Since the velocity is constant along the z – axis, the net of the momentum across the annulus is zero arranging and dividing equation (3.2) by (2π L Δr) gives the following [51]:

$$\frac{d}{dr}(r\tau_{rz}) = \frac{(P_0 - P_L)}{L} r \quad \dots (3.3)$$

Integrating equation (3.3) as follows:

$$\tau_{rz} = \frac{(P_0 - P_L)}{2L} r + \frac{C_1}{r} \quad \dots (3.4)$$

Using the boundary condition at r = 0, τ_{rz} = 0 which leads to make the shear stress to reach infinity therefore C_1 must be zero

$$\tau_{rz} = \frac{(P_0 - P_L)}{2L} r \quad \dots (3.5)$$

The shear stress is defined as follows:

$$\tau_{rz} = -\mu \frac{du_z}{dr} \quad \dots (3.6)$$

Substituting equation (3.5) into (3.6) and rearranging the equation as follows:

$$\frac{du_z}{dr} = \frac{(P_0 - P_L)}{2\mu L} r \qquad \ldots (3.7)$$

Integrating equation (3.7)

$$u_z = \frac{(P_0 - P_L)}{4\mu L} r^2 + C_2 \qquad \ldots (3.8)$$

By using the boundary condition at the center of the pipe at velocity zero (u_z=0) and the radius at (r=R), then equation (3.8) will be as follows:

$$C_2 = -\frac{(P_0 - P_L)}{4\mu L} R^2 \qquad \ldots (3.9)$$

Substituting equation (3.9) into (3.8) and rearranging as follows:

$$u_z = \frac{(P_0 - P_L)}{4\mu L} R^2 \left(1 - \left(\frac{r}{R}\right)^2\right) \qquad \ldots (3.10)$$

Equation (3.10) is the velocity distribution inside pipe as a function of the radius of pipe.

The average velocity is obtained by the following expression [50] as follows:

$$u = \frac{\int_0^{2\pi}\int_0^R r\, u_z\, dr\, d\theta}{\int_0^{2\pi}\int_0^R r\, dr\, d\theta} \qquad \ldots (3.11)$$

Substitution of equation (3.10) into (3.11) and integrating gives

$$u = \frac{\Delta P d_t^2}{32 \mu L} \qquad \ldots (3.12)$$

Rearranging equation (3.12) gives

$$\frac{\Delta P}{L} = \frac{32 \mu u}{d_t^2} \qquad \ldots (3.13)$$

Where equation (3.13) is the **Hagen-Poiseuille** equation, which is a physical law concerning the voluminal laminar stationary flow of Newtonian fluid through a cylindrical tube with constant circular cross-section [52].

Considering a unit volume packed bed, the volumes occupied by the voids and the solid particles are ε and (1-ε) respectively, where **ε** is the void fraction or porosity of the bed. Let S is the surface area per unit volume of the solid material in the bed. Thus the total surface area (S_B) in a packed bed of unit volume is (1 - ε) S [16].

An equivalent pore diameter d_{pore} for flow through the bed can be defined as four times the cross-sectional flow area divided by the appropriate flow perimeter. For random packing, this is equal to four times the volume occupied by the fluid divided by the surface area of particles in contact with the fluid. Thus, the equivalent pore diameter is [16]:

$$d_{pore} = \frac{4\varepsilon}{(1-\varepsilon)S} \qquad \dots (3.14)$$

If the free space in the bed is assumed to consist of a series of tortuous channels, then equation (3.12) for flow through a bed may be rewritten by the substitution of the equivalent diameter [16]:

$$u_1 = \frac{\Delta P}{32\,\mu\,L}\left(\frac{16\,\varepsilon^2}{(1-\varepsilon)^2 S^2}\right) \qquad \dots (3.15)$$

The average velocity through the pore channels (u_1) is defined as the superficial velocity (u) divided by the porosity of the bed [22].

$$u_1 = \frac{u}{\varepsilon} \qquad \dots (3.16)$$

Substituting equation (3.16) in equation (3.15), therefore equation (3.15) will be as follows:

$$u = \frac{\Delta P}{2\,\mu\,L}\left(\frac{\varepsilon^3}{(1-\varepsilon)^2 S^2}\right) \qquad \ldots (3.17)$$

Replacing equation (3.17) by the following equations:

$$u = \frac{\Delta P}{K_c\,\mu\,L}\left(\frac{\varepsilon^3}{(1-\varepsilon)^2 S^2}\right) \qquad \ldots (3.18)$$

$$\frac{\Delta P}{L} = K_c \frac{(1-\varepsilon)^2 S^2 \mu u}{\varepsilon^3} \qquad \ldots (3.19)$$

$$\frac{\Delta P}{L} = K_c \frac{(1-\varepsilon)^2 \mu u}{\varepsilon^3} \frac{36}{d_p^2} \qquad \ldots (3.20)$$

Where K_c is **Kozeny's** constants and given in figure (2.3) [16].

Replacing equation (3.20) by the following equation:

$$\frac{\Delta p}{L} = K_1 \frac{(1-\varepsilon)^2 \mu u}{\varepsilon^3 d_p^2} \qquad \ldots (3.21)$$

where K_1 is a dimensionless empirical constant [6]. Equation (3.21) is known as the **Carmen-Kozeny** equation and has been successfully used to calculate pressure drop for laminar flow through packed bed. **Carman** [46] applied this equation to experimental results on flow through packed beds and found that K_1=180.

At high Reynolds number the kinetic-energy losses become significant, which was found by modifying the kinetic-energy term [5].

$$\frac{\Delta P}{\rho} = \frac{u_1^2}{2} \qquad \ldots (3.22)$$

If the energy loss is to occur repeatedly in a unit channel length, then equation (3.22) will be as follows:

$$\frac{\Delta P}{L} = \frac{n}{2}\rho u_1^2 \qquad \text{… (3.23)}$$

Where n = the number of repetitive kinetic-energy losses in a unit length

ΔP = pressure drop due to kinetic-energy losses

In the channels under consideration, the expansions in channel width probably occur at distance roughly equivalent to the channel diameter. This follows because the particulate bed has been replaced by a model consisting of many parallel, circular ducts. The diameter of these ducts will be proportional to the particle diameter, and the fact of one expansion occurring for each particle is thus approximated. Using n proportional to $1/d_t$ gives [5]:

$$\frac{\Delta P}{L} = K\frac{\rho u_1^2}{d_t} \qquad \text{… (3.24)}$$

Equation (3.24) must be converted in term of d_p and u by substituting the equation of specific surface area ($S=6/d_p$) and the intestinal velocity from equation (3.16), therefore equation (3.24) will be as follows [5]:

$$\frac{\Delta P}{L} = K\frac{\frac{\rho u^2}{\varepsilon^2}}{\frac{4\varepsilon d_p}{6(1-\varepsilon)}} = K_2\frac{\rho u^2}{d_p}\frac{(1-\varepsilon)}{\varepsilon^3} \qquad \text{… (3.25)}$$

where K_2 is a dimensionless empirical constant that through many experiments was determined to equal to 1.75 [6].This equation was first derived by **Burke and Plummer** [53] to express the pressure drop of turbulent flow through packed bed.

Equation (3.21) for the pressure drop caused by form drag and equation (3.25) for the pressure drop caused by kinetic-energy losses may be added to obtain the total pressure drop resulting from flow through the bed [5].

$$\frac{\Delta P}{L} = K_1 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu u}{d_p^2} + K_2 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho u^2}{d_p} \qquad \ldots (3.26)$$

Where u is the fluid velocity, ΔP is the pressure drop, L is the length of the bed and K_1 and K_2 are factors which depend on both fluid and porous medium properties.

The expression for $K_1$and K_2 has been studied by many investigators, the most widely used expression is that given by **Ergun in 1952** [54] as shown below:

$$\frac{\Delta P}{L} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu u}{d_p^2} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho u^2}{d_p} \qquad \ldots (3.27)$$

Ergun equation has been modified by **Duplessis in 1994** by using the pore diameter instead of particle diameter and using the tortuosity factor, then equation (3.27) will be as follows [15]:

$$\frac{\Delta P}{L} = 36 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 2.05 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \ldots (3.28)$$

Where τ is the medium tortuosity and d_{pore} is the equivalent pore diameter.

A semi- empirical equation has been based on the same formula of **Duplessis** equation for the different types of packing systems by using experiments data from literature for water and air flow through packed bed.

The semi-empirical equation is comparable with **Duplessis** equation, and can be written as follows:

$$\frac{\Delta P}{L} = b_1 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + b_2 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \ldots (3.29)$$

where b_1 and b_2 are constants which can be evaluated from experimental data taken from literature by statistical fitting. The above equation can be used for each type of packing system by using experimental data used statistical fitting to find the constants of equation.

3.2 The Tortuosity Factor

The most important parameter needed to represent in this work is the tortuosity of sphere packing which is included in equation (3.29) as one of the main parameters.

The empirical equation of tortuosity can be calculated using the following equation [31]:

$$\tau = \sqrt{\left(1 - \ln \varepsilon^2\right)} \qquad \ldots (3.30)$$

Equation (3.30) is one of the expressions that show the dependence of tortuosity in porosity of packed bed.

Equation (3.30) can be proposed to add more accurate empirical formula on the tortuosity in equation (3.29). The new form of the tortuosity which can be written in the following expression:

$$\tau = \sqrt{b_1 - b_2 \ln(\varepsilon)^2} \qquad \ldots \qquad (3.31)$$

The constants of equation (3.31) can be evaluated from experimental data taken from literatures [3, 4, 7, 8, 56, 57, 58, 59, 60, 61, and 62] by using statistical fitting.

Chapter Four

Results and discussion

4.1 The Semi-Empirical Equations Constants

Equation (3.29) in chapter three has been fitted using experimental data obtained from literatures [3, 4, 7, 8, 56, 57, 58, 59, 60, 61, and 62] as shown in table 4.1, in order to calculate the different constant in it. This has been done for water and air flow through packed bed of different types and sizes of packing. The resulted constants are presented in tables 4.2 and 4.3.

Table 4.1 Experimental data for different types and sizes of packing

Types of packing materials	system	Particle diameter (cm)	Bed diameter (cm)	Height of packing (cm)
Glasses, pea gravel, black marbles [3]	Water	0.25,0.635, 1.095, 1.27	8.89, 15.24	38.1, 48.26, 45.72
Pea gravel, glass marbles [4]	Water	1.27	8.89, 15.24	46.99
Plastic marbles, pea gravel [7]	Water	0.655, 1.27	8.89, 15.24	40.64
Acrylic balls [8]	Water	0.635	8.826	28.25
Glasses [56]	Water, air	0.42, 0.51, 0.6, 0.79, 0.99	7.64	15.15
Glasses [57]	Water, air	0.42, 0.51, 0.61, 0.79, 1.01	7.62	20
Glasses [58]	Water, air	0.24, 0.42, 0.82, 0.61, 1.03	7.64	15.15, 20
Black marbles [59]	Water	1.9	14.616	61.6, 67.3
Acrylic balls [60]	Water	0.655, 1.27	8	49.53
Acrylic balls [61]	Water	0.636, 1.27	8	48.26, 50.8
Acrylic balls [62]	Water	0.653, 1.27	8	50.8

Table 4.2 Constants of equation 3.29 for air flow through packed bed

System type	B1	B2
Mono sphere	48.49076	2.220827
Binary	36.06787	2.020557
Ternary	49.91447	1.852967
Quaternary	8.77095	2.657649
Quinary	0.06354	2.555163
Generalized for multi sized	-5.47872	2.726732

Table 4.3 Constants of equation 3.29 for water flow through packed bed

System type	B1	B2
Mono sphere	54.38942	1.588934
Binary	148.8836	0.834330
Ternary	65.24735	1.880320
Quaternary	7.089078	2.604021
Quinary	108.3983	2.118838
Generalized for multi sized	39.33526	2.248336

The tortuosity used in equation 3.29 is taken from formula 3.31 after fitting for water and air through packed bed. The resulting constants are written in table 4.3.

Table 4.4 Tortuosity constants for air and water flow in packed bed

System type	B1	B2
Mono sphere	1.693034	0.173894
Binary	1.666232	0.151646
Ternary	0.737275	0.708423
Quaternary	0.484121	0.869721
Quinary	4.348614	-1.04003
Generalized for multi sized	1.821618	0.126665

4.2 Calculation of Pore Diameter

The effective pore diameter for granular packing can be determined by using the following equation [55]:

$$d_{pore} = \frac{2}{3} d_p \frac{\varepsilon}{(1-\varepsilon)} \qquad \ldots (4.1)$$

Equation (4.1) can be modified to add more accurate formula of equation (3.29). The particle diameter and porosity of the bed were used to write a new form of pore diameter.

The calculation of pore diameter was based on analysis of experimental data taken from literatures [3, 4, 7, 8, 56, 57, 58, 59, 60, 61, and 62]. Table 4.4 shows the diameter of pore for different sizes of packing systems.

Table 4.5 Diameter of pore for different sizes of packing systems

Air Flow through Packed Bed			
A. Mono Size			
Run	d_{pore} (m)	d_p	ε
1.	**0.00478**	**0.01**	**0.4181**
2.	**0.00367**	**0.0079**	**0.4088**
3.	**0.00272**	**0.0061**	**0.4005**
4.	**0.00485**	**0.0101**	**0.4186**
5.	**0.00363**	**0.0079**	**0.4082**
6.	**0.00271**	**0.0061**	**0.3998**
B. Binary Size			
7.	**0.00407**	**0.0089**	**0.4079**
8.	**0.00332**	**0.0075**	**0.3986**
9.	**0.00227**	**0.0055**	**0.3817**
C. Ternary Size			
10.	**0.00293**	**0.0071**	**0.3822**
11.	**0.00259**	**0.0065**	**0.3727**
12.	**0.00253**	**0.0065**	**0.3696**
D. Quaternary Size			
13.	**0.00217**	**0.0055**	**0.371**

E. Quinary size			
14.	**0.0024**	**0.0061**	**0.3695**
15.	**0.0014**	**0.0048**	**0.2977**
Water Flow through Packed Bed			
A. Mono sphere			
16.	**0.00564**	**0.0127**	**0.4**
17.	**0.00576**	**0.0127**	**0.4048**
18.	**0.00573**	**0.0127**	**0.4037**
19.	**0.0079**	**0.01778**	**0.4**
B. Binary Size			
20.	**0.00393**	**0.01016**	**0.367**
21.	**0.00274**	**0.00726**	**0.3612**
C. Ternary Size			
22.	**0.00356**	**0.00765**	**0.4111**
23.	**0.00319**	**0.0071**	**0.4023**
D. Ternary Size			
24.	**0.00356**	**0.00765**	**0.4111**
25.	**0.00319**	**0.0071**	**0.4023**
E. Quaternary Size			
26.	**0.00216**	**0.0055**	**0.3711**
F. Quinary Size			
27.	**0.00231**	**0.0061**	**0.3623**

The following steps can be considered to calculate pore diameter in packed bed:

1. Dependence on particle diameter:

It is noticed that pore diameter increases with increasing in particle diameter at constant porosity so that pore diameter was proportional with particle diameter as shown in runs (16 and 19) i.e.:

$$d_{pore} \alpha \; d_p \qquad \text{… (4.2)}$$

The resulting equation will be as follows:

$$d_{pore} = k_p d_p \qquad \text{… (4.3)}$$

2. Dependence on porosity of packed bed:

It is noticed that the pore diameter decreases with decreasing in porosity at constant particle diameter so that pore diameter was directly proportional with porosity as shown in runs (2 and 5), (9 and 13), (11 and 12), (14 and 27), and (17 and 18) i.e.:

$$d_{pore} \alpha \ \varepsilon \qquad \text{… (4.4)}$$

The resulting equation will be as follows:

$$d_{pore} = k_{\varepsilon} \varepsilon \qquad \text{… (4.5)}$$

Combining equations (4.5) and (4.3) gives:

$$d_{pore} = k_p k_{\varepsilon} d_p \varepsilon \qquad \text{… (4.6)}$$

Replacing equation (4.6) by the following equation:

$$d_{pore} = b_1 \ d_p^{b2} \ \varepsilon^{b_3} \qquad \text{… (4.7)}$$

where b_1, b_2 and b_3 are constants and can be evaluated by using statistical fitting. The Pore diameter used in equation 3.29 is taken from formula (4.7) after fitting for water and air through packed bed. The resulting constants are written in table 4.5.

Table 4.5 Pore diameter formula constants for air and water flow through packed bed

System type	b_1	b_2	b_3
Mono sphere	1.947981	0.999378	1.614425
Binary	1.955938	0.999649	1.619048
Ternary	2.003437	1.000455	1.640400
Quaternary	1.497049	1.198670	0.307693
Quinary	1.325409	1.487273	0.943397
Generalized for multi sized	1.961229	1.000699	1.616058

4.3 The Effect of Different Parameters on the pressure drop on the proposed General Equation

This section shows the effect of different parameter on pressure drop using equation 3.29 after the substitution of the constants for the general equation multi-sized particles systems. The systems include all different types of spherical particles packing sizes namely mono, binary, ternary, quaternary and quinary.

The important parameters affecting the pressure drop in the equation was found to be porosity, pore diameter, tortuosity, and bed length.

The fluid physical properties used in all fluid flow equations were taken from experiments held at temperature of (32°C) for air flow and (25°C) for water flow through packed bed [43]. The physical properties (density and viscosity) are shown in table 4.6.

Table 4.6 Physical Properties of Fluids [43].

Type of fluid	Density (kg/m^3)	Viscosity (kg/m.s)
Water	995.647	$0.8*10^{-3}$
Air	1.1582	$1.88*10^{-5}$

4.3.1 Effect of Pore Diameter on Pressure Drop:

Figure 4.1 and 4.2 shows the variation of velocity on calculated pressure drop values for water and air flow through packed bed respectively. It is noticed that an increase in the pore diameter causes decrease in the pressure drop; this is due to the fact that when the pore diameter increases the resistance of fluid flow decreases which lead to decrease in pressure drop. For example for air flow through packed bed at velocity 0.28 m/s when the pore diameter is 0.001 m the pressure drop is 117.5028 Pa, while for the same velocity with pore diameter of 0.0018 m the pressure drop is 56.9185 Pa.

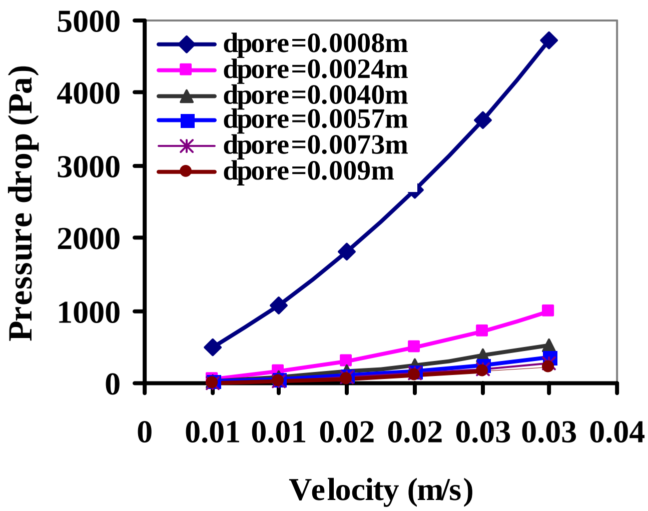

Figure 4.1 Pressure drop vs. velocity at different pore diameters of particles for tortuosity of 1.43, porosity 0.3 and bed length 0.2 m.

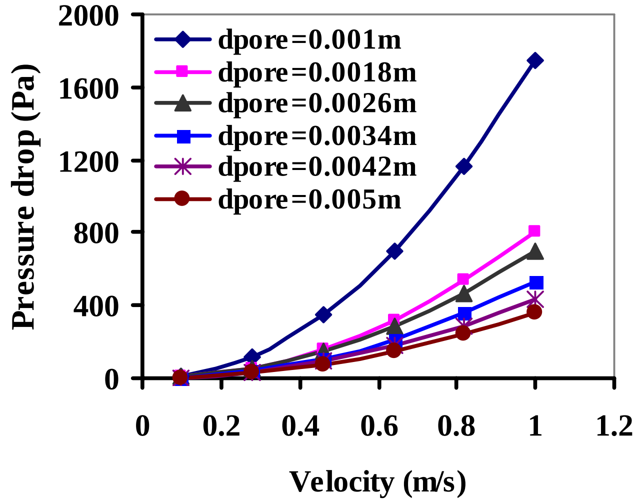

Figure 4.2 Pressure drop vs. velocity at different pore diameters of particles for tortuosity of 1.4, porosity 0.3 and bed length 0.15 m.

4.3.2 Effect of Porosity on Pressure Drop:

Figure 4.3 and 4.4 shows the variation of velocity on calculated pressure drop values for water and air flow through packed bed respectively. It is noticed that when the porosity increases the pressure drop decreases, where the void fraction between particles become larger this leads to less resistance to fluid flow through the bed [63]. For example for water flow through packed bed at velocity 0.01 m/s when the porosity is 0.3 the pressure drop is 1084.439 Pa, while for the same velocity with porosity of 0.32 the pressure drop is 953.1203 Pa.

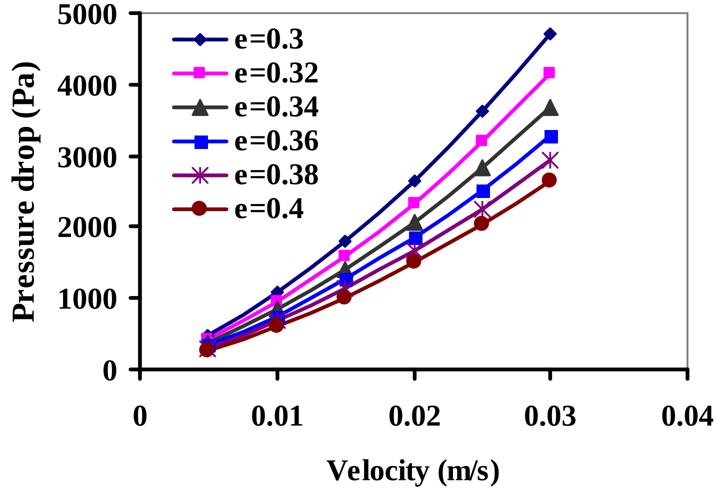

Figure 4.3 Pressure drop vs. velocity at different porosities for tortuosity of 1.43, pore diameter 0.8 cm and bed length 20 m.

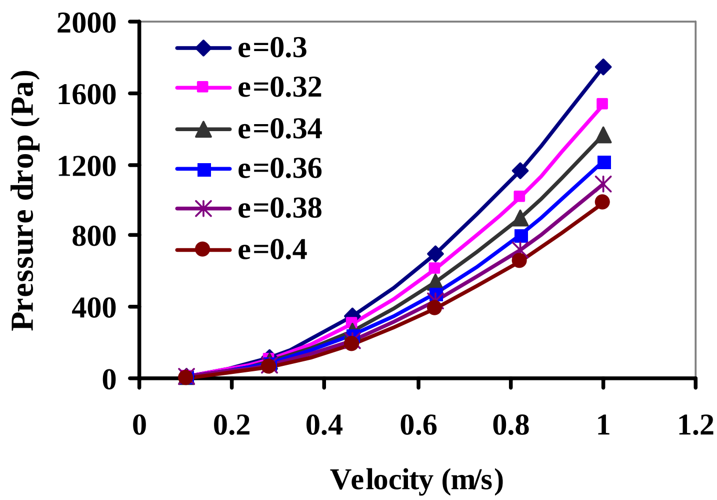

Figure 4.4 Pressure drop vs. velocity at different porosities for tortuosity of 1.4, pore diameter 0.1 cm and bed length 15 cm.

4.3.3 Effect of Packing Height on Pressure Drop:

Figure 4.5 and 4.6 shows the variation of velocity on calculated pressure drop values for water and air flow through packed bed respectively. It is noticed that whenever the length of the packing height increases the fluid flow resistance increases this leads to an increase in pressure drop, as shown by the work of Coluson 1949[49]. For example for water flow through packed bed at velocity 0.01 m/s when the packing height is 0.2 m the pressure drop is 1084.439, while for the same velocity with packing height of 0.3 m the pressure drop increased to 1626.659 Pa, further increase in the packing height to 0.6 m for the same velocity the pressure drop increased to 3253.317 Pa.

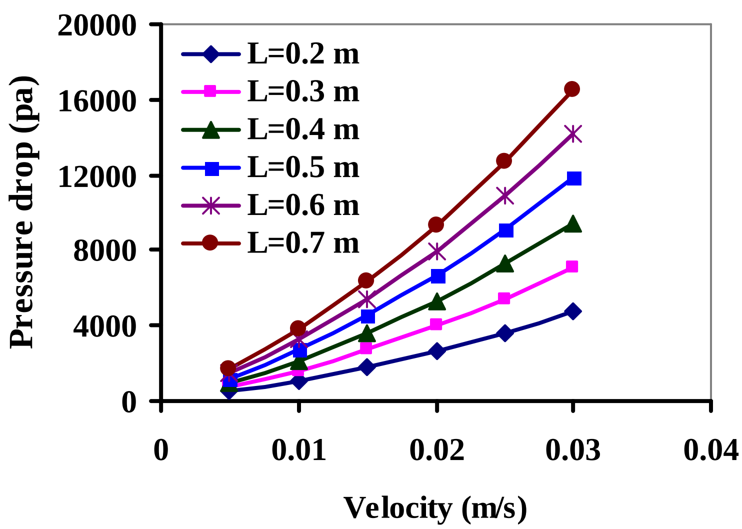

Figure 4.5 Pressure drop vs. velocity at different bed lengths for tortuosity of 1.43, pore diameter 0.08 cm and porosity 0.3.

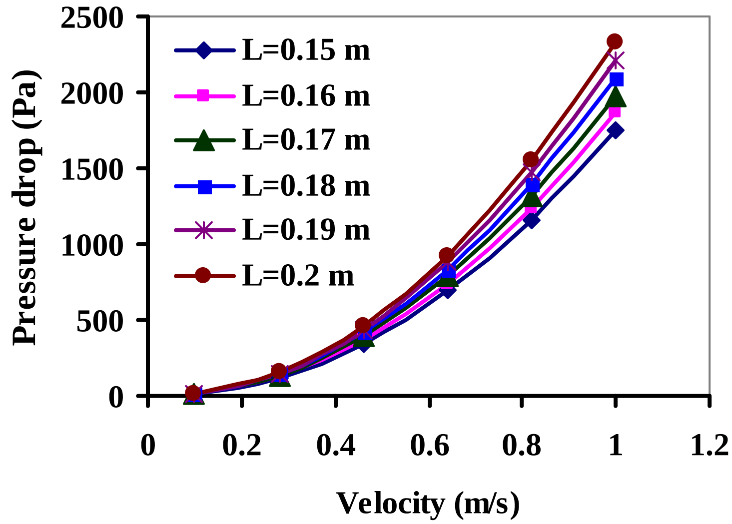

Figure 4.6 Pressure drop vs. velocity at different bed lengths for tortuosity of 1.4, pore diameter 0.1 cm and porosity 0.3.

4.3.4 Effect of Tortuosity on Pressure Drop:

Figure 4.7 and 4.8 shows the variation of velocity on calculated pressure drop values for water and air flow through packed bed respectively. It is noticed that that whenever the tortuosity of the packing increases the voidage of packing decreases, which leads to increase pressure drop [31]. For example for water flow through packed bed at velocity 0.01 m/s when the tortuosity is 1.43 the pressure drop is 1084.439, while for the same velocity with tortuosity of 1.46 the pressure drop increased to 1182.494 Pa.

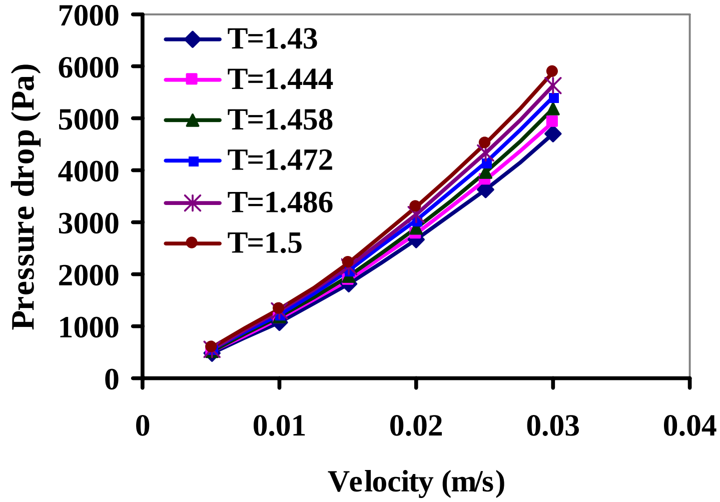

Figure 4.7 Pressure drop vs. velocity at different tortuosities for bed length 0.2 m, pore diameter 0.0008m and porosity 0.3.

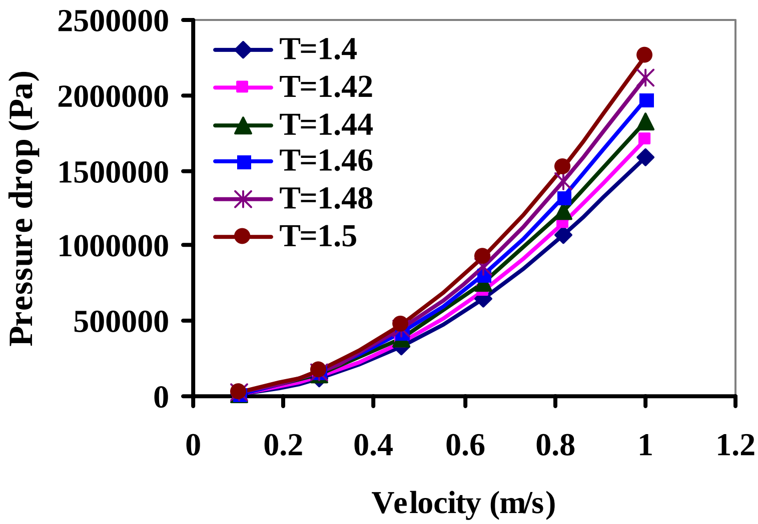

Figure 4.8 Pressure drop vs. velocity at different tortuosities for bed length 0.15 m, pore diameter 0.001m and porosity 0.3.

4.4 Comparisons between Proposed Equation, Ergun Equation and Experimental Data.

4.4.1 Singular Equations Results for Different Sizes of Packing

4.4.1.1 Mono Size Spherical Particle System

A. Air Flow Through Packed Bed

The values of pressure drop versus velocity for air flow through packed beds of mono size particles are plotted in figures 4.9 to 4.13.

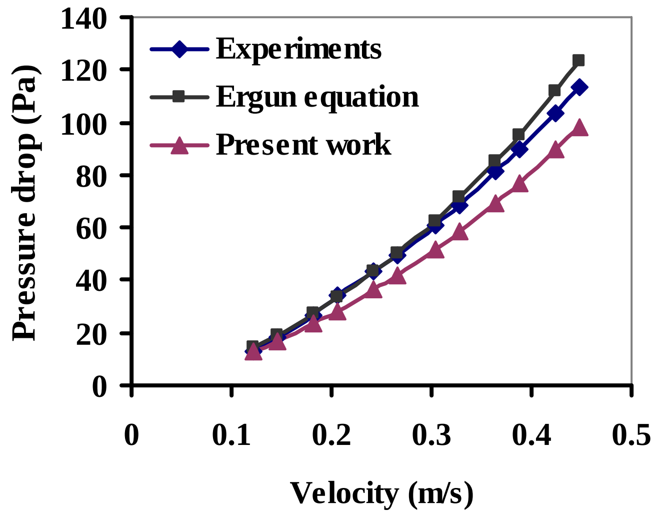

Figure 4.9 Pressure drop vs. velocity for spherical particle diameter of 0.61 cm, pore diameter of 0.27 cm, tortuosity of 1.4182, bed porosity of 0.4005, packing height of 15.15 cm and bed diameter of 7.62 cm [58] (Appendix A.5)

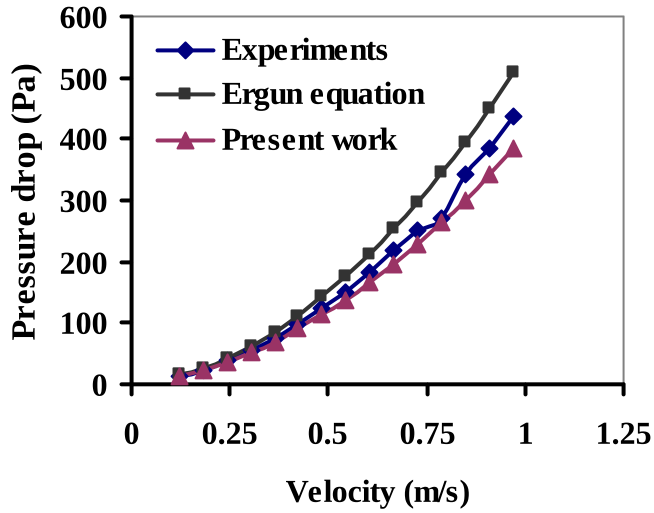

Figure 4.10 Pressure drop vs. velocity for spherical particle diameter of 0.61 cm, pore diameter of 0.27 cm, tortuosity of 1.4184, bed porosity of 0.3998, packing height of 15.15 cm and bed diameter of 7.62 cm [56] (Appendix A.6)

It can be noticed that the pressure drop values in figure 4.9 are range in 13.2473-98.0898 Pa which are less than those in figure 4.10 (range from 13.3468-384.429) Pa, because the porosity in figure 5.9 (0.4005) is greater than in figure 5.10 which is (0.3998), as the porosity decreased the pressure drop increases [16].

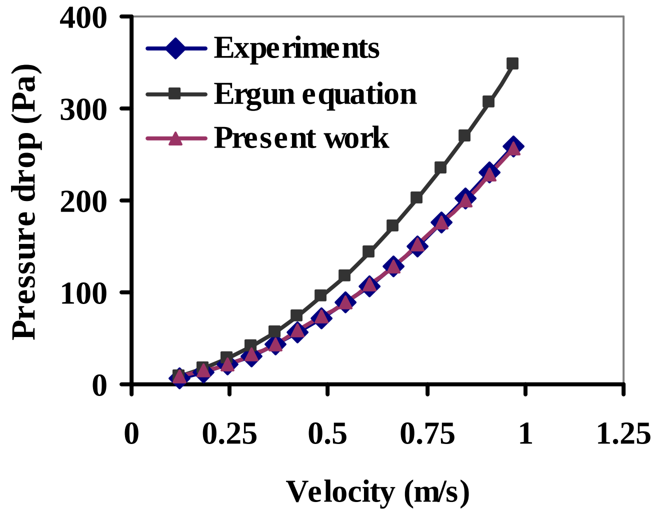

Figure 4.11 Pressure drop vs. velocity for spherical particle diameter of 0.7955 cm, pore diameter of 0.37 cm, tortuosity of 1.4157, bed porosity of 0.4088, packing height of 15.15 cm and bed diameter of 7.62 cm [56] (Appendix A.4)

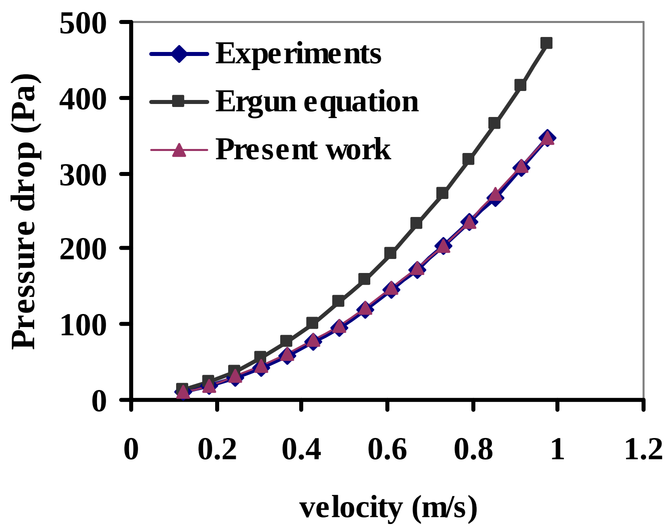

Figure 4.12 Pressure drop vs. velocity for spherical particle diameter of 0.7955 cm, pore diameter of 0.37 cm, tortuosity of 1.4157, bed porosity of 0.4088, packing height of 20 cm and bed diameter of 7.62 cm [57] (Appendix A.2)

Figure 4.11 and 4.12 shows that as the packing height increased the pressure drop increased, this is because when the packing height increased the fluid flow resistance increased and this leads to an increase in the pressure drop. The packing height increased from (15.15) cm in Fig. 5.11 to 20 cm in Fig. 5.12, which led to increase the pressure drop values from the range of (7.8332-257.286) Pa in Fig 5.11, to the range of (10.6549-347.817) Pa in Fig 5.12, for the same porosity of 0.4088, bed diameter of 7.62cm, and particle diameter of 0.7955cm with pore diameter of 0.37 cm.

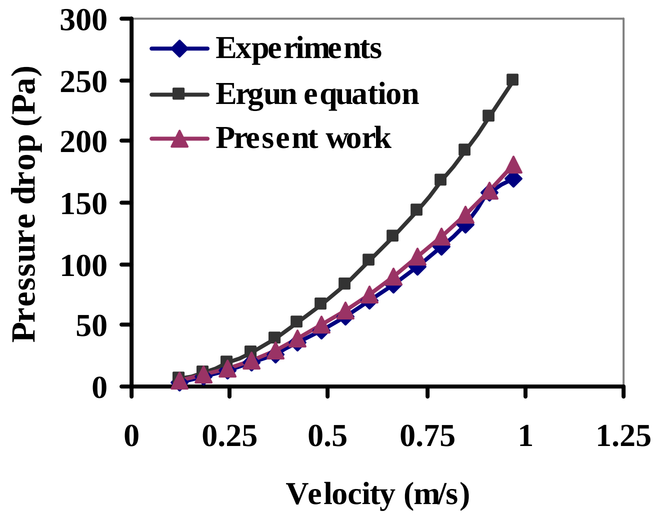

Figure 4.13 Pressure drop vs. velocity for spherical particle diameter of 0.9987 cm, pore diameter of 0.48 cm, tortuosity of 1.4129, bed porosity of 0.4181, packing height of 15.15 cm and bed diameter of 7.62 cm [56] (Appendix A.7)

The wall affect on the bed porosity and increase its value, this appears in Fig. 4-13 where the bed porosity increases to a value of 0.4181 this wall effect may be due to that the ratio of bed diameter (7.62 cm) to the particles diameter (0.9987 cm) is less than the supposed ratio $(\frac{D}{d_p} \geq 10)$[41].

The best fitting for the experimental data for mono size systems for spherical particles are represented by the following equation.

$$\frac{\Delta P}{L} = 48.49076\frac{\tau(\tau-1)^2}{\varepsilon^2}\frac{\mu u}{d_{pore}^2} + 2.220827\frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)}\frac{\rho u^2}{d_{pore}} \qquad \text{… (4.9)}$$

The average percentage error was found 7.25509% between experimental work and the proposed equation.

B. Water Flow Through Packed Bed

The values of pressure drop versus velocity for water flow through packed beds of mono size particles were plotted in figures 4.14 to 4.17

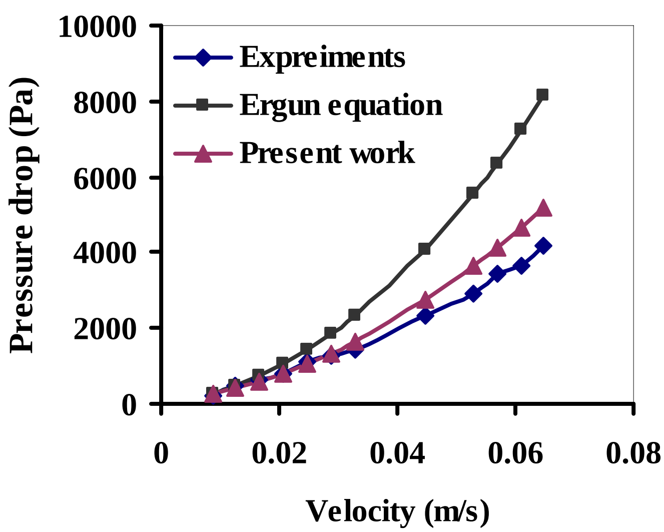

Figure 4.14 Pressure drop vs. velocity for spherical particle diameter of 0.64 cm, pore diameter of 0.24 cm, tortuosity of 1.4309, bed porosity of 0.3609, bed diameter of 8.883 cm and packing height of 45.72 cm [3] (Appendix B.4)

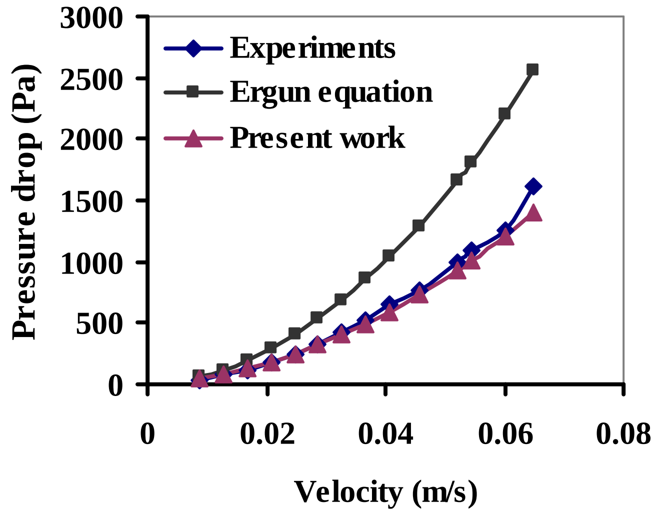

Figure 4.15 Pressure drop vs. velocity for spherical particle diameter of 1.27 cm, pore diameter of 0.57 cm, tortuosity of 1.4172, bed porosity of 0.4037, bed diameter of 8.883 cm and packing height of 45.72 cm [3] (Appendix B.8)

It can noticed that in Figure 4.14 and 4.15 show that as the pore diameter decreased from 0.57 cm in fig.4.15 to 0.24 cm in fig.4.14, the bed porosity will decrease from 0.4037 to 0.3609, which led to increase the pressure drop values from the range of (45.6247-1400.02) Pa (fig.4.15) to the range of (246.8429-5183.159) Pa (fig.4.14), this is because when the porosity decreased the fluid flow resistance increased this is leads to an increase in pressure drop for the same bed diameter of 8.883 cm and packing height of 45.72 cm.

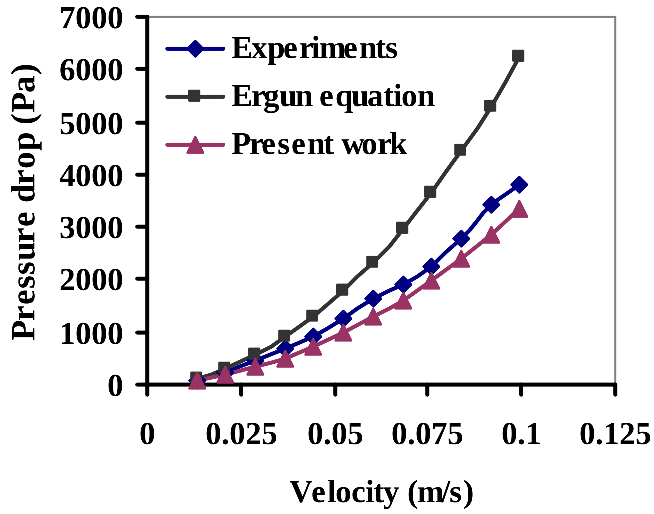

Figure 4.16 Pressure drop vs. velocity for black marble diameter of 1.27 cm, pore diameter of 0.57 cm, tortuosity of 1.4183, bed porosity of 0.4, packing height of 46.99 cm and bed diameter of 8.89 cm [4] (Appendix B.7)

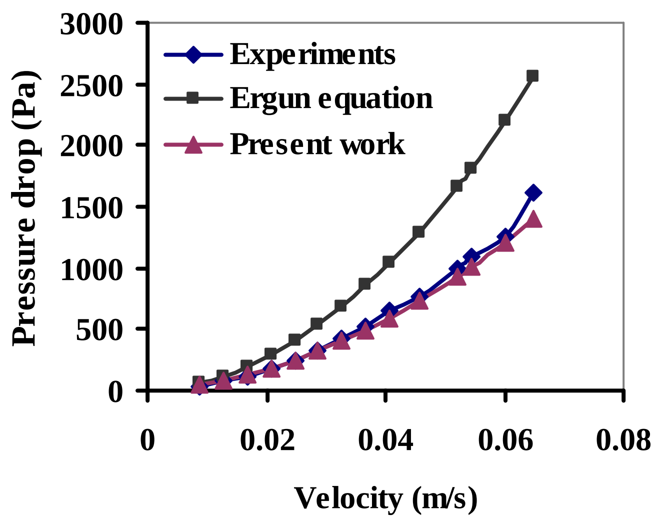

Figure 4.17 Pressure drop vs. velocity for spherical particle diameter of 1.27 cm, pore diameter of 0.57 cm, tortuosity of 1.4172, bed porosity of 0.4, packing height of 45.72 cm and bed diameter of 8.89 cm [3] (Appendix B.8)

Figure 4.16 show that the proposed model results of pressure drop-velocity curve is close to the values of experimental results curve. The values of pressure drop of the model results in fig. 4.16 (range from 87.8937-3330.22) Pa are close to those of experimental data results (range from 74.658-3807.558) Pa, for velocity (range from 0.0131-0.0996) m/s. This means that the proposed equation is very close to the experimental results.

The best fitting for the experimental data for mono size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L}=54.38942\frac{\tau(\tau-1)^2}{\varepsilon^2}\frac{\mu u}{d_{pore}^2}+1.5889\frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)}\frac{\rho u^2}{d_{pore}} \qquad \ldots (4.10)$$

The average percentage error was found 11.0591% between experimental work and the proposed equation.

4.4.1.2 Binary Sizes Spherical Particle System

A. Air Flow Through Packed Bed

The values of pressure drop versus velocity for air flow through packed beds of binary sized spherical particles were plotted in figures 4.18 to 4.20

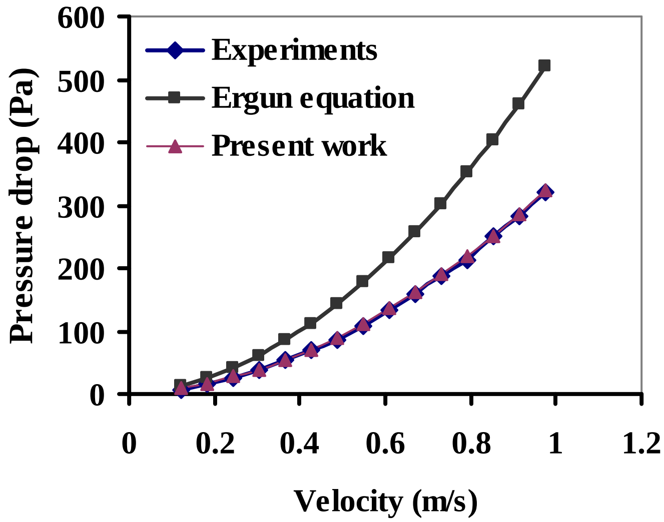

Figure 4.18 Pressure drop vs. velocity for spherical particle diameters of (dp_1=0.79, dp_2=1.01, dp_{eff}=0.89 cm), pore diameter of 0.37 cm, tortuosity of 1.3990, bed porosity is 0.3832, bed diameter is 7.64 cm and packing height is 20 cm [57] (Appendix A.9)

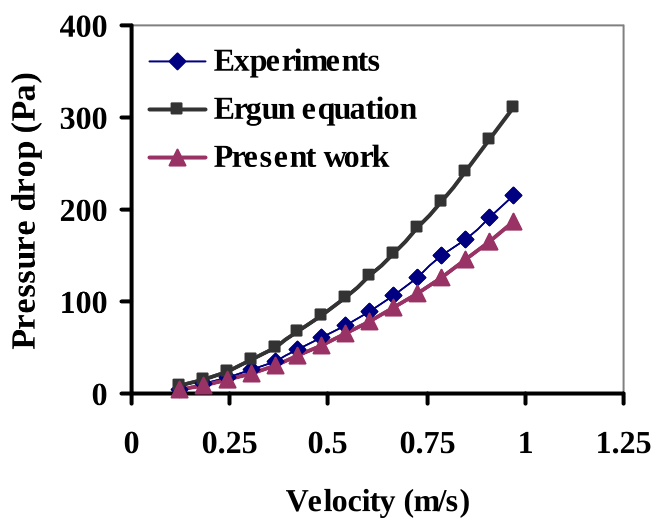

Figure 4.19 Pressure drop vs. velocity for spherical particle diameters of (dp1=0.9987 and dp2=0.7955 cm, with dp_{eff}=0.886 cm), fraction of (x_1=0.5, x_2=0.5), pore diameter of 0.41 cm, tortuosity of 1.3992, bed porosity of 0.4079, bed diameter is 7.64 cm and packing height is 15.15 cm [56] (Appendix A.10)

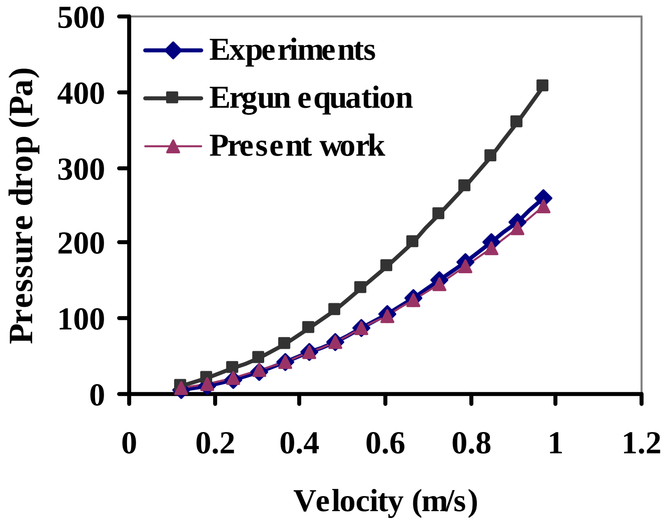

Figure 4.20 Pressure drop vs. velocity for spherical particle diameters of (dp_1=0.9987, dp_2=0.6015, dp_{eff}=0.7508 cm), pore diameter of 0.33 cm, tortuosity of 1.3947, bed porosity is 0.3986, bed diameter is 7.64 cm and packing height is 15.15 cm [56] (Appendix A.8)

Figure 4.19 and 4.20 show that as pore diameter decreased from 0.41 cm (fig.4.19) to 0.33 cm (fig.4.20), the bed porosity decreased from 0.4079 to 0.3986, which led to increase the pressure drop values from the range of (5.0892-187.56) Pa (fig.4.19) to the range of (7.3059-248.659) Pa (fig.4.20), this is because when the porosity decreased this leads to an increase in pressure drop for the same bed diameter of 7.64 cm and packing height of 15.15 cm.

The best fitting for the experimental data for binary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 36.06787 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 2.0205 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \ldots (4.11)$$

The average percentage error was found 9.9593% between experimental work and the proposed equation.

B. Water Flow through Packed Bed

The values of pressure drop versus velocity for water flow through packed beds of binary sized spherical particles were plotted in figures 4.21 and 4.22

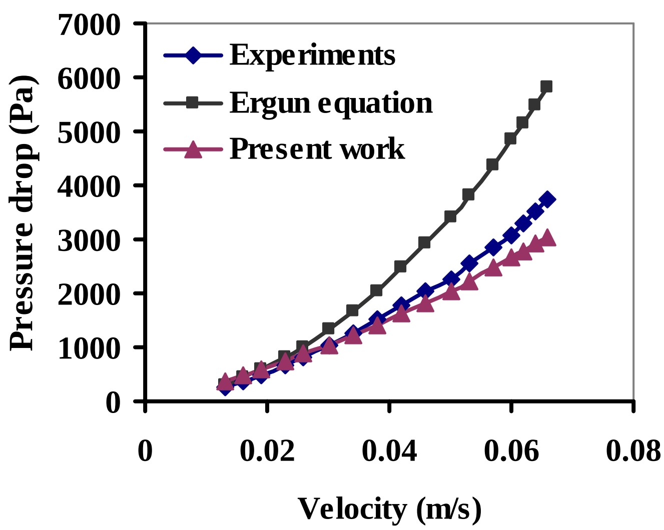

Figure 4.21 Pressure drop vs. velocity for Acrylic balls of particles diameters (dp1=0.636 and dp2=1.27 cm, with dp_{eff}=0.907cm), fraction of (x_1=0.4, x_2=0.6), pore diameter of 0.35cm, tortuosity of 1.4044, bed porosity of 0.3645, packing height of 48.26 cm and bed diameter of 8 cm [61](Appendix B.15)

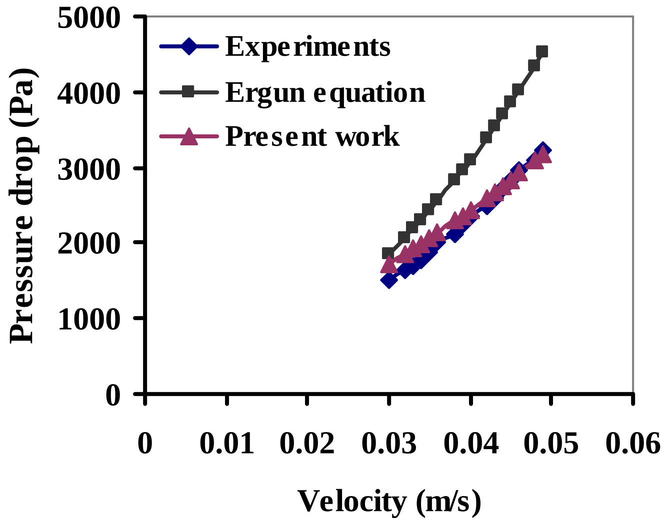

Figure 4.22 Pressure drop vs. velocity for Acrylic balls of particles diameters (dp1=0.636 and dp2=1.27 cm, with dp_{eff}=0.7065 cm), fraction of (x_1=0.8, x_2=0.2), pore diameter of 0.27 cm, tortuosity of 1.4055, bed porosity of 0.3609, packing height of 48.26 cm and bed diameter of 8 cm [61](Appendix B.14)

Figure 4.22 show that the value of porosity (0.3609) is less than in fig.4.21 (0.3645), this is due to that the weight fraction of small particles in fig. 4.22 (0.8) is less than it in figure 4.21 (0.4), this is because in binary system the particles with smaller sizes tend to fill the voids between the larger sizes particles. [56]

The best fitting for the experimental data for binary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 148.8836 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 0.8343 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \dots (4.12)$$

The average percentage error was found 12.6548% between experimental work and the proposed equation.

4.4.1.3 Ternary Sized Spherical Particle System

A. Air Flow Through Packed Bed

The values of pressure drop versus velocity for air flow through packed beds of ternary sized spherical particles were plotted in figures 4.23 to 4.26

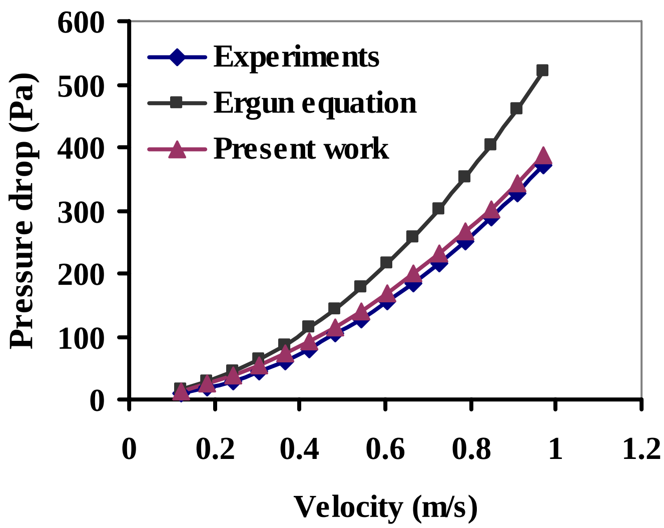

Figure 4.23 Pressure drop vs. velocity for spherical particles diameters of (0.9987, 0.7955 and 0.509cm, with dp_{eff}=0.7104 cm), pore diameter of 0.29 cm, tortuosity of 1.4525, bed porosity of 0.3796, packing height of 15.15 cm and bed diameter of 7.64 cm [56] (Appendix A.15)

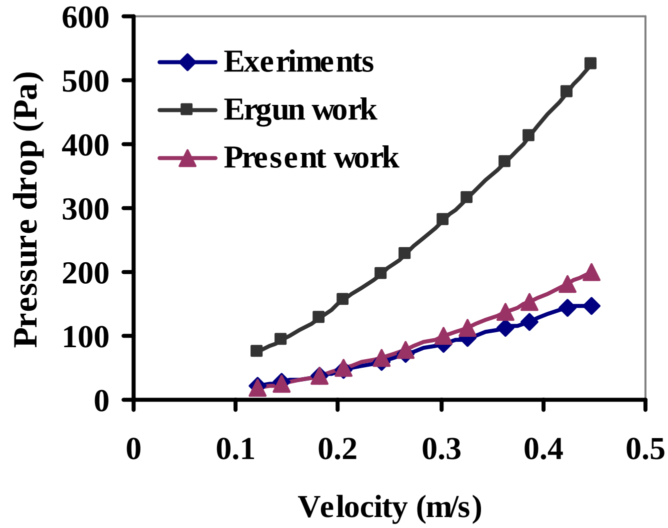

Figure 4.24 Pressure drop vs. velocity for spherical particles diameters of (0.24, 0.42 and 0.82cm, with dp_{eff}=0.3862 cm), pore diameter of 0.13 cm, tortuosity of 1.5014, bed porosity of 0.3428, packing height of 15.15 cm, bed diameter of 7.64 cm [58] (Appendix A.12)

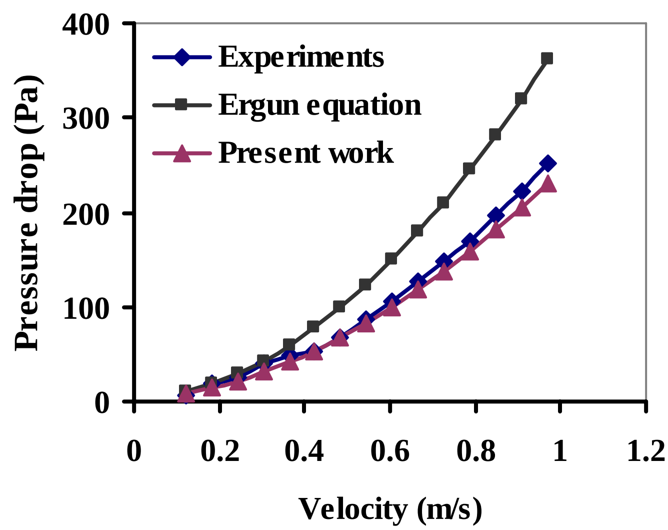

Figure 4.25 Pressure drop vs. velocity for spherical particles diameters of (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651cm), pore diameter of 0.35 cm, tortuosity of 1.4156, bed porosity of 0.3899, packing height of 15.15 cm and bed diameter of 7.64 cm [56] (Appendix A.13)

The above figures show that tortuosity increased from 1.4156 cm (fig.4.25) to the 1.5014 cm (fig.4.24), the bed porosity decreased from 0.3899 to 0.3428, which led to increase the pressure drop values from the (range of 7.8542 -231.42) Pa (fig.4.25) to (the range of (13.2949-697.8937) Pa (fig.4.24), as tortuosity of porous and granular media increased the void fraction of packed bed will be decrease, this is leads to increase in pressure drop for the same bed diameter of 7.64 cm and packing height of 15.15 cm [31].

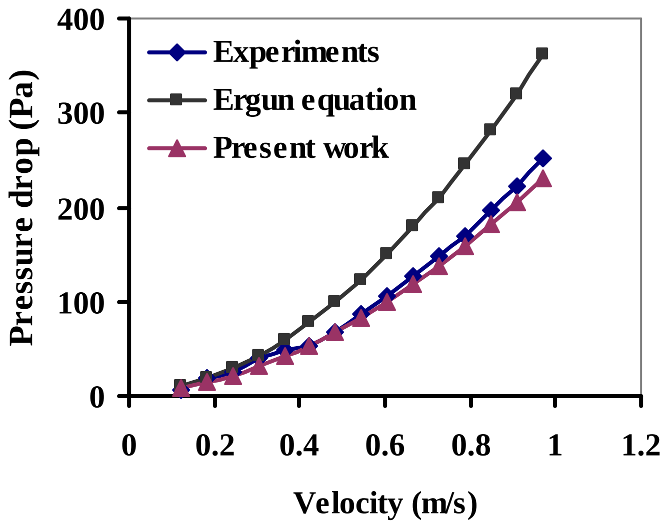

Figure 4.26 Pressure drop vs. velocity for spherical particles diameters of (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651cm), pore diameter of 0.35 cm, tortuosity of 1.4156, bed porosity of 0.405, packing height of 15.15 cm and bed diameter of 7.64 cm [56] (Appendix A.14)

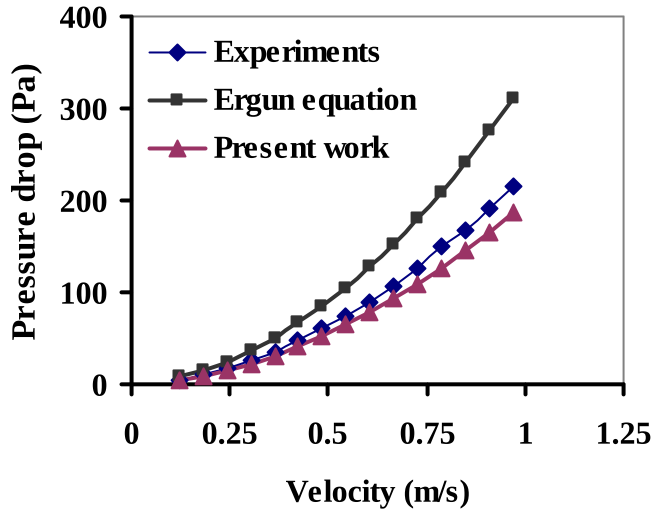

Figure 4.27 Pressure drop vs. velocity for spherical particle diameters of (dp1=0.9987 and dp2=0.7955 cm, with dp_{eff}=0.886 cm), fraction of (x_1=0.5, x_2=0.5), pore diameter of 0.41 cm, tortuosity of 1.3992, bed porosity of 0.4079, bed diameter of 7.64 cm and packing height of 15.15 cm [56] (Appendix A.10)

The porosity highly affects the pressure drop and inversely proportional to it [66]; this is appeared in figure 4.26 and 4.27. This figure shows that the bed porosity decreased from 0.4079 in fig.4.27 to 0.405 in fig.4.26, which led to increase the pressure drop values from the range of (5.0892-187.5597) Pa in binary size particles (fig.4.27) to the range of (7.8542-231.42) Pa in ternary size particles (fig.4.26) for the same bed diameter of 7.64 cm and packing height of 15.15 cm.

The best fitting for the experimental data for ternary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 49.91447 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 1.85297 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \quad \ldots (4.13)$$

The average percentage error was found 12.9554% between experimental work and the proposed equation.

B. Water Flow Through Packed Bed

The values of pressure drop versus velocity for water flow through packed beds of ternary size particles were plotted in figures 4.28 and 4.29

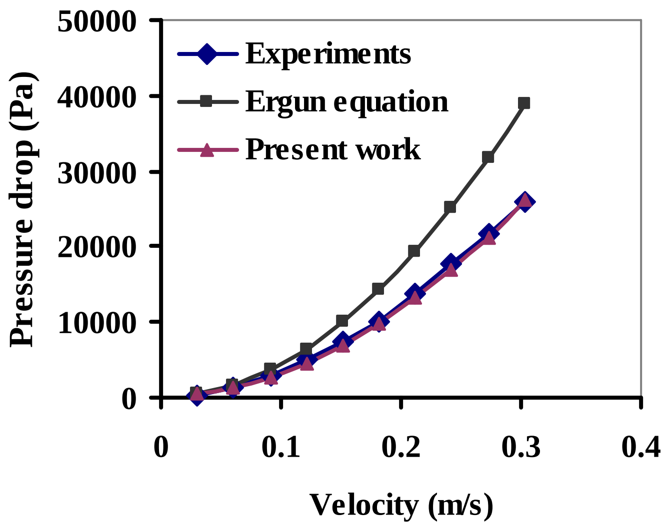

Figure 4.28 Pressure drop vs. velocity for spherical particles diameter of (0.9987, 0.7955 and 0.421, with dp_{eff}=0.6477 cm), pore diameter of 0.28 cm, tortuosity of 1.4366, bed porosity of 0.3921, packing height of 15.15 cm and bed diameter of 7.62 cm [56](Appendix B.26)

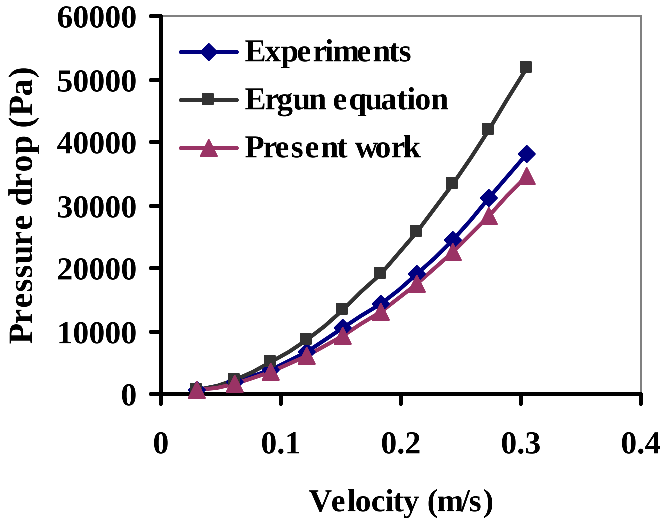

Figure 4.29 Pressure drop vs. velocity for spherical particles diameter of (0.51, 0.79 and 1.01, with dp_{eff}=0.6536 cm), pore diameter of 0.28 cm, tortuosity of 1.4373, bed porosity of 0.392, packing height of 20 cm and bed diameter of 7.62 cm [57](Appendix B.24)

Figure 4.28 shows that the proposed model results of pressure drop-velocity curve lay on the experimental results curve. The values of pressure drop of the model results in fig.4.28 (range from 402.7382-26086.89) Pa are close to those of experimental data results (range from 390.071-25886.5) Pa, for the same velocity (range from 0.0303-0.303) m/s. This means that the proposed model is very close to the experimental results.

The best fitting for the experimental data for ternary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 65.24735 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 1.88032 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \ldots (4.14)$$

The average percentage error was found 10.5479% between experimental work and the proposed equation.

4.4.1.4 Quaternary Sized Spherical Particle System

A. Air Flow Through Packed Bed

The values of pressure drop versus velocity for air flow through packed beds of quaternary size particles were plotted in figures 4.30 and 4.31

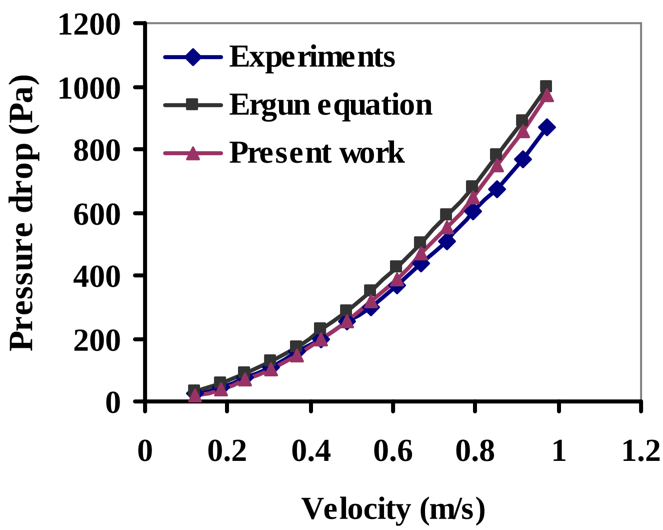

Figure 4.30 Pressure drop vs. velocity for spherical particles diameters of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.552 cm), pore diameter of 0.22 cm, tortuosity of 1.4862, bed porosity of 0.371, packing height of 20 cm and bed diameter of 7.62 cm [57] (Appendix A.18)

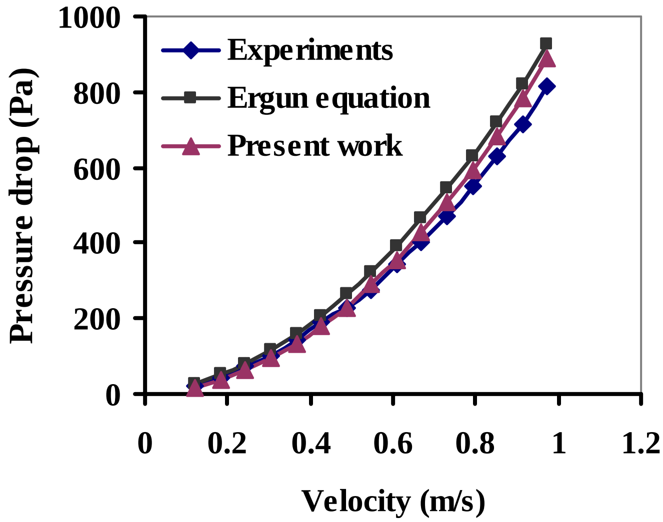

Figure 4.31 Pressure drop vs. velocity for spherical particles diameters of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.5738 cm), pore diameter of 0.23 cm, tortuosity of 1.4807, bed porosity of 0.3745, packing height of 20 cm and bed diameter of 7.62 cm [57] (Appendix A.17)

The best fitting for the experimental data for ternary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 8.77095 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 2.6576 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \dots (4.15)$$

The average percentage error was found 8.88463% between experimental work and the proposed equation.

B. Water Flow Through Packed Bed

The values of pressure drop versus velocity for water flow through packed beds of quaternary size particles were plotted in figure 4.32

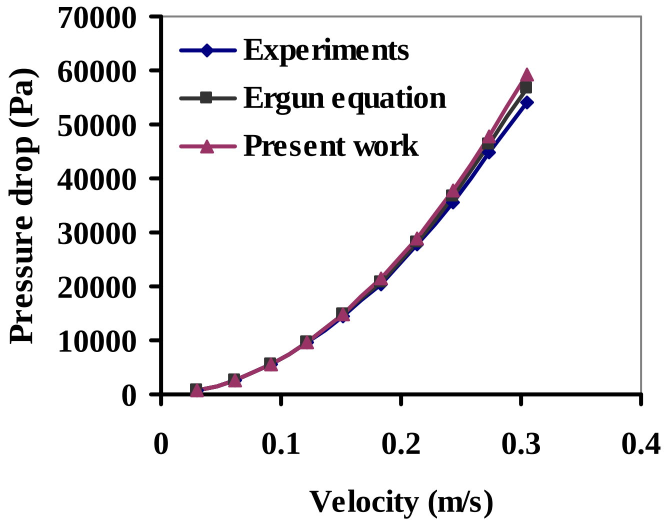

Figure 4.32 Pressure drop vs. velocity for glass spherical diameter of (0.42, 0.51, 0.61 and 0.79, with dp_{eff}=0.55 cm), pore diameter of 0.22 cm, tortuosity of 1.4861, bed porosity of 0.3711, packing height of 15.15 cm and bed diameter of 7.62 cm [57] (Appendix B.30)

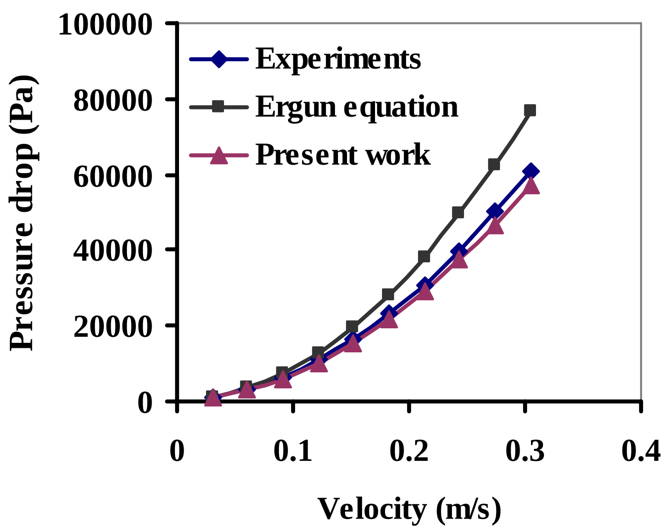

Figure 4.33 Pressure drop vs. velocity for spherical particles diameter of (0.61, 0.79 and 1.01cm, with dp_{eff}=0.54cm), pore diameter of 0.21 cm, tortuosity of 1.4630, bed porosity of 0.3715, packing height of 20 cm and bed diameter of 7.62 cm [57] (Appendix B.21)

Figures 4.32 and 4.33 shows that the pressure drop increased with increased in packing height. The packing height increased from 15.15 cm (Fig. 4.32) to 20 cm (Fig. 4.33), which led to increase the pressure drop values from the range of (625.8377-59110.91) Pa (Fig 4.32), to the range of (970.0841-57264.93) Pa (Fig 4.33), for the approximately near value of porosity is 0.3711 in quaternary size particles figure 4.30 and 0.3715 in ternary size particles figure 4.31 for same bed diameter of 7.62 cm and pore diameter of 0.28cm.

The best fitting for the experimental data for quaternary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 7.0890 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 2.6040 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \ldots (4.16)$$

The average percentage error was found 8.17105% between experimental work and the proposed equation.

4.4.1.5 Quinary Sized Spherical Particle System

A. Air Flow Through Packed Bed

The values of pressure drop versus velocity for air flow through packed beds of quinary size particles were plotted in figure 4.34

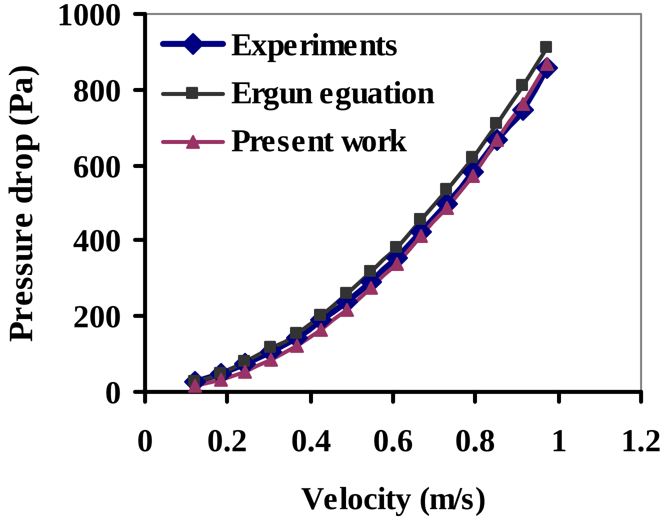

Figure 4.34 Pressure drop vs. velocity for spherical particles diameters of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.607 cm), pore diameter of 0.24 cm, tortuosity of 1.509, bed porosity of 0.3694, packing height of 20 cm and bed diameter of 7.62 cm [57] (AppendixA.20)

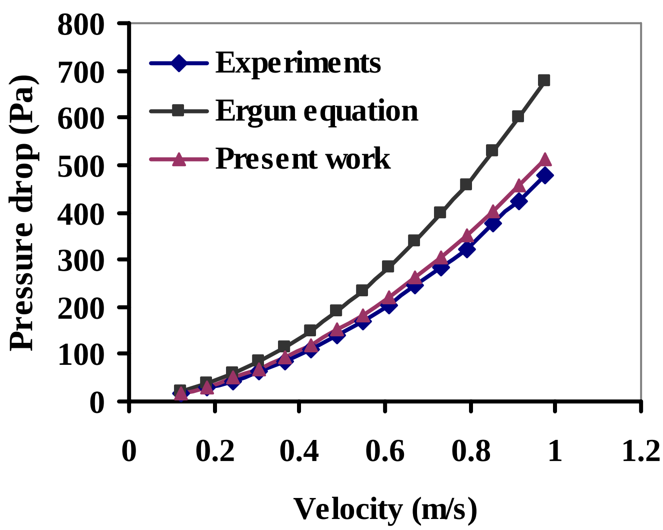

Figure 4.35 Pressure drop vs. velocity for spherical particle diameter of 0.61cm, pore diameter of 0.27 cm, tortuosity of 1.4184, bed porosity of 0.3998, packing height of 20 cm and bed diameter of 7.62 cm [57] (AppendixA.3)

It can be noticed that the pressure drop values in figure 4.35 mono size particles (range from 17.8431-402.292) Pa are less than those in figure 4.34 quinary size particles (range from 13.602-665.655) Pa, this is because that porosity in figure 4.35 (0.3998) with pore diameter (0.27) cm is greater than in figure 4.34 (0.3694) with pore diameter (0.24) cm, as the porosity decreased the pressure drop will be increase.

The best fitting for the experimental data for quinary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 0.06354 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 2.5551 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \dots (4.17)$$

The average percentage error was found 12.8196% between experimental work and the proposed equation.

B. Water Flow Through Packed Bed

The values of pressure drop versus velocity for water flow through packed beds of quinary size particles were plotted in figure 4.36

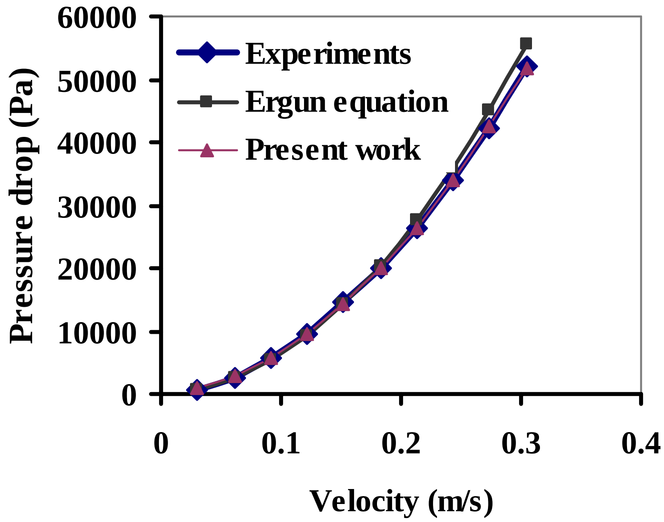

Figure 4.36 Pressure drop vs. velocity for spherical particles diameter diameter of (0.42, 0.51,0.61,0.79 and 1.01 cm, with dp_{eff}=0.61 cm), pore diameter of 0.24 cm, tortuosity of 1.4956, bed porosity of 0.3623, packing height of 15.15 cm and bed diameter of 7.62 cm [57] (Appendix B.31)

It can be noticed from the above figure that the proposed equation gave good fitting to the experiment rather than Ergun equation, while the results of Ergun lies above them; this is due to the differences in beds dimensions, packing shapes and sizes used by Ergun [16].

The best fitting for the experimental data for quinary size systems for spherical particles are represented by the following equation:

$$\frac{\Delta P}{L} = 108.3983 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 2.1188 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \ldots (4.18)$$

The average percentage error was found 4.24209% between experimental work and the proposed equation

4.4.2 Results of General Equation for Different Sizes of Packing Systems

A. Air Flow Through Packed Bed

The results of the general equation are presented in this section. This presentation takes into account a comparison with Ergun equation and experimental data. The results of the general equation include mono, binary, ternary, quaternary and quinary spherical particles sizes.

The values of pressure drop versus velocity for air flow through packed beds were plotted in Fig. 4.37 for mono spherical particles, Fig. 4.38 for binary spherical particles, Fig. 4.39 for ternary spherical particles, Fig. 4.40 for quaternary spherical particles and Fig. 4.41 for quinary spherical particles systems.

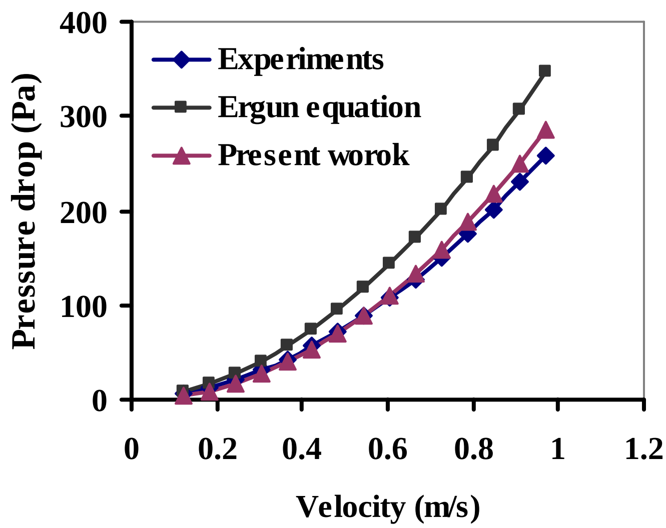

Figure 4.37 Pressure drop vs. velocity for spherical particles diameter of 0.7955 cm, pore diameter of 0.37 cm, tortuosity of 1.4312, bed porosity of 0.4088, packing height of 15.15 cm and bed diameter of 7.62 cm [56]

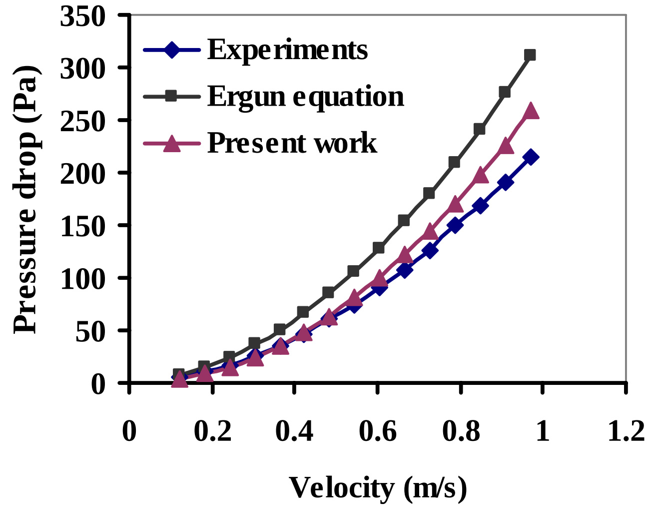

Figure 4.38 Pressure drop vs. velocity for spherical particle diameters of (dp1=0.9987 and dp2=0.7955 cm, with dp_{eff}=0.886 cm), fraction of (x_1=0.5, x_2=0.5), pore diameter of 0.41 cm, tortuosity of 1.4314, bed porosity of 0.4079, bed diameter of7.64 cm and packing height of 15.15 cm [56]

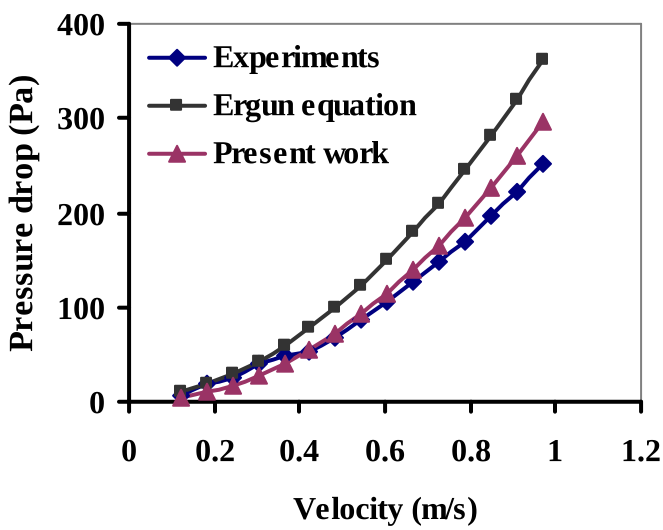

Figure 4.39 Pressure drop vs. velocity for spherical particles diameters of (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651 cm), pore diameter of 0.35, tortuosity of 1.4311, cm, bed porosity of 0.3899, packing height of 15.15 cm and bed diameter of 7.64 cm [56]

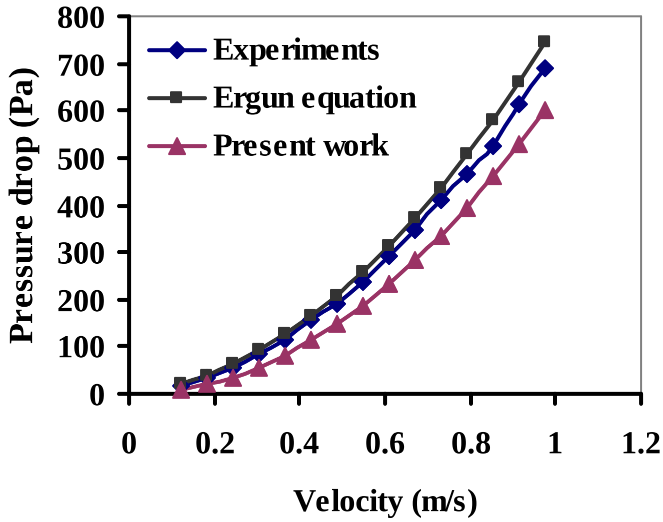

Figure 4.40 Pressure drop vs. velocity for spherical particles diameters of (0.42, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.6373 cm), pore diameter of 0.27 cm, tortuosity of 1.4366, bed porosity of 0.3843, packing height of 20 cm and bed diameter of 7.62 cm [57]

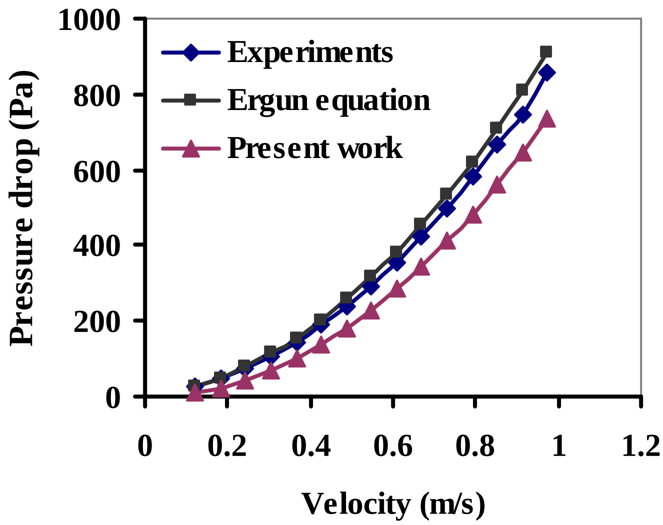

Figure 4.41 Pressure drop vs. velocity for spherical particles diameters of (0.24, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.607 cm), pore diameter of 0.24 cm, tortuosity 1.4401, bed porosity of 0.3694, packing height of 20 cm and bed diameter of 7.62 cm [57]

The proposed equation (3.29) was fitted for air flow through packed beds of multi sized particles, so the general form of the proposed equation for air flow through packed beds will be as follows:

$$\frac{\Delta P}{L} = -5.47872\frac{\tau(\tau-1)^2}{\varepsilon^2}\frac{\mu u}{d_{pore}^2} + 2.7267\frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)}\frac{\rho u^2}{d_{pore}} \qquad \text{… (4.19)}$$

The average percentage errors were found 13.57423% between experimental work and the proposed equation.

Equation 4.19 shown above can be used for all types of packing systems.

It could be noticed from figures (4.37 to 4.41) that the proposed equation gave a good fitting to the experimental results. This is due to that Ergun designed his equation using completely different procedures than experimental data work, so it was no surprise when its failure was confirmed. Ergun used pea gravel for the packed bed and air for the fluid; while the experiments were glass is used for the packed bed and air for the fluid [63]. So there is a cretin deviation between Ergun results and experimental results. This deviation was also found between the proposed equation results and the Ergun equation.

The general equation can be used for any system of packing, while the singular equation for only one types of packing, and can not be used for another type. Figures (4.37 to 4.41) show the results of the general equation for multi sized particles.

B. For Water Flow Through Packed Bed

The following presentation of results and comparisons are based on general equation fittings for all systems considered in the present work. This system considered includes mono spherical particles, binary spherical particles, ternary spherical particles, quaternary spherical particles and multi-sizes of spherical particles systems.

The proposed equation (3.29) was fitted for water flow through packed beds of multi-sized spherical particles, so the general form of the proposed equation for water flow through packed bed will be as follows:

$$\frac{\Delta P}{L} = 108.3983 \frac{\tau(\tau-1)^2}{\varepsilon^2} \frac{\mu u}{d_{pore}^2} + 2.1188 \frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)} \frac{\rho u^2}{d_{pore}} \qquad \ldots (4.20)$$

The average percentage error was found 12.9576% between the experimental work and the proposed equation.

The values of pressure drop versus velocity for water flow through packed beds were plotted in Fig. 4.42 for mono spherical particles, Fig. 4.43 for binary spherical particles, Fig. 4.44 for ternary spherical particles, Fig. 4.45 for quaternary spherical particles and Fig. 4.46 for quinary spherical particles sizes.

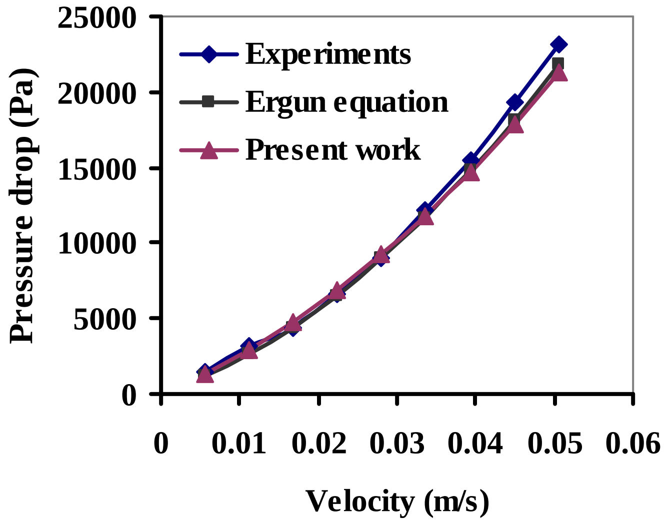

Figure 4.42 Pressure drop vs. velocity for pea gravel of particles diameter 0.25 cm, pore diameter of 0.8 cm, tortuosity of 1.4506, bed porosity of 0.360902 and packing height of 8.89 cm [3]

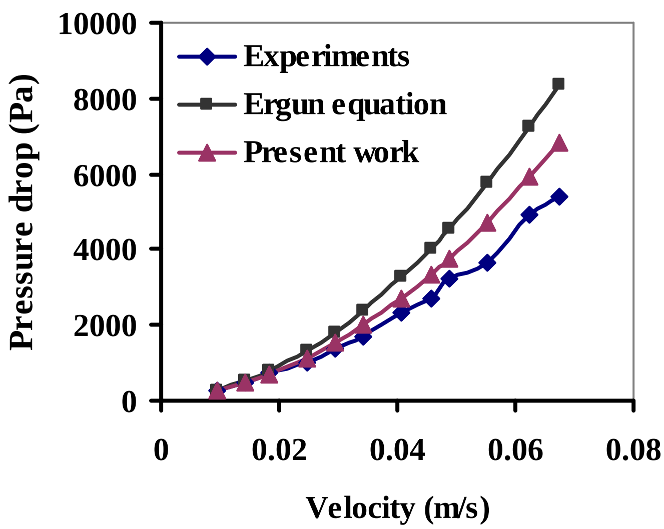

Figure 4.43 Pressure drop vs. velocity for Acrylic balls of particles diameter(dp1=0.636 and dp2=1.72 cm, with dp_{eff}=0.7257 cm), fraction of (x_1=0.75, x_2=0.25), with pore diameter of 0.27 cm, tortuosity of 1.4421, bed porosity of 0.3612, packing height of 50.8cm and bed diameter of 8 cm [62]

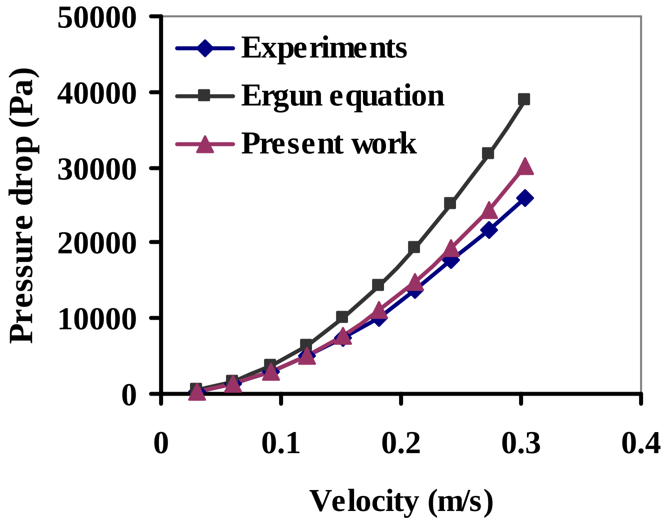

Figure 4.44 Pressure drop vs. velocity for spherical particles diameter (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.647 cm), pore diameter of 0.28 cm, tortuosity of 1.4349, bed porosity of 0.3921, packing height of 15.15 cm and bed diameter of 7.62 cm [56]

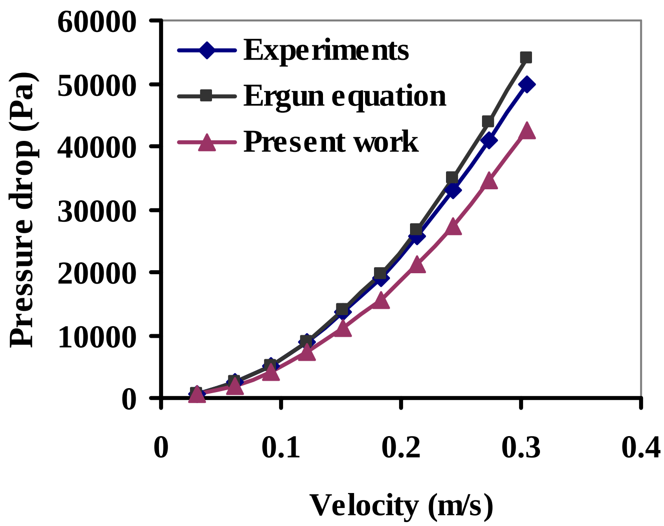

Figure 4.45 Pressure drop vs. velocity for spherical particles diameter of (0.42, 0.51, 0.61 and 1.01 cm, , with dp_{eff}=0.5738 cm), pore diameter of 0.23 cm, tortuosity of 1.4395, bed porosity of 0.3719, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

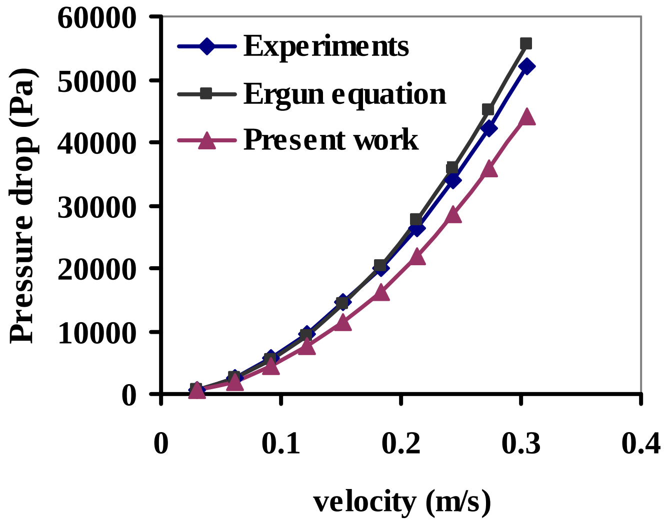

Figure 4.46 Pressure drop vs. velocity for spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.61 cm), pore diameter of 0.23 cm, tortuosity of 1.4418, bed porosity of 0.3623, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

It can be noticed from figures (4.41 to 4.46) that the proposed equation gave a good fitting to the experiment. The proposed equation results of pressure drop-velocity curves are close to the experimental results curves; this may be due to:

1. The difference of bed dimensions (diameter and height of bed) [68].
2. The difference of void fraction (difference of packing shape and size) [12].
3. Other reasons of this large deviation from Ergun equation, that Ergun's equation does not take in to consideration wall effects, because Ergun considered that in to packed beds it is generally assumed that the diameter of the packing is close to that of the column; therefore, there are no wall effects [59].

Chapter Five

Conclusions and Recommendations for Future Work

5.1 Conclusions

1. The proposed equations describe the effects of different parameters on pressure drop of fluid flow through packed beds, such that fluid velocity, height of packing, type of packing particles, pore size, bed porosity, tortuosity and bed diameter, compared with the experimental results.

2. Pore diameter formulas have been written for all equations used in the calculation. These formulas depend mainly on porosity and particle diameter of the bed. The calculated results of these formulas had been compared with experimental results taken from documented literature data; the comparisons show that the pore diameter formula deviate's for water flow through packed bed with small average percentage error of 0.328%, and for air flow through packed bed with percentages of average errors 0.3897%.

3. The tortuosity of porous media increased with decreased in porosity of packed bed, this leads to increase in pressure drop.

4. It was found that proposed equations is very close to experimental results rather than Ergun equation; this is due to the difference of bed dimensions (diameter and height of bed), the difference of void fraction (difference of

packing shape and size) and other reasons of this large deviation from Ergun equation, that Ergun's equation does not take in to consideration wall effects, because Ergun considered that in to packed beds it is generally assumed that the diameter of the packing is close to that of the column; therefore, there are no wall effects.

5.2 Recommendations for Future Work

The following suggestions could be considered for future work:

1. The proposed equation can be extended to include non spherical particles systems.
2. Study the flow of two phases through the packed bed.
3. Studying the effect of the surface roughness of the material on sphericity and its effects on the pressure drop of fluid flow through packed bed.

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Appendix A

Air Flow through Packed Bed

A.1 Singular Equations Results for Different Types of Packing

A.1.1 Mono Size Spherical Particles System

Table A.1 Calculation of pressure drop for spherical particle diameter of 1.01cm, pore diameter of 0.48 cm, tortuosity of 1.4128, bed porosity of 0.4128, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.122	7.072	6.48302
0.183	13.358	12.2138
0.244	21.216	19.6041
0.305	31.431	28.6538
0.366	44.004	39.3631
0.426	56.576	51.5158
0.487	71.506	65.5168
0.548	86.436	81.1774
0.609	106.08	98.4975
0.67	125.725	117.477
0.731	149.298	138.116
0.792	172.872	160.415
0.853	196.445	184.373
0.914	220.019	209.991
0.975	251.45	237.268

Table A.2 Calculation of pressure drop for spherical particle diameter of 0.79 cm, pore diameter of 0.37 cm, tortuosity of 1.4157, bed porosity of 0.4082, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.122	10.215	10.6549

0.183	18.859	19.5079
0.244	29.074	30.7113
0.305	42.432	44.2651
0.366	58.934	60.1692
0.426	77.007	78.1055
0.487	94.294	98.6719
0.548	117.867	121.589
0.609	145.37	146.856
0.67	172.872	174.473
0.731	204.303	204.441
0.792	235.734	236.76
0.853	267.166	271.428
0.914	306.455	308.447
0.975	345.744	347.817

Table A.3 Calculation of pressure drop for spherical particle diameter of 0.61cm, pore diameter of 0.27 cm, tortuosity of 1.4184, bed porosity of 0.3998, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.122	15.716	17.8431
0.183	28.288	31.7333
0.244	44.004	48.9359
0.305	62.862	69.4509
0.366	86.436	93.2782
0.426	110.009	119.946
0.487	141.441	150.344
0.548	168.943	184.055
0.609	204.303	221.077
0.67	243.592	261.412
0.731	282.881	305.06
0.792	322.17	352.02
0.853	377.175	402.292
0.914	424.322	455.877
0.975	479.326	512.774

Table A.4 Calculation of pressure drop for spherical particle diameter of 0.7955 cm, pore diameter of 0.37 cm, tortuosity 1.4157 bed porosity of 0.4088, packing height of 15.15 cm and bed diameter of 7.62 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	6.7188	7.8332
0.182	13.437	14.4041
0.242	21.5	22.5819
0.303	30.906	32.6391
0.364	43.004	44.4539
0.424	56.4381	57.7895
0.485	71.2195	73.0906
0.545	88.6884	89.8555
0.606	107.501	108.643
0.667	127.657	129.188
0.727	150.506	151.111
0.788	176.033	175.142
0.848	201.564	200.494
0.909	229.787	228.011
0.97	258.08	257.286

Table A.5 Calculation of pressure drop for spherical particle diameter of 0.61 cm, pore diameter of 0.27 cm, tortuosity 1.4182, bed porosity of 0.4005, packing height of 15.15 cm and bed diameter of 7.62 cm [58]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	12.814	13.2473
0.145	18.002	17.0398
0.181	26.771	23.4517
0.206	33.876	28.4150
0.242	43.077	36.2972

0.266	49.266	42.0341
0.303	60.865	51.6340
0.327	68.653	58.3511
0.363	81.043	69.1498
0.387	89.687	76.8310
0.424	103.569	89.4284
0.448	113.264	98.0898

Table A.6 Calculation of pressure drop for spherical particle diameter of 0.6105 cm, pore diameter of 0.27 cm, tortuosity 1.4184, bed porosity of 0.3998, packing height of 15.15 cm and bed diameter of 7.62 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	12.6313	13.3468
0.182	24.1877	23.8154
0.242	37.6254	36.5582
0.303	55.0943	51.9999
0.364	76.5945	69.9487
0.424	98.0948	90.0490
0.485	123.6263	112.971
0.545	150.5016	137.963
0.606	182.7519	165.858
0.667	219.0336	196.260
0.727	252.6277	228.609
0.788	270.0966	263.984
0.848	341.1362	301.225
0.909	384.3166	341.573
0.97	435.3797	384.429

Table A.7 Calculation of pressure drop for spherical particle diameter of 0.9987 cm, pore diameter of 0.48 cm, tortuosity of 1.4129, bed porosity of 0.4181, packing height of 15.15 cm and bed diameter of 7.62 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.121	4.0312	4.96081
0.182	8.0625	9.36749
0.242	13.4376	14.9482
0.303	20.1564	21.8889
0.364	26.8752	30.1072
0.424	36.2816	39.4369
0.485	45.6879	50.1892
0.545	57.7818	62.0115
0.606	69.8757	75.2979
0.667	83.3134	89.8617
0.727	98.0948	105.433
0.788	114.2199	122.531
0.848	131.6889	140.595
0.909	158.5642	160.227
0.97	169.3143	181.136

A.1.2 Binary Sized Spherical Particles System

In the binary system, the mixture contains two sizes of spherical particles. The most noticeable effect from mixing two sizes of particles is the decrease in porosity with respect to mono size mixture.

Table A.8 Calculation of pressure drop for spherical particles diameters of (dp_1=0.9987, dp_2=0.6015, dp_{eff}=0.7508 cm), pore diameter of 0.33 cm, tortuosity of 1.3947, bed porosity of 0.3986, bed diameter is 7.64 cm and packing height of 15.15 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1211	5.9125	7.3059

0.1817	11.8251	13.5076
0.2424	19.6189	21.4219
0.303	29.5628	31.0227
0.3635	41.6567	42.3015
0.4241	55.0943	55.2957
0.4847	69.8757	69.988
0.5453	87.3447	86.3784
0.6059	106.157	104.467
0.6665	127.658	124.254
0.7271	150.502	145.738
0.7877	174.689	168.921
0.8482	201.565	193.76
0.9088	228.44	220.336
0.9695	258.003	248.659

Table A.9 Calculation of pressure drop for spherical particles diameters of (dp_1=0.79, dp_2=1.01, dp_{eff}=0.89 cm), pore diameter of 0.37 cm, tortuosity of 1.3990, bed porosity is 0.3832, bed diameter is 7.64 cm and packing height is 20 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1218	7.858	9.090363
0.1827	15.716	16.99465
0.2436	25.931	27.13833
0.3046	39.289	39.54359
0.3655	53.433	54.16976
0.4264	70.72	71.03533
0.4873	86.436	90.14029
0.5482	108.438	111.4847
0.6091	133.583	135.0684
0.67	157.156	160.8916
0.7309	188.587	188.9542
0.7918	212.161	219.2562
0.8528	251.45	251.8528
0.9137	282.881	286.6373
0.9746	322.17	323.6611

Table A.10 Calculation of pressure drop for spherical particle diameters of (dp1=0.9987 and dp2=0.7955 cm, with dp_{eff}=0.886 cm), fraction of (x_1=0.5, x_2=0.5), pore diameter of 0.41 cm, tortuosity 1.3992, bed porosity is0.4079, bed diameter is 7.64 cm and packing height is 15.15 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1211	5.1063	5.0892
0.1817	10.4813	9.6014
0.2424	17.4689	15.435
0.303	25.5315	22.571
0.3635	35.7441	31.004
0.4241	47.0317	40.76
0.4847	60.4694	51.827
0.5453	73.907	64.205
0.6059	90.0322	77.894
0.6665	107.501	92.894
0.7271	126.314	109.21
0.7877	149.158	126.83
0.8482	167.971	145.73
0.9088	190.815	165.97
0.9695	215.002	187.56

Table A.11 Calculation of pressure drop for spherical particle diameters of (dp1=0.601 and dp2=0.7955 cm, with dp_{eff}=0.688 cm), fraction of (x_1=0.5, x_2=0.5), pore diameter of 0.26 cm, tortuosity 1.4049, bed porosity is 0.3628, bed diameter is 7.64 cm and packing height is 20 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1218	12.573	17.69582
0.1827	25.145	31.96128
0.2436	40.861	49.83844
0.3046	59.719	71.36555
0.3655	83.293	96.47204
0.4264	111.581	125.1902

0.4873	141.441	157.5201
0.5482	168.943	193.4617
0.6091	204.303	233.015
0.67	243.592	276.18
0.7309	290.739	322.9566
0.7918	337.886	373.345
0.8528	385.033	427.4367
0.9137	432.18	485.0544
0.9746	495.042	546.2838

Table A.12 Calculation of pressure drop for spherical particle diameters of (dp1=0.9987 and dp2=0.509 cm, with dp_{eff}=0.551 cm), fraction of (x_1=0.5, x_2=0.5),pore diameter of 0.23 cm, tortuosity of 1.3994, bed porosity of 0.3817, bed diameter of 7.64 cm and packing height of 15.15 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1211	12.0938	14.7619
0.1817	24.1877	26.2806
0.2424	40.3129	40.581
0.303	59.1256	57.616
0.3635	86.0009	77.372
0.4241	108.845	99.9143
0.4847	137.064	125.213
0.5453	172.002	153.267
0.6059	206.94	184.077
0.6665	244.565	217.643
0.7271	287.566	253.964
0.7877	331.91	293.042
0.8482	378.942	334.805
0.9088	421.942	379.39
0.9695	489.13	426.811

A.1.3 Ternary Sized Spherical Particles System

In the ternary system, the mixture contains three sizes of spherical particles.The percentage of each size is equal 1/3 from the total packing.

Table A.13 Calculation of pressure drop for spherical particles diameters of (0.9987, 0.7955 and 0.6015 cm, with dp_{eff}=0.7651cm), pore diameter of 0.35 cm, tortuosity of 1.4156, bed porosity of 0.3899, packing height of 15.15 cm and bed diameter of 7.64 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	5.644	7.85429
0.182	19.56	14.0849
0.242	25.35	21.6983
0.303	40.31	30.9485
0.364	49.59	41.721
0.424	53.75	53.8019
0.485	68.53	67.594
0.545	86.99	82.6451
0.606	104.8	99.4568
0.667	126.3	117.791
0.727	147.8	137.309
0.788	169.3	158.663
0.848	197.5	181.152
0.909	223.1	205.525
0.97	252.6	231.42

Table A.14 Calculation of pressure drop for spherical diameters of (0.51, 0.79 and 1.01 cm, with dp_{eff}=0.7115 cm), pore diameter of 0.29 cm, tortuosity 1.4491, bed porosity of 0.3822, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.122	15.72	18.08638
0.183	29.86	31.81816

0.244	48.72	48.67567
0.305	72.29	68.65891
0.366	100.96	91.76788
0.426	125.67	117.5473
0.487	157.42	146.8565
0.548	209.89	179.2914
0.609	253.16	214.852

Table A.15 Calculation of pressure drop for spherical particles diameters of (0.9987, 0.7955 and 0.509cm, with dp_{eff}=0.7104 cm), pore diameter of 0.29 cm, tortuosity of 1.4525, bed porosity of 0.3796, packing height of 15.15 cm and bed diameter of 7.64 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	8.6	14.17619
0.182	17.47	24.99275
0.242	29.56	38.03186
0.303	45.69	53.72817
0.364	61.81	71.8845
0.424	80.63	92.14307
0.485	103.5	115.1791
0.545	127.7	140.2375
0.606	154.5	168.1533
0.667	185.4	198.5291
0.727	215	230.8069
0.788	249.9	266.0625
0.848	287.6	303.14
0.909	327.9	343.2753
0.97	370.9	385.8707

A.1.4 Quaternary Sized Spherical Particles System

In the packing of quaternary size particles, the mixture contains four sizes of spherical particles. The percentage of each size is equal 1/4 from the total packing.

Table A.16 Calculation of pressure drop for spherical particles diameters of (0.42, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.6373 cm), pore diameter of 0.26 cm, tortuosity of 1.4655, bed porosity of 0.3843, packing height of 20 cm and bed diameter of 7.62 cm [58]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1218	18.859	13.29498
0.1827	35.36	27.86102
0.2436	56.576	47.7061
0.3046	86.436	72.87581
0.3655	113.94	103.2876
0.4264	157.16	138.9785
0.4873	188.59	179.9484
0.5482	235.73	226.1973
0.6091	290.74	277.7252
0.67	345.74	334.5322
0.7309	408.61	396.6182
0.7918	463.61	463.9833
0.8528	526.47	536.751
0.9137	612.91	614.6828
0.9746	691.49	697.8937

Table A.17 Calculation of pressure drop for spherical particles diameters of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.5738 cm), pore diameter of 0.23 cm, tortuosity 1.4807, bed porosity of 0.3745, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1218	22.788	17.3251
0.1827	43.218	36.034
0.2436	73.863	61.4405
0.3046	102.15	93.6028
0.3655	145.37	132.415

0.4264	188.59	177.926
0.4873	227.88	230.134
0.5482	275.02	289.039
0.6091	345.74	354.642
0.67	400.75	426.943
0.7309	471.47	505.941
0.7918	550.05	591.637
0.8528	628.63	684.187
0.9137	715.06	783.289
0.9746	817.21	889.089

Table A.18 Calculation of pressure drop for spherical particles diameters of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.552 cm), pore diameter of 0.22 cm, tortuosity of 1.4862, bed porosity of 0.371, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1218	25.145	19.09688
0.1827	47.147	39.60297
0.2436	78.578	67.41414
0.3046	110.009	102.5941
0.3655	157.156	145.0274
0.4264	196.445	194.7659
0.4873	251.45	251.8094
0.5482	298.597	316.158
0.6091	369.317	387.8118
0.67	440.037	466.7706
0.7309	510.758	553.0345
0.7918	605.051	646.6035
0.8528	675.772	747.6492
0.9137	770.065	855.8404
0.9746	872.217	971.3367

A.1.5 Quinary Sized Spherical Particles System

In the packing of quinary sized particles, the mixture contains five sizes of spherical particles. The percentage of each size is equal 1/5 from the total packing.

Table A.19 Calculation of pressure drop for spherical particles diameters of (0.24, 0.42, 0.82, 0.61 and 1.03 cm, with dp_{eff}=0.4818 cm), pore diameter of 1.28 cm, tortuosity of 1.3521, bed porosity of 0.2977, packing height of 15.15 cm and bed diameter of 7.64 cm [58]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	22.447	15.079
0.145	29.279	21.6402
0.181	43.919	33.69808
0.206	49.775	43.63619
0.242	68.319	60.20008
0.266	78.079	72.72004
0.303	97.599	94.33724
0.327	107.359	109.8612
0.363	126.879	135.3632
0.387	146.399	153.842
0.424	161.039	184.6456
0.448	167.871	206.1283

Table A.20 Calculation of pressure drop for spherical particles diameters of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.607 cm), pore diameter of 0.24 cm, tortuosity of 1.509, bed porosity of 0.3694, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1218	24.359	13.602
0.1827	45.575	30.5838
0.2436	73.863	54.3529
0.3046	106.08	84.965

0.3655	145.37	122.32
0.4264	188.587	166.462
0.4873	235.734	217.391
0.5482	290.739	275.107
0.6091	353.602	339.611
0.67	424.322	410.902
0.7309	495.042	488.98
0.7918	581.478	573.846
0.8528	667.914	665.655
0.9137	746.492	764.106
0.9746	856.501	869.344

A.2 Results of General Equation for Different Sizes of Packing Systems

The following results are for the general equation for all systems considered in the present work. The general equation constants are shown in Tables A.21 to A.25 below represent the results of pressure drop through packed bed using the general equation for mono sizes spherical particles, binary sized spherical particles, ternary sized spherical particles, quaternary sized spherical particles and multi sized spherical particles respectively.

Table A.21 Calculation of pressure drop for spherical particle diameter of 1.01cm, pore diameter of 0.48 cm, tortuosity of 1.4291, bed porosity of 0.4186, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.122	7.072	3.95746
0.183	13.358	9.1866
0.244	21.216	16.5827
0.305	31.431	26.1457
0.366	44.004	37.8757

0.426	56.576	51.5273
0.487	71.506	67.5557
0.548	86.436	85.751
0.609	106.08	106.113
0.67	125.725	128.642
0.731	149.298	153.338
0.792	172.872	180.202
0.853	196.445	209.232
0.914	220.019	240.428
0.975	251.45	273.792

Table A.22 Calculation of pressure drop for spherical particle diameters of (dp_1=0.7955, dp_2=0.59 and dp_{eff}=0.551 cm), pore diameter of 0.23 cm, tortuosity of 1.4372, bed porosity is 0.3817, bed diameter is 7.64 cm and packing height is 15.15 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1211	12.0938	6.95041
0.1817	24.1877	16.834
0.2424	40.3129	31.0171
0.303	59.1256	49.4531
0.3635	86.0009	72.1208
0.4241	108.845	99.0951
0.4847	137.064	130.342
0.5453	172.002	165.862
0.6059	206.94	205.654
0.6665	244.565	249.719
0.7271	287.566	298.057
0.7877	331.91	350.667
0.8482	378.942	407.453
0.9088	421.942	468.602
0.9695	489.13	534.135

Table A.23 Calculation of pressure drop for spherical particles diameters of (0.9987, 0.7955 and 0.509 cm, with dp_{eff}=0.7104 cm), pore diameter of 0.29 cm, tortuosity of 1.4377, bed porosity of 0.3796, packing height of 15.15 cm and bed diameter of 7.64 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	8.6	5.77074
0.182	17.47	13.7983
0.242	29.56	25.0413
0.303	45.69	39.8745
0.364	61.81	58.1386
0.424	80.63	79.4504
0.485	103.5	104.52
0.545	127.7	132.526
0.606	154.5	164.402
0.667	185.4	199.708
0.727	215	237.783
0.788	249.9	279.895
0.848	287.6	324.664
0.909	327.9	373.581
0.97	370.9	425.93

Table A.24 Calculation of pressure drop for spherical particles diameters of (0.42, 0.51, 0.61 and 0.79 cm, with dp_{eff}=0.552 cm), pore diameter of 0.22 cm, tortuosity of 1.4397, bed porosity of 0.371, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.1218	25.145	10.2734
0.1827	47.147	24.9489
0.2436	78.578	45.9837
0.3046	110.009	73.4278
0.3655	157.156	107.191
0.4264	196.445	147.314

0.4873	251.45	193.796
0.5482	298.597	246.638
0.6091	369.317	305.838
0.67	440.037	371.398
0.7309	510.758	443.317
0.7918	605.051	521.595
0.8528	675.772	606.376
0.9137	770.065	697.383
0.9746	872.217	794.749

Table A.25 Calculation of pressure drop for spherical particles diameters of (0.24, 0.42, 0.82, 0.61 and 1.03 cm, with dp_{eff}=0.4818 cm), pore diameter of 0.13 cm, tortuosity of 1.4590, bed porosity of 0.2977, packing height of 15.15 cm and bed diameter of 7.64 cm [58]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.121	22.447	17.7775
0.145	29.279	27.4502
0.181	43.919	45.7746
0.206	49.775	61.1934
0.242	68.319	87.2752
0.266	78.079	107.207
0.303	97.599	141.921
0.327	107.359	167.024
0.363	126.879	208.493
0.387	146.399	238.683
0.424	161.039	289.212
0.448	167.871	324.574

Appendix B

Water Flow through Packed Bed

B.1 Singular Equations Results for Different Types of Packing

B.1.1 Mono Size Spherical Particles System

Table B.1 Calculation of pressure drop for black marbles of particles diameter 1.9cm, pore diameter of 0.86 cm, tortuosity of 1.4169, bed porosity of 0.4047, packing height of 61.6cm and bed diameter of 14.606 cm [59]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0435	929.57	560.785
0.0465	1003.6	635.578
0.0504	1099.2	739.776
0.0551	1216.4	875.813
0.0599	1333.6	1026.55
0.0646	1450.9	1185.71
0.0685	1548.6	1326.46
0.0715	1626.9	1440.09
0.0745	1705.2	1558.38

Table B.2 Calculation of pressure drop for black marbles of particles diameter 1.905cm, pore diameter of 0.69 cm, tortuosity of 1.4338, bed porosity of 0.3523, packing height of 48.26 cm and bed diameter of 15.24 cm [3]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0058	24.9066	36.1004
0.0116	74.719801	96.2645
0.0175	174.3462	180.492
0.0233	323.7858	288.784
0.0291	423.4122	421.139
0.0349	622.66501	577.559
0.0407	871.73101	758.049

Table B.3 Calculation of pressure drop for pea gravel of particles diameter 0.25 cm, pore diameter of 0.08 cm, tortuosity of 1.4425, bed porosity of 0.3279, packing height of 38.1 cm and bed diameter of 8.89 cm [3]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0056	1394.7696	1676.779
0.0112	3237.858	3533.602
0.0168	4308.8418	5570.469
0.0225	6575.3425	7787.38
0.0281	8941.4695	10184.33
0.0337	12229.141	12761.33
0.0393	15442.092	15518.38
0.0449	19277.709	18455.46
0.0505	23163.138	21572.59

Table B.4 Calculation of pressure drop for spherical particle diameter of 0.635 cm, pore diameter of 0.24 cm, tortuosity of 1.4309, bed porosity of 0.3609, bed diameter of 8.883 cm and packing height of 45.72 cm [3]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0087	223.7106	246.8429
0.0127	467.75852	407.7228
0.0167	650.79446	598.0972
0.0207	793.15575	817.9661
0.0247	1098.2157	1067.329
0.0287	1260.9143	1346.187
0.0328	1443.9502	1654.54
0.0448	2338.7926	2756.563
0.0528	2928.5751	3638.718
0.0568	3416.6709	4124.038
0.0608	3660.7189	4638.851
0.0649	4189.4894	5183.159

Table B.5 Calculation of pressure drop for spherical particles diameter of 0.6223 cm, pore diameter 0.22cm, tortuosity 1.4346, bed porosity of 0.35, packing height of 28.2575 cm and bed diameter of 8.8265 cm [8]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0051	249.07817	109.189
0.0136	498.15634	366.873
0.0179	747.23451	531.2
0.0221	996.31268	719.183
0.0281	1245.3909	1022.1
0.0306	1494.469	1166.12
0.0349	1743.5472	1425.08
0.0375	1992.6254	1591.8
0.0417	2241.7035	1888.61
0.0451	2490.7817	2143.08
0.0477	2739.8599	2343.88
0.0519	2988.9381	2697.46
0.0545	3238.0162	2920.96
0.057	3487.0944	3152.98
0.0604	3736.1726	3475.59
0.063	3985.2507	3727.48
0.0647	4234.3289	3900.14
0.0673	4483.4071	4166.23
0.069	4732.4853	4348.35
0.0715	4981.5634	4628.63
0.0732	5230.6416	4820.22
0.0766	5479.7198	5214.74
0.0792	5728.7979	5520.57
0.0817	5977.8761	5834.92
0.0843	6226.9543	6157.78
0.0868	6476.0324	6489.16
0.0885	6725.1106	6714.81
0.0902	6974.1888	6944.25
0.0928	7223.267	7295.498
0.0945	7472.3451	7534.396
0.097	7721.4233	7899.841

0.0988	7970.5015	8148.202
0.1005	8219.5796	8400.349
0.103	8468.6578	8785.665
0.1047	8717.736	9047.274
0.1064	8966.8142	9312.668
0.1081	9215.8923	9581.847
0.1098	9464.9705	9854.812
0.1115	9714.0487	10131.56

Table B.6 Calculation of pressure drop for black marbles diameter of 0.1.9 cm, pore diameter of 0.86 cm, tortuosity 1.4169, bed porosity of 0.4047, packing height of 67.3cm and bed diameter of 14.606cm [59]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.042	892.57	573.728
0.045	966.57	652.896
0.048	1040.6	737.155
0.0528	1157.8	882.561
0.0575	1275	1037.57
0.0623	1392.3	1208.77
0.067	1509.5	1389.04
0.07	1587.8	1510.64
0.073	1666	1637.33

Table B.7 Calculation of pressure drop for marble diameter of 1.27 cm, pore diameter of 0.57 cm, tortuosity 1.4183, bed porosity of 0.4, packing height of 46.99 cm and bed diameter of 8.89 cm [4]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0131	74.658	87.8937
0.0210	223.974	191.557
0.0288	447.948	333.44
0.0367	696.808	513.541
0.0446	920.782	731.861
0.0524	1244.3	988.4

0.0603	1642.476	1283.16
0.0681	1891.336	1616.13
0.0760	2239.74	1987.33
0.0839	2787.232	2396.74
0.0917	3409.382	2844.37
0.0996	3807.558	3330.22

Table B.8 Calculation of pressure drop for spherical particle diameter of 1.27 cm, pore diameter of 0.57 cm, tortuosity of 1.4172, bed porosity of 0.4037, bed diameter of 8.883 cm and packing height of 45.72 cm [3]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0087	40.6747	45.6247
0.0127	81.3493	81.5343
0.0167	122.024	126.803
0.0207	183.036	181.43
0.0247	244.048	245.417
0.0287	325.397	318.762
0.0328	427.084	401.466
0.0368	528.771	493.53
0.0408	650.794	594.952
0.0456	772.818	729.012
0.052	996.529	928.724
0.0544	1098.22	1009.79
0.060	1260.91	1212.06
0.0649	1606.65	1400.02

B.1.2 Binary sized spherical particles system

Table B.9 Calculation of pressure drop for Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm, and dp_{eff}=1.016 cm), fractions of (x_1=0.25, x_2=0.75), pore diameter of 0.39 cm, tortuosity of 1.4037, bed porosity of 0.367, packing height of 49.53cm and bed diameter of 8cm [60]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0087	99.544	195.9456
0.0111	149.316	257.3869
0.0159	273.746	389.8529
0.0191	373.29	485.2625
0.0239	522.606	639.0253
0.0287	746.58	805.566
0.0324	920.782	942.6623
0.0352	1094.98	1051.458
0.0384	1294.07	1181.12
0.0439	1617.59	1417.245
0.0488	1965.99	1641.742
0.0540	2289.51	1894.548
0.0584	2662.80	2120.173
0.0617	2886.78	2296.439
0.0681	3409.38	2655.502

Table B.10 Calculation of pressure drop for Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm, and dp_{eff}=0.7257cm), with fractions of (x_1=0.75, x_2 =0.25), pore diameter of 0.27 cm, tortuosity of 1.4054, bed porosity of 0.3612, packing height of 50.8cm, bed diameter of 8 cm [62]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0095	248.86	457.9945
0.0143	497.72	718.5425
0.0183	721.69	950.6095

0.0247	1020.3	1350.174
0.0295	1393.6	1672.671
0.0344	1692.2	2022.064
0.0408	2314.4	2509.117
0.0456	2687.7	2897.23
0.0488	3235.2	3166.841
0.0552	3633.4	3732.145
0.0625	4907.1	4419.403
0.0673	5404.8	4895.956

Table B.11 Calculation of pressure drop for Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm with dp_{eff}=1.1545 cm), fraction of (x_1=0.1, x_2=0.9), pore diameter of 0.45 cm, tortuosity of 1.4025, bed porosity of 0.3709, packing height of 40.64cm and bed diameter of 8.001cm [7]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0087	84.658	119.3389
0.0107	116.99	150.8795
0.0127	149.32	183.9552
0.0151	199.09	225.6724
0.0175	248.86	269.6001
0.0191	286.19	300.1133
0.0207	323.52	331.609
0.0231	385.73	380.6947
0.0255	447.95	431.9909
0.0283	547.49	494.6303
0.0311	647.04	560.2786
0.0352	796.35	661.8348
0.0372	871.01	713.7155
0.0392	945.67	767.1313
0.0416	1045.2	833.2565
0.044	1144.8	901.5923

Table B.12 Calculation of pressure drop for spherical particle diameter of (dp_1=0.635 cm, dp_2=1.27cm with dp_{eff}=0.907 cm), fraction of (x_1=0.4, x_2=0.6), pore diameter of 0.36 cm, tortuosity of 1.4024, bed porosity of 0.371308 and packing height of 48.26cm [3]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.01268	162.6986	338.6028
0.01911	325.3972	545.4736
0.02633	589.7825	806.6713
0.03436	955.8544	1132.243
0.04239	1362.601	1495.029
0.04961	1830.359	1853.353
0.05683	2338.793	2241.819
0.06245	2684.527	2564.799
0.06647	3070.936	2806.663

Table B.13 Calculation of pressure drop for spherical particle diameter of (dp_1=0.635 cm, dp_2=1.27cm with dp_{eff}=0.907 cm), fraction of (x_1=0.5, x_2=0.5), pore diameter of 0.31 cm, tortuosity of 1.4074, bed porosity of 0.354482 and packing height of 48.26cm [3]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.01028	198.1586	391.2661
0.0118	277.422	459.514
0.01349	396.3171	529.6843
0.01509	475.5806	601.777
0.0167	515.2123	675.7921
0.01911	554.844	790.4193
0.02071	594.4757	869.2403

Table B.14 Calculation of pressure drop for Acrylic balls of particles diameters ($dp1=0.636$ and $dp2=1.27$ cm, with $dp_{eff}=0.7065$ cm), fraction of ($x_1=0.8$, $x_2=0.2$), pore diameter of 0.27 cm, tortuosity of 1.4055, bed porosity of 0.3609, packing height of 48.26 cm and bed diameter of 8 cm [61]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.03	1500	1708.64
0.032	1650	1849.15
0.033	1686.5	1920.66
0.034	1767.4	1992.99
0.035	1885.6	2066.16
0.036	2003.8	2140.15
0.038	2122	2290.64
0.039	2240.2	2367.13
0.04	2358.4	2444.45
0.042	2476.6	2601.59
0.043	2594.8	2681.41
0.044	2713	2762.05
0.045	2843.8	2843.53
0.046	2974.6	2925.84
0.048	3105.3	3092.95
0.049	3236.1	3177.76

Table B.15 Calculation of pressure drop for Acrylic balls of particles diameters ($dp1=0.636$ and $dp2=1.27$ cm, with $dp_{eff}=0.9071$ cm), fraction of ($x_1=0.4$, $x_2=0.6$), pore diameter of 0.35cm, tortuosity of 1.4044, bed porosity of 0.3645, packing height of 48.26 cm and bed diameter of 8 cm [61]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.013	243.98	384.5322
0.016	371.08	488.2017
0.019	498.18	597.4705
0.023	659.94	751.8722
0.026	821.69	874.2059
0.03	1045.7	1046.028

0.034	1269.6	1227.803
0.038	1518.5	1419.533
0.042	1767.4	1621.218
0.046	2018.6	1832.856
0.05	2269.7	2054.449
0.053	2565.8	2227.176
0.057	2861.9	2466.188
0.06	3073.4	2651.98
0.062	3285	2778.952
0.064	3521.4	2908.413
0.066	3757.8	3040.362

Table B.16 Calculation of pressure drop for glass of particles diameters (dp1=0.7955 and dp2=0.509 cm, with dp_{eff}=0.6208 cm), fraction of (x_1=0.5, x_2=0.5), pore diameter of 0.25cm, tortuosity of 1.3999, bed porosity of 0.38, packing height of 15.15 cm and bed diameter of 8 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0303	497.4584	521.6059
0.0606	815.603	1264.677
0.0909	1631.21	2229.214
0.1211	2836.88	3410.938
0.1511	4255.32	4802.662
0.1817	5957.45	6445.878
0.2121	7943.26	8302.017
0.2424	10212.8	10373.88
0.2726	12624.1	12659.28
0.303	15319.1	15182.01

B.1.3 Ternary sized spherical particles system

Table B.17 Calculation of pressure drop for glass spherical particles diameter of (0.9987, 0.7955, 0.6015cm, and dp_{eff}=0.765 cm), pore diameter of 0.36 cm, tortuosity of 1.4131, bed porosity of 0.4111, packing height of 15.15cm and bed diameter of 7.62cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0303	205.674	241.685
0.0606	638.298	803.307
0.0909	1489.36	1684.87
0.1211	2482.27	2881.87
0.1511	3900.71	4385.63
0.1817	5673.76	6242.56
0.2121	7234.04	8410.47
0.2424	9716.31	10891.7
0.2726	11702.1	13683.1
0.303	14184.4	16814

Table B.18 Calculation of pressure drop for glass of particles diameter (0.9987, 0.6015, 0.421 cm, with dp_{eff}=0.595 cm, pore diameter of 0.25 cm, tortuosity of 1.4475, bed porosity of 0.3835, packing height of 15.15 cm, bed diameter of 7.62 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0303	390.071	516.9208
0.0606	1418.44	1634.839
0.0909	2695.04	3353.755
0.1211	4680.85	5665.024
0.1511	7092.2	8552.106
0.1817	9574.47	12103.88
0.2121	13404.3	16239.4
0.2424	16453.9	20963.3
0.2726	20212.8	26269.64
0.303	25177.3	32214.1

Table B.19 Calculation of pressure drop for glass of particles diameter (0.9987, 0.509 and 0.421cm, with dp_{eff}=0.562 cm, pore diameter of 0.23 cm, tortuosity of 1.4549, bed porosity of 0.3777, packing height of 15.15 cm, bed diameter of 7.62 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0303	453.901	613.976
0.0606	1489.36	1918.56
0.0909	3191.49	3913.751
0.1211	5673.76	6589.55
0.1511	8510.64	9926.885
0.1817	12056.7	14028.41
0.2121	16312.1	18800.59
0.2424	20567.4	24248.82
0.2726	25673.8	30366.26
0.303	30851.1	37217.1

Table B.20 Calculation of pressure drop for spherical particles diameters of (0.42, 0.61 and 0.79cm, with dp_{eff}=0.5627 cm, pore diameter of 0.23 cm, tortuosity of 1.4569, bed porosity of 0.3762, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0305	767	836.9819
0.0609	2596	2609.087
0.0914	5564	5327.975
0.1218	9520	8975.816
0.1523	14465	13576.61
0.1827	20400	19100.19
0.2132	27323	25582.88
0.2436	34989	32982.2
0.2741	43767	41346.8
0.3046	52792	50653.9

Table B.21 Calculation of pressure drop for spherical particles diameter of (0.61, 0.79 and 1.01cm, with dp_{eff}=0.535cm), pore diameter of 0.21 cm, tortuosity of 1.4630, bed porosity of 0.3715, packing height of 20cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0305	890	970.0841
0.0609	2967	2991.975
0.0914	6429	6078.975
0.1218	11003	10210.84
0.1523	16443	15414.76
0.1827	23120	21656.6
0.2132	30909	28977.43
0.2436	39687	37329.24
0.2741	50319	46766.99
0.3046	61076	57264.93

Table B.22 Calculation of pressure drop for spherical particles diameter of (0.51, 0.61 and 1.01 cm, with dp_{eff}=0.6165 cm), pore diameter of 0.26 cm, tortuosity of 1.4451, bed porosity of 0.3854, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0305	606	636.8543
0.0609	2102	2023.364
0.0914	4698	4168.651
0.1218	8036	7058.647
0.1523	12611	10712.37
0.1827	17309	15105.85
0.2132	22996	20268
0.2436	29796	26164.97
0.2741	37090	32835.56
0.3046	45127	40261.6

Table B.23 Calculation of pressure drop for glass spherical particles diameter of (0.9987, 0.7955 and 0.509, with dp_{eff}=0.71 cm), pore diameter of 0.32cm, tortuosity of 1.4239, bed porosity of 0.4023, packing height of 15.15 cm and bed diameter of 7.62 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0303	283.688	304.1342
0.0606	992.908	996.5844
0.0909	2127.66	2077.351
0.1211	3546.1	3540.946
0.1511	5106.38	5376.781
0.1817	7730.5	7641.495
0.2121	9929.08	10283.58
0.2424	13120.6	13305.92
0.2726	15248.2	16704.69
0.303	18439.7	20515.56

Table B.24 Calculation of pressure drop for spherical particles diameter of (0.51, 0.79 and 1.01, with dp_{eff}=0.6536 cm), pore diameter of 0.28 cm, tortuosity of 1.4373, bed porosity of 0.3915, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0305	495	534.6457
0.0609	1855	1718.831
0.0914	3956	3560.346
0.1218	6553	6047.117
0.1523	10385	9195.502
0.1827	14218	12984.86
0.2132	19163	17440.11
0.2436	24356	22532.05
0.2741	31156	28294.18
0.3046	38080	34710.81

Table B.25 Calculation of pressure drop for spherical particles diameter of (0.42, 0.51 and 0.509, with dp_{eff}=0.5061 cm), pore diameter of 0.19 cm, tortuosity of 1.4708, bed porosity of 0.3655, packing height of 20cm and bed diameter of 7.62cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0305	1014	1158.955
0.0609	3338	3530.265
0.0914	7047	7129.532
0.1218	11745	11933.15
0.1523	17680	17972.73
0.1827	24480	25208.66
0.2132	32269	33688.55
0.2436	41665	43356.8
0.2741	52050	54277
0.3046	62807	66419.36

Table B.26 Calculation of pressure drop for spherical particles diameter of (0.9987, 0.7955 and 0.421, with dp_{eff}=0.647 cm), pore diameter of 0.28 cm, tortuosity of 1.4366, bed porosity of 0.3921, packing height of 15.15 cm and bed diameter of 7.62 cm [56]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0303	390.071	402.7382
0.0606	1418.44	1295.688
0.0909	2836.88	2678.849
0.1211	4964.54	4545.232
0.1511	7446.81	6881.409
0.1817	9929.08	9759.375
0.2121	13829.8	13113.61
0.2424	17730.5	16947.82
0.2726	21631.2	21257.18
0.303	25886.5	26086.89

B.1.4 Quaternary sized spherical particles system

Table B.27 Calculation of pressure drop for spherical particles diameter of (0.42, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.6373 cm), pore diameter of 0.26 cm, tortuosity of 1.4808, bed porosity of 0.3747, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0305	631	498.4059
0.0609	2164	1937.612
0.0914	4574	4327.088
0.1218	7913	7651.164
0.1523	12116	11931.71
0.1827	16814	17140.65
0.2132	22625	23312.27
0.2436	29301	30406.08
0.2741	36596	38468.77
0.3046	44632	47478.54

Table B.28 Calculation of pressure drop for spherical particles diameter of (0.42, 0.51, 0.79 and 1.01 cm, with dp_{eff}=0.6063 cm), pore diameter of 0.24 cm, tortuosity of 1.4826, bed porosity of 0.3733, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0305	767	539.354
0.0609	2596	2093.695
0.0914	5564	4673.249
0.1218	9520	8261.101
0.1523	14589	12880.85
0.1827	20152	18502.22
0.2132	27447	25162.17
0.2436	35236	32817.04
0.2741	43890	41517.2
0.3046	53410	51239.13

Table B.29 Calculation of pressure drop for spherical particles diameter of (0.42, 0.51, 0.61 and 0.79 cm with dp_{eff}=0.5519 cm), pore diameter of 0.23 cm, tortuosity of 1.4848, bed porosity of 0.3719, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0305	729	587.6626
0.0609	2473	2277.263
0.0914	5193	5079.918
0.1218	9025	8977.248
0.1523	13600	13994.89
0.1827	18916	20099.95
0.2132	25592	27332.59
0.2436	33134	35645.38
0.2741	40923	45093.01
0.3046	49948	55649.95

Table B.30 Calculation of pressure drop for glass spherical diameter of (0.42, 0.51, 0.61 and 0.79, with dp_{eff}=0.55 cm), pore diameter of 0.22 cm, tortuosity of 1.4861, bed porosity of 0.3711, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0305	791	625.8377
0.0609	2658	2421.779
0.0914	5564	5399.639
0.1218	9644	9539.891
0.1523	14589	14869.77
0.1827	20400	21354.34
0.2132	27941	29036.24
0.2436	35483	37865.11
0.2741	44879	47899.04
0.3046	54152	59110.91

B.1.5 Quinary Sized Spherical Particles System

Table B.31 Calculation of pressure drop for spherical particles diameter of (0.42, 0.51,0.61,0.79 and 1.01 cm, with dp_{eff}=0.61 cm), pore diameter of 0.24 cm, tortuosity of 1.4956, bed porosity of 0.3623, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0305	767	967.969
0.0609	2596	2868.544
0.0914	5687	5714.229
0.1218	9644	9486.363
0.1523	14465	14209.76
0.1827	19905	19853.46
0.2132	26334	26454.58
0.2436	34000	33969.83
0.2741	42159	42448.66
0.3046	52050	51867.89

B.2 Results of General Equation for Different Sizes of Packing

Table B.32 Calculation of pressure drop for spherical particles diameter of 0.635 cm, pore diameter of 0.24 cm, tortuosity of 1.4422, bed porosity of 0.360902 and packing height of 45.72cm [3]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0087	223.7106	235.1505
0.0127	467.75852	412.8103
0.0167	650.79446	634.0107
0.0207	793.15575	898.7517
0.0247	1098.2157	1207.033
0.0287	1260.9143	1558.856
0.0328	1443.9502	1954.218
0.0448	2338.7926	3401.551
0.0528	2928.5751	4584.142
0.0568	3416.6709	5240.748
0.0608	3660.7189	5940.895
0.0649	4189.4894	6684.583

Table B.33 Calculation of pressure drop for Acrylic balls of particles diameter (dp_1=0.655cm, dp_2=1.27cm, and dp_{eff}=0.7065 cm) with fraction of (x_1=0.8, x_2=0.2), pore diameter of 0.27 cm, tortuosity of 1.4422, bed porosity of 0.3609, packing height of 48.26 cm and bed diameter of 8 cm [61]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.03	1500	1547.881
0.032	1650	1733.12
0.033	1686.5	1829.586
0.034	1767.4	1928.615
0.035	1885.6	2030.209
0.036	2003.8	2134.366
0.038	2122	2350.373
0.039	2240.2	2462.222
0.04	2358.4	2576.636

0.042	2476.6	2813.154
0.043	2594.8	2935.259
0.044	2713	3059.928
0.045	2843.8	3187.161
0.046	2974.6	3316.959
0.048	3105.3	3584.245
0.049	3236.1	3721.734

Table B.34 Calculation of pressure drop for spherical particles diameter of (0.51, 0.79 and 1.01 cm, with dp_{eff}=0.6536 cm), pore diameter of 0.28 cm, tortuosity of 1.4350, bed porosity of 0.3915, packing height of 20 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0305	495	511.9452
0.0609	1855	1794.142
0.0914	3956	3855.027
0.1218	6553	6681.085
0.1523	10385	10290.91
0.1827	14218	14660.83
0.2132	19163	19819.59
0.2436	24356	25733.37
0.2741	31156	32441.08
0.3046	38080	39924.52

Table B.35 Calculation of pressure drop for spherical particles diameter of (0.42, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.6373 cm), pore diameter of 0.25 cm, tortuosity of 1.4389, bed porosity of 0.3747, packing height of 15.15 cm, bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (present work)
0.0305	631	483.9075

0.0609	2164	1678.92
0.0914	4574	3592.898
0.1218	7913	6213.293
0.1523	12116	9557.343
0.1827	16814	13603.12
0.2132	22625	18377.24
0.2436	29301	23848.4
0.2741	36596	30052.59
0.3046	44632	36973

Table B.36 Calculation of pressure drop for spherical particles diameter of (0.42, 0.51, 0.61, 0.79 and 1.01 cm, with dp_{eff}=0.61 cm), pore diameter of 0.23 cm, tortuosity of 1.4418, bed porosity of 0.3623, packing height of 15.15 cm and bed diameter of 7.62 cm [57]

U (m/s)	ΔP(Pa) (experiments)	ΔP(Pa) (presents work)
0.0305	767	592.2992
0.0609	2596	2032.647
0.0914	5687	4330.52
0.1218	9644	7470.849
0.1523	14465	11474.3
0.1827	19905	16314.61
0.2132	26334	22023.63
0.2436	34000	28563.92
0.2741	42159	35978.51
0.3046	52050	44247.29

الخلاصة

تـم صـياغة معـادلات شـبه عمليـة لجريـان الموائـع خـلال عمـود حـشوي بالاعتمـاد علـى التركيـب الاحصائي للنتائج العمليه. تم استخدام نوعان من الموائع (الماء و الهواء) بشكل منفصل في كل مرة (جريان طور واحد). تم استخدم عده حجوم من الحشوات، وتم دراسة كل واحد منها بشكل منفصل.

تم دراسة العوامل المختلفة التي تؤثر على هبوط الضغط عند جريان الموائع في عمود حشوي كل على حده، هذه العوامل هي سرعة جريان الموائع، مسامية الحشوة، قطر العمود الحشوي،قطر الفتحه في العمود الحشوي، طول الحشوة في العمود الحشوي، الالتوائيه في العمود الحشوي و طول الحشوة في العمود الحشوي.

تـم افتـراض معادلـة شـبه عمليـة لجريـان الموائـع لكـل نـوع محـدد مـن الحـشوات المـستخدمة وتـسمى بالمعادلـة المنفـردة (احاديـه، ثنائيـه، ثلاثيـة، رباعيـة، خماسـية و حـشوة متعـددةالحجوم). وقـد كتبـت عـشرة معادلات احادية، خمسه منها لجريان الماء وخمسة منها لجريان الهواء خلال العمود الحشوي.

كمـا تم افتـراض معادلـة عامـة لجريـان الموائـع خـلال العمـود الحـشوي تـصلح لكافـة انـواع الحـشوات وهي:

1. لجريان الهواءخلال العمود الحشوي

$$\frac{\Delta P}{L} = -5.47872\frac{\tau(\tau-1)^2}{\varepsilon^2}\ \frac{\mu u}{d_{pore}^2} + 2.7267\frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)}\frac{\rho u^2}{d_{pore}}$$

بنسبة الخطأ 13.5742%.

2. لجريان الماءخلال العمود الحشوي

$$\frac{\Delta P}{L} = 108.3983\frac{\tau(\tau-1)^2}{\varepsilon^2}\ \frac{\mu u}{d_{pore}^2} + 2.1188\frac{\tau(\tau-1)}{\varepsilon^2(3-\tau)}\frac{\rho u^2}{d_{pore}}$$

بنسبة الخطأ 12.9576%.

تم اقتراح صيغه التجريبية لحساب قطر الفتحه وقد استخدمت لكافـة معـادلات الجريـان، وتـم مقارنـه نتائج الحسابات لهذه الطريقه مع نتائج التجارب الماخوذه من الجداول الموثقه وكانت المعادلـه العامـه لحساب قطر الفتحه للماء والهواء هي:

$$d_{pore} = 1.9612\ \ d_p\ \varepsilon^{1.61605}$$

تم ايجاد نسبة الخطأ لجريان الهواء 0.3897% ولجريان الماء 0.328%.

شكر وتقدير

في البداية اشكر الله عز وجل الذي وفقني لاكمال متطلبات هذا البحث. وانا انهي بحثي لا يسعني واعترافا بالفضل الا ان اتقدم بوافر الشكر والامتنان للاستاذ المشرف الدكتور محمد نصيف لطيف لاقتراحه موضوع الرسالة واشرافه عليها ولمواصلته ومتابعته العلمية للبحث وما ترتب على ذلك من توجيهات قيمه واراء سديدة.

كما اتقدم بجزيل الشكر الى رئيس قسم الهندسة الكيمياوية، و جميع اساتذة قسم الهندسة الكيمياوية لمساعدتهم القيمة لي طيلة فترة الدراسة ولمدهم يد العون لي خلال اعداد هذه الرسالة.

واتقدم بشكري وامتناني الى عمادة كليه الهندسه في جامعة النهرين،لمساعدتهم ودعمهم الدائم لي طيلة فترة الدراسة.

كما اتقدم بالشكر الجزيل الى كافة العاملين في المكتبة المركزية بجامعة النهرين، وذلك لمساعدتهم لي في الحصول على بعض المصادر المستعملة في البحث.

وشكري الجزيل الى جميع زملائي و زميلاتي الذين مدو يد العون عند حاجتي اليها في البحث.

ولا انسى ان اتقدم بجزيل الشكر والتقدير الى من ساندني وساعدني على تخطي الصعوبات خلال فترة البحث الى الذين لا مثيل لهم في الدنيا الى أبي وأمي الأعزاء، أخوتي وأخواتي.

مروة ناظم عباس

حساب انخفاض الضغط خلال العمود الحشوي

لنظام الماء والهواء

رسالة

مقدمة إلى كلية الهندسة في جامعة نهرين

وهي جزء من متطلبات نيل درجة ماجستير علوم

في الهندسة الكيمياوية

من قبل

مروة ناظم عباس

(بكالوريوس علوم في الهندسة الكيمياوية ٢٠٠٥)

ربيع الاول ١٤٢٩

اذار ٢٠٠٨