THE THEORETICAL INVESTIGATION OF DRAG COEFFICIENT AND SETTLING VELOCITY CORRELATIONS

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Submitted to the College of Engineering of Nahrain University in Partial Fulfillment of the Requirements for the Degree of Master of Science in Chemical Engineering

by
Henaa Kamel Khalaf
(B.Sc. in Chemical Engineering 2005)

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I certify that this thesis entitled " **THE THEORETICAL INVESTIGATION OF DRAG COEFFICIENT AND SETTLING VELOCITY CORRELATIONS**" was prepared by **"Henaa Kamel Khalaf"** under my supervision at Nahrain University / College of Engineering in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering.

Signature :

Name : **Assist. Prof. Dr. Muhannad A. R. Mohammed**

(Supervisor)

Date : / /

Signature :

Name : **Prof. Dr. Qasim J. Slaiman**

(Head of Department)

Date : 19 / 11 / 2009

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Signature:
Name: **Assist. Prof. Dr. Muhannad A. R. Mohammed**
(Supervisor)
Date: / /

Signature:
Name: **Assist. Prof. Basim O. Hasan**
(Member)
Date: 5/11/2009

Signature:
Name: **Assist. Prof. Malek M. Mohammed**
(Member)
Date: / /

Signature:
Name: **Assist. Prof. Majid I. Abdulwahab**
(Chairman)
Date: 15/11/2009

Approval of the College of Engineering

Signature: M. J. Jweeg
Name: Prof. Dr. Muhsin J. Jweeg
(Dean)
Date: 24/11/2009

Abstract

The aim of this research is to find general empirical equations for predicting drag coefficient and settling velocity (for polymeric system of Car boxy Methyl cellulose CMC and Hydroxyl Ethel cellulose HEC as non-Newtonian fluids and water as Newtonian fluid), and to compare these equations with the other equations in literatures to show the accuracy of the developed equations.

Empirical equation was found to relate drag coefficient C_d and Reynolds number Re_p for a wide range of Re_p from 0.001 to 10000 and for spherical and irregular shaped particles for Newtonian and non-Newtonian fluids as follows: $C_d = \frac{24.00987}{Re_p} + \frac{54.4537}{\exp(4.629538\psi)}$

Where; ψ is the sphericity, which is the ratio of the surface area of sphere having the same volume of the particle to the surface area of the particle.

Settling velocity correlations were obtained for spherical and irregular shaped particles and for Newtonian and non- Newtonian fluids. These correlations consider size, surface condition, density of fluid, and density of particles as follows:

a- for spherical particles

$$v_s = 12.0049 \frac{\mu_{eq}}{\rho_f d_p} \sqrt{1 + 17.12 \frac{(\rho_p - \rho_f)}{\rho_f (d_p \rho_f / \mu_{eq})^2}} - 1$$

$$v_s(t) = 49.7 \sqrt{\frac{d_p(\rho_p - \rho_f)}{\rho_f}} \qquad \text{(for turbulent regime)}$$

for irregular shaped particles

$$V_s = 86.4(\mu_{eq}/ \rho_f d_p) [(1+0.126 d_p (\rho_p-\rho_f)/ \rho_f (d_p \rho_f / \mu_{eq})^2)^{0.5}\_1]$$

$$V_s (t) = 30.7[dp (\rho_p-\rho_f)/ \rho_f]^{0.5} \quad \text{(for turbulent regime).}$$

Comparison between the present C_d-Re_p correlation and the available correlations in the literatures showed that the present equation was the most applicable for all flow slip regimes and it was close to the experimental data. Graphs were plotted to show the effect of the factors which affect settling velocity such as diameter of particles, density difference between the particle and the fluid. Comparison of settling velocity correlation with other available in literature was used. The comparison showed that the present correlation is in good agreement with previous works.

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Notations

Variable Notations

Symbols		Notations	Unit
A_P	=	Projected area of the particle in a plane perpendicular to the direction of the flow	m^2
C_d		Particle drag coefficient	dimensionless
D		Inside pipe diameter.	m
D_P		Particle diameter.	m
D_S		Dimensionless diameter.	dimensionless
F		Force.	N
F_B		Bouncy force.	N
F_D		Drag force.	N
F_G		Gravity force.	N
g		Acceleration due to gravity	m/s^2
k		Power-Law consistency index, Shape factor	$kg.s^n /100m^2$
k		Constant volume.	m^3
L		Length.	m
n		Power-Law flow behavior index.	dimensionless
Re_P		Particle Reynolds' number.	dimensionless
u *		Dimensionless velocity.	dimensionless
V		Solid particle volume.	m^3
V		velocity.	m/s

V_S	Settling velocity.	m/s

Abbreviations

HEC	Hydroxy Ethyl Cellulose.
CMC	Carboxy Meyhyl Cellulose.
cp	Centipoise.
ppg	Pound per gallon.

Greek Letters

μ	Newtonian fluid viscosity	p
μ_a	Apparent viscosity	p
μ_e	Effective viscosity	p
μ_{eq}	Equivalent viscosity	p
ρ	Density	kg/m^3
ρ_P	Density of particle	kg/m^3
ρ_f	Density of fluid	kg/m^3
Φ	Speed	dimensionless
ψ	Sphericity	dimensionless
τ_p	Shear stress on the particle	Pa
γ_p	Shear rate of the particle	s^{-1}

Subscripts

F	Fluid
P	particle
S	Specific, settling
eq	Equivalent

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Chapter One

Introduction

In settling process the particles are separated from the fluid by gravitational forces acting on them. Applications of settling include removal of solids from liquids sewage wastes, settling of crystals from the mother liquor, separation of liquid-liquid mixture from a solvent-extraction stage in a settler, and settling of slurry from a soybean leaching process. The particles can be solid particles or liquid drops. The fluid can be a liquid or gas and it may be at rest or in motion.

In some processes of settling and sedimentation the purpose is to remove the particles from the fluid stream so that the fluid is free of particle contaminants. In other processes the particles are recovered as the product, as in recovery of the dispersed phase in liquid-liquid extraction. In some cases the particles are suspended in fluids so that the particles can be separated into fractions differing in size or density.

When a particle is at a sufficient distance from the walls of the container and from other particles so that its fall is not effected by them, the process is called free settling. Interference is less than 1% if the ratio of the particle diameter to the container diameter is less than 1:200 or if the particle concentration is less than 0.2 vol% in the solution [1].

The problem of particle settling is found in many industrial processes. The extremes of interest are given by the examples of the paint industry, which is concerned with colloidally sized particles settling in highly viscous polymer fluids, and the oil drilling industry which is interested in particles of millimeter or centimeter size, settling in polymers or clay based fluids which can be easily and efficiently pumped [2].

The design of sluge settling, slurry pipelines and fluidized beds all require knowledge of settling velocity of solid in liquid. Hydraulic transport slurry system for coal transportation, thickeners, mineral processing, solid-liquid mixing, water waste processing, cement industries, drilling for oil and gas, geothermal drilling, all are typical examples of the applications of settling idea [3, 4].

Generally, there are many factors that affect solid particle settling velocity, which are; shape and size of the particle, density difference between the particle and the fluid and the rheological properties of the fluid [5].

There are two types of solid particles settle fluids: Isometric particles or regular shaped particles (which have perpendicular axes of symmetry, follow vertical paths, such as spheres, disks and cylinders), and non isometric particles or irregular shaped particles.

Most data available in the literature involves smooth spherical particles, or regularly shaped particles like disks or cylinders. Other studies have been done on irregularly shaped particles, which they do not have any geometrical shape, especially for settling these particles in non-Newtonian fluids [3, 6].

The shape of irregular particles can be expressed using different approaches like sphericity as the shape factor (Ψ), which is the ratio of the surface area of sphere having same volume of the particle to the surface area of the particle [6].

The aims of this research are:-

1- Develop an equation to calculate drag coefficient as a function of Reynolds number(for polymeric system of Car boxy Methyl cellulose CMC and Hydroxyl Ethel cellulose HEC as non-Newtonian fluids and water as Newtonian fluid), and study the factors which affect C_d-Re_p relationship such as particle diameter, fluid and particle densities, particle shape and surface condition.

2-Develop an equation to calculate settling velocity of regular and irregular particles.

For this purpose, experimental data were used to get empirical equations and compared with the other equations in literature to show the accuracy of the developed equations.

Chapter Two

Literature Survey and Theoretical Aspects

2.1 Fluid dynamics

Fluid dynamics is a branch of general mechanics that deals with the motions of practical in a surrounding fluid (liquid or gas).

According to this definition there are two cases:

1- The particle movement in a stagnant fluid.

2- The particle suspension in a moving fluid.

The basic theory of particle dynamic is to study the motion of a single particle in a given fluid, thus the particle is freely in its motion.

The motion of large number of particle in confined fluid are not freely in its movement, but, if the concentration of these particles are low, this motion can be approximated by a single particle motion. The motion of single particle is called "free settling", while the other motion is called "hindered settling"[6, 7].

2.2 Mechanics of particle motion through fluids

Many processing steps, especially mechanical separation, involve the movement of solid particles or liquid drops through a fluid.

The movement of particle through a fluid requires external force acting on the particle. This force may come from a density difference between the particle and the fluid or it may be the result of electric or magnetic fields.

Three forces act on a particle moving through a fluid: (1) the external force, gravitational or centrifugal; (2) the buoyant force, which acts parallel to the external force but in the opposite direction; and (3) the drag force, which appears whenever there is relative motion between the particle and the fluid, the drag force acts to oppose the motion and acts parallel to the direction of movements but the opposite direction.

In the general case, the direction of movement of the particle relative to the fluid may not be parallel to the direction of the external and buoyant forces, and the drag force then makes an angle with the other two. In this situation, which is called two-dimensional motion, the drag must be resolved into components, which complicates the treatment of particle mechanics [7].

2.3 Drag force on a body

There are two forces act on bodies in flow. Skin friction, the stress parallel to the body, where the no-slip condition applies and a form drag, the force due to the pressure (or normal stresses) on the body. The sum of both is the drag, the force that needs to be applied to keep the body moving at constant speed [8].

2.4 Force balance on settling particles

For a rigid particle moving in a fluid, there are three forces acting on the body as we denoted before: gravity acting downward, buoyant force acting upward, and resistance or drag force acting in opposite direction to the particle motion.

A particle of mass m (kg) falling at a velocity v (m/s) relative to the fluid will be considered. The solid particle has a density ρ_p (kg/m^3) solid and the liquid is ρ (kg/m^3). The buoyant force F_b in N on the particle is

$$Fb = \frac{m\,\rho\,g}{\rho_p} = V_p\,\rho\,g \qquad (2\text{-}1)$$

Where $\frac{m}{\rho_p}$ is the volume V_p in m^3 of the particle and g is the gravitational acceleration in m/s^2

The gravitational or external force F_g in N on the particle is

$$F_g = mg \tag{2-2}$$

The drag force F_D on a body in N may be derived from the fact that, like in flow of fluids, the drag force or frictional resistance in proportional to the velocity head $v^2/2$ of the fluid displaced by the moving body. This must be multiplied by the density of the fluid and by a significant area A, such as the projected area of the particle. This defined as:

$$F_D = C_D \frac{v^2}{2} \rho A \tag{2-3}$$

Where the drag coefficient C_D is the proportionality constant and is dimensionless.

By balancing the above forces the free settling velocity or terminal settling velocity v_t can be obtained.

$$v_t = \sqrt{\frac{2g(\rho_p-\rho)m}{A\,\rho_p C_D \rho}} \tag{2-4}$$

For spherical particle $m = \pi D_p^3 \rho_p / 6$ and $A = \pi D_p^2/4$. Substituting these into Eq. (3-6), the following equation was obtained [1]

$$v_t = (4g\, D_p(\rho_p - \rho_f)/3\, C_d\, \rho)^{0.5} \tag{2-5}$$

2.5 Drag Coefficient

Basically, the drag coefficient represents the fraction of the kinetic energy of the settling velocity that is used to overcome the drag force on the particle [9].

From the above definition of drag coefficient, it was obtained

$$C_d = (F_D/A_P)/(\rho v^2/2g) \tag{2-6}$$

2.5.1 Drag coefficient for sphere

Consider a smooth sphere immersed in a flowing fluid and at a distance from the solid boundary of the stream to be at a uniform velocity. Define a plane perpendicular to the direction of flow, as shown in fig 3-1. Denote the projected area by A_p . For a sphere, the projected area is that of a circle, or $(\pi/4)\ D_p^2$, where D_p is the diameter of the particle.

From dimensional analysis, the drag coefficient of a smooth solid in an incompressible fluid depends upon a Reynolds number and the necessary shape factors. For a given shape

$C_D = \O(N_{Rep})$ [7]

In the laminar flow region, called the Stokes law region for $N_{Re}<1$, the drag coefficient is

$$C_d = 24\ \mu / D_p\ \rho v \qquad (2\text{-}7)$$

Where μ is the viscosity of the fluid in P_a.s or kg/m.s (lbm/ft.s). Subsituting this into Eq.(2-7) for laminar flow,

$$v_t = \frac{g\ (\rho_p - \rho) Dp^2}{18\mu} \qquad (2\text{-}8)$$

For other shape of particles, drag coefficient differ from that given by equ.(2-7)[7].

In the turbulent Newton law region a Reynolds number of about 1000 to $2*10^5$, the drag coefficient is approximately constant at $C_D = 0.44$.

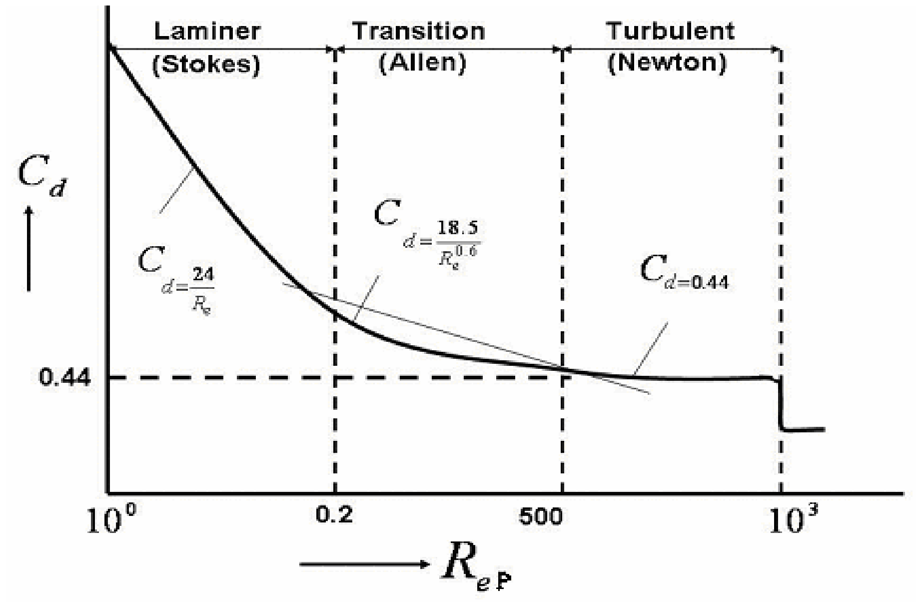

Fig 2.1 C_d-Re_p relationship [6]

2.5.2 Drag coefficient for non spherical particles

For particles having shapes other than spherical, it is necessary to specify the size and geometrical form of the body and its orientation with respect to the direction of the fluid. On major dimension is chosen as the characteristic length, and other important dimensions are given as ratios to the chosen one. Such ratios are called shape factor. Thus for short cylinders, the diameter is usually chosen as the defining dimension, and the ratio of length to diameter is a shape factor. The orientation between the particle and the stream also is specified. For a cylinder, the angle formed by the axis of the cylinder and the direction of fluid is sufficient. Then the projected area is determined and can be calculated. For cylinder so oriented that its axis is perpendicular to the flow, A_p is LD_p, where L is the length of the cylinder. For cylinder with its axis parallel to the direction of flow, A_p is $(\pi/4)\ D_p^2$, the same as for a sphere of the same diameter [7].

In fig 2.2 the drag coefficient for a spherical and other types of particles was plotted.

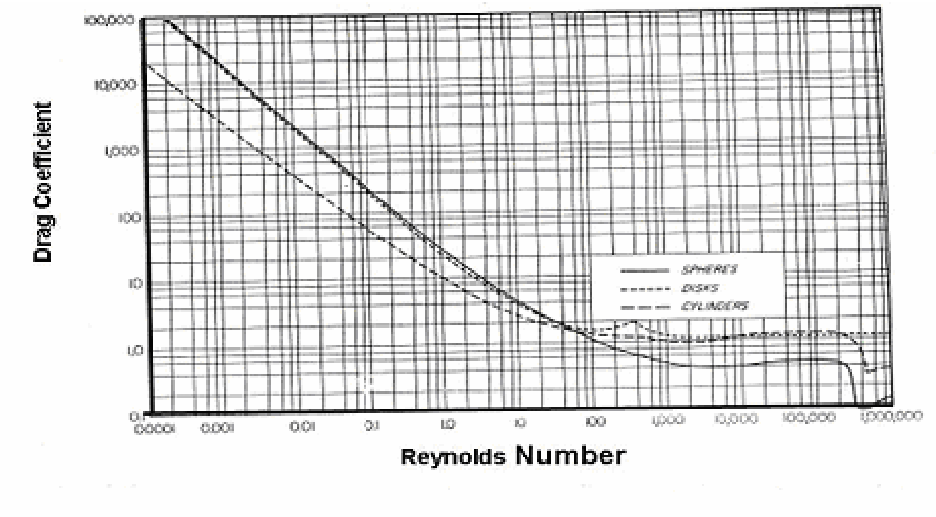

Fig 2.2 drag coefficient for different shapes of particles[5]

2.6 Particle Reynolds number R_{ep}

Generally Reynolds number is defined as the ratio of inertial forces to viscous forces. It is used to indicate whether the boundary layer around a particle is turbulent or laminar, and the drag exerted will depend on this [9, 10]. Particle Reynolds number is given by the following formula;

$$R_{ep} = \frac{\rho_f v_s D p}{\mu} \quad (2\text{-}9)$$

For Newtonian fluids the viscosity (μ) is constant, while the problem arises for non-Newtonian fluids, where the viscosity varies with shear rate and the duration of shear. Therefore, an expression of equivalent viscosity (μ_{eq}) can be used, which represents the viscosity of the fluid around the particle during its movement.

The equivalent viscosity is defined as the ratio of shear stress on the particle surface to the average shear rate of the particle [11, 12].

$$\mu_{eq} = 478.8(\tau_p / \gamma_p) \quad (2.10)$$

Where μ_{eq} is equivalent viscosity in centipoises. τ_p is the shear stress on the particle surface in P_a , γ_p is the shear rate of the particle in s^{-1}.

There are several equations that relate shear stress to shear rate for non-Newtonian fluids. According to that, there are several forms of equivalent viscosities, depending on the type of the non-Newtonian model. In this study Power-Law models equation is only used and the equivalent viscosity of this model as follows [1]:

$$\mu_{eq} = 478.8(0.6\ \frac{v_s}{D_p})^{n-1} \qquad (2\text{-}11)$$

Where, n is the flow behavior index.

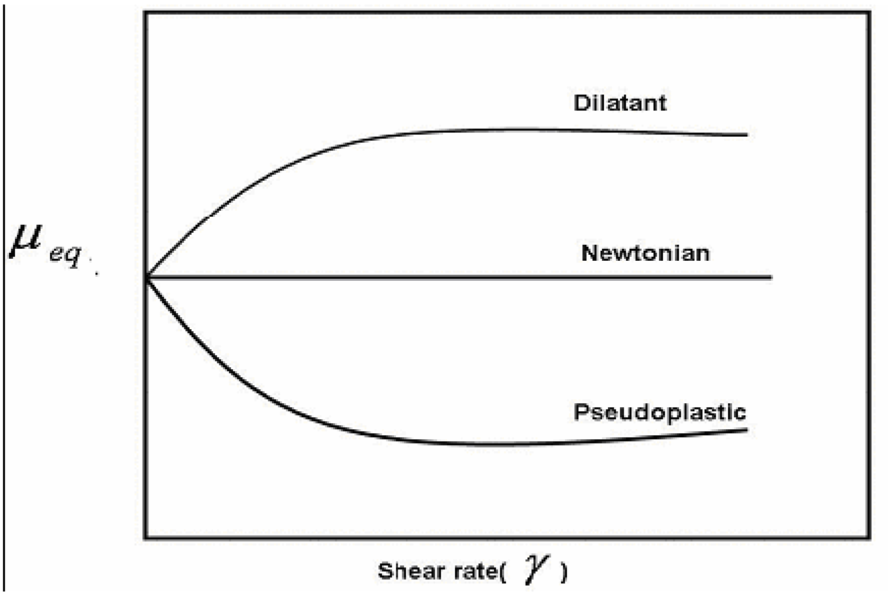

Fig (2.3) equivalent viscosity for Newtonian and non Newtonian fluid[13]

2.7 Flow regimes

Single-Phase flow can be either laminar, plug, or turbulent. It is usually important to know which of these flow regimes is present in a particular situation.

2.8 Flow curve and types of fluid

The relationship between the shear stress and shear rate in a real fluid are part of the science of rheology. A plot of shear stress versus shear rate is called "Flow Curve" or "rheogram".

Generally, fluids may be classified according to the observed flow curve or rheogram into:

2.8.1 Newtonian fluids:

A Newtonian fluid is defined by a straight line relationship between the shear stress (τ) to the shear rate (γ) with a slope equal to the viscosity of the fluid;

$$\tau = \mu \gamma \qquad (2\text{-}12)$$

In this type of fluid, viscosity is constant and is only influenced by changes in temperature and pressure, as shown in fig (2-4). Examples of Newtonian fluids include oil and water.

2.8.2 Non Newtonian fluids:

For many fluids a plot of shear stress (τ) against shear rate (γ) does not give a straight line. These are the so-called non-Newtonian fluids. Plots of τ against γ are experimentally determined using viscometer.

The term viscosity has no meaning for a non-Newtonian fluid unless it is related to a particular shear rate γ . An apparent viscosity μ_a can be defined as follows:

$$\mu_a = \frac{\tau}{\gamma} \qquad (2\text{-}13)$$

When the apparent viscosity μ_a decreases with an increase in shear rate γ the fluid is said to be pseudo plastic. When μ_a increases with an increase in γ the fluid is said to be dilatants.

Another type of non-Newtonian fluid is the Bingham Plastic. A plot of τ against γ on Cartesian coordinates for a Bingham Plastic is a straight line having an intercept τ_B on the shear stress axis called the yield stress. τ_B is the stress which must be exceeded before flow starts. The fluid at rest contains a three dimensional structure of sufficient rigidity to resist any stress less than the yield stress. When this stress is exceeded, the system behaves as a Newtonian fluid under a shear stress $\tau - \tau_B$. For Bingham Plastic, the slope of the shear stress, shear rate plot is called the coefficient of rigidity.

Pseudo plastic, dilatants and Bingham Plastic are example of time independent non-Newtonian fluid, i.e. the apparent viscosity depends only on the rate of shear at any particular moment and not on the time for which the shear rate is applied.

For certain class of fluids the apparent viscosity continues to change as a function of the time for which the particular shear rate is applied. These are known as time dependent non-Newtonian materials. Fluids which become more pseudo plastic with time at a constant shear rate are known as thixotropic fluids. There structure progressively breaks down with time at constant shear rate are known as thixotropic fluids. Their structure progressively breaks down with time at a constant shear rate [14]. Fluids which become more dilatants with time at a constant shear rate are known as rheopectic fluids. In general, the apparent viscosity of rheopectic fluids increases with time to a constant rate of shear. Most rheopectic fluids revert very quickly to their original viscosity if left to stand.

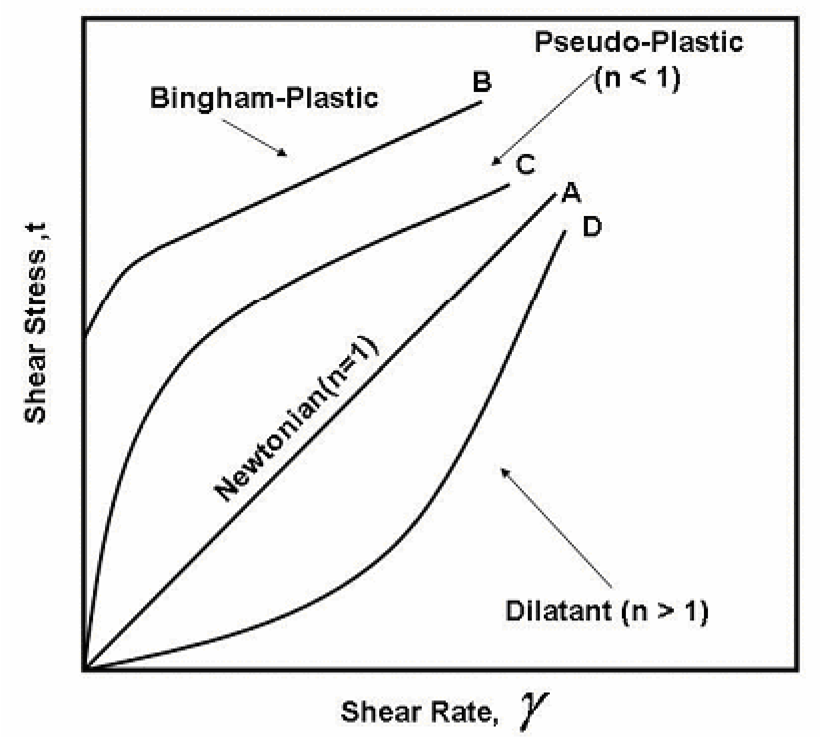

Figure 2.4 Shear stress vs. shear rate for Newtonian and non- Newtonian fluids [13].

2.9 Rheological- models for non- Newtonian fluids

There are many rheological correlations which describe shear stress shear rate relationship. All these relationships are empirical equations. Like Bingham-Plastic model, Power-Law model, Modified Power-Law model, Casson model, Robertson-Stiff model, Ellis equation, Reiner-Philippof equation, Modified Robertson-Stiff model and others. The most common model which is used in this study is Power-law model. The Power-law model was chosen for determining the rheological properties of non-Newtonian fluids used in this study due to fact that this model is more applicable and most widely used.

The flow curve of Power-Law model can be described by an empirical equation which is:

$$\tau = k\gamma^n \qquad (2\text{-}14)$$

A plot of **τ** vs. **γ** on log-log paper gives a straight line with a slop of n and intercept of k at **γ** =1.0, where n and k are rheological parameters of Power-Law fluid. The parameter n is the flow behavior index, which discusses the degree of non- Newtonian behavior. As n becomes far from unity, this means a greater non- Newtonian behavior. The parameter k is the consistency index, which is an indication of the thickness of the fluid. As k increases, this means thickness of fluid increases. The parameter n and k can be determined approximately using a Fann-VG reading as follows;

$$n = 3.32 \log \theta600/\theta300 \qquad (2\text{-}15)$$

$$k = \theta600/(1022)^n \qquad (2\text{-}16)$$

Where; θ_{600} =dial reading at 600 rpm, and

; θ_{300} = dial reading at 300 rpm.

2.10 C_d-Re_p relationship

Dimensional analysis of the general problem of particle motion under condition of equilibrium shows that there is unique relationship between two dimensionless groups, the particle Reynolds number and the drag coefficient [15].

The relationship between c_d and Re for a sphere can be divided into three regions; laminar, transition and turbulent. In laminar region; the drag coefficient has a linear relationship with particle Reynolds number, this means it is inversely proportional to particle Reynolds number, while in turbulent region, the drag coefficient is independent or slightly dependent on particle Reynolds number. This means that in laminar- slip regime, the main retarding forces of the liquid affect upon the falling particles, (or in the other words, the resistance to the particles falling in the liquid) are the viscous forces of the liquid, and the inertial forces are negligible. In turbulent regime, the resistance to the particle settling in the liquid is caused by the inertial forces of the liquid and the viscous forces are negligible. Between these two regions is the transition region, where the resistance to the particle settling in the liquid is caused by both viscous and inertial forces.

In general, as the particle Reynolds number increases, the drag coefficient will decrease, at very high Re, C_d will approach a small constant value.

2.11 Factors affect on C_d- Re_p relationship

There are many factors which affect the C_d-Re_p relationship such as; settling velocity, particle diameter, rheological properties of fluid, density of particle and fluid and the shape of particle.

2.11.1 Terminal settling velocity

Terminal settling velocity is the most important factor affecting C_d-Re_p relationship because it enters in the evaluation of these two quantities.

The terminal settling velocity of any solid particles depends mainly on various factors such as; particle diameter, density and rheological properties of fluid, and density and shape of solid particle [5].

Consider a solid particle falling from a rest in a stationary fluid under the action of gravity. At first, the particle will accelerate as it does in a vacuum, but unlike in a vacuum, its acceleration will be retarded due to friction with the surrounding fluid. As frictional force increases with the velocity, this force will eventually reaches a value equals to that of the gravitational force. From this point on, the two forces balanced and the particle continue to fall with constant velocity. Since this velocity is attained at the end of the acceleration period, it is called terminal settling velocity [16].

In practice, the acceleration is a very short duration, often of the order of a small fraction of second. It is therefore customary to ignore this period in all practical problems concerned with settling processes, and the terminal settling velocity then becomes the only important factor in this kind of problem. Its magnitude is closely related to the physical properties of the fluid and the particle.

The settling behavior at low Reynolds number is known as laminar slip and that of high Reynolds number as turbulent regime, between these two regions transition regime. In laminar region, the settling velocity is affected by viscosity and the rheology of fluid. While in the turbulent region, the settling velocity is affected by the density of the fluid and the surface characteristics of particle [9].

The irregularly shaped particles settle at lower velocity than do spherical particles because the symmetrical and geometrical shaped, in other words; decrease in sphericity and increase in projected area will increase the drag so they tend to orient and take different trajectories in a preferred direction during their fall, this preferred orientation is not generally predictable, depending on the position of their center of gravity relative to the center of force since these two centers must fall on the same line of direction of motion, also increase roughness of particle surface increase drag [10, 17].

Many investigators studied terminal settling velocities of particles in Newtonian fluids. Consider a spherical particle of density ρ_p, falling in a stationary fluid of density ρ_f under the action of gravity. The gravitational force acts on the particle even when it is at rest, and remains constant during the whole period of fall. Let d_p be the diameter of the particle, then it's volume is $v_p = 4/3\pi\, r_p^3$ (2-17)

and $[\rho_p(\pi\, d_p^3/6)]$ is it's mass. From Newton's second law of motion, using the absolute system of units

$$F_G = g\,(\pi D_P^3\, \rho_p/6 - \pi D_P^3\, \rho_f/6) \qquad (2\text{-}18)$$

Where g is the gravitational acceleration.

The last term of this equation represents the buoyancy effect. This effect may be ignored if the density of fluid is negligible or small compared with the density of solid particle, as in the case when the fluid is a gas. Otherwise equation (2-18) becomes

$$F_G = (\pi D_P^3/6)\,(\rho_p - \rho_f)\, g \qquad (2\text{-}19)$$

For particle settling at its terminal velocity the opposing forces are in a balance, so that by combining with equation of $F_D = 3\pi D_P \mu V$ (2-20)

Let v_s be the terminal settling velocity then substituting in the above equation for F_D and F_G from the equations (2-19) and (2-20) so:

$$Vs = g D_P^2(\rho_p - \rho_f)/18\mu \qquad (2\text{-}21)$$

This equation is restricted to the region where Stoke's law applied [16]. The assumptions made in the derivation of Stoke's low of settling velocity are:1- The particle must be spherical particles, smooth and rigid; there must be no slip between it and the liquid.

2. The particle must move as it would in a fluid of infinite extent.
3. The terminal velocity must have been reached.
4. The fluid must be homogeneous compared with size of particle.
5. The settling speed must be low so that only viscous forces are brought into Play.

In any actual situation the assumptions listed above will not usually be valid and correction factors are sometimes necessary [18].

Equ (2-21) is valid at Re<1 and for spherical particles in which;

$$C_d = 24/Re \qquad (2\text{-}22)$$

While for turbulent- slip regime (Re>800), Newton's equation is

$$V_s = 1.74[gd_p\,(\rho_p - \rho_f)/\,\rho_f]^{0.5} \qquad (2\text{-}23)$$

For region (1<Re<1000), Allen's equation is used;

$$Vs = 0.2[g(\rho_p - \rho_f)/\,\rho_f]^{0.72}[D_p^{1.18}/(\mu/\,\rho_f)^{0.45}] \qquad (2\text{-}24)$$

These equations applied only for spherical particles. For particles with nominal or equivalent diameter D_P [19],

$$V_S = 113.4[D_P(\rho_p - \rho_f)/\,\rho_f\,C_d]^{0.5} \qquad (2\text{-}25)$$

Chien[9] developed the following settling velocity equation which is depended on the type of the non-Newtonian fluid for irregular particle. It covers all slips regimes:

$$V_S = 120(\mu_e/D_p\,\rho_f)[(1+0.0727\,D_p((\rho_p - \rho_f)/\,\rho_f)(D_p\,\rho_f/\,\mu_e) - 1]^{0.5} \qquad (2\text{-}26)$$

μ_e = effective viscosity of Non-Newtonian fluid.

Chien suggested different effective viscosity values for each type of non-Newtonian fluids. The different values of effective viscosity of non-Newtonian fluids are due to the shear stress-shear rate relationship. Chien’s correlation considers size, surface condition, and density of the particle and density and viscosity of the fluid [9].

Benoit Camenen[20] presented a simple and general formula for the settling velocity taking into account the shape and roundness of particles.

$$W_s = (\nu/d)[(1/4(A/B)^{2/m} + (4d^3_*/3B)^{1/m} - (1/2)(A/B)^{1/m}]^m \qquad (2\text{-}27)$$

Where A, B, m =coefficients for the C_d equation.

d*=dimensionless particle diameter.

d=diameter of particle .

ν=kinematic viscosity of the fluid.

The basic principle for predicting settling velocity is by balancing the buoyancy with the drag force. Settling velocities of the particles can be determined experimentally by direct measurements of the velocity. Experimental estimation of settling velocities for irregularly shaped particles in non- Newtonian fluids so difficult due to the interactions effects of particles shapes and sizes, and the rheological properties of the non-Newtonian fluids on these velocities.

2.11.2 The particle shape factor

The particle shape has an important effect on C_d-Re_prelationship. This effect is introduced as sphericity which is the ratio of particle volume to the volume of a sphere having the same surface area having the same surface area as particle. It is found that as the sphericity of the particle becomes smaller (i.e, far from unity value for a sphere shape), the drag coefficient will increase, especially at high particle Reynolds numbers.

Many attempts have been made to show the effect of particles shape on C_d-Re_P relationship [9, 15, 21, 22, 23, and 24].

Heywood [24] used the projected diameter which is equal to the diameter of the circle having an area equal to the projected area of the particle when placed in the most stable position.

Zeidler[21] shows that the sphericity shape factor has a minor effect on C_d-Re_p relationship, since a single curve is drown for particles of various sphericities (which is less than one). While the spherical particles have a lower drag coefficient values than that of the other shape particles.

Chein [9] shows that the sphericity shape factor has an important effect on C_d-Re_p relationship, since many curves were drown for particles of various sphericities, while the spherical particles have a lower drag coefficient values than that of the other shapes.

2.11.3 The effect of rheological properties of fluids

Rheological properties include viscosity and flow behavior index of the fluid. The true general idea is that an increase in the viscosity of the fluid will decrease the settling velocity of the particles in the laminar flow around the particles. In turbulent flow, the viscosity has no effect on C_d-Re_p relationship, and due to that, the viscous forces will be insignificant while the inertial forces will be dominated.

The flow behaviors of non- Newtonian fluids obey different rheological models, which represent the shear stress-shear rate flow curve.

2.12 Treatment of the movement of particles through fluids

The problem of a sphere moving very slowly through a stationary fluid was first solved by Stokes in 1851.

Most practical applications of stokes flow involve determination of the settling velocity; i.e. the velocity with which small solid or liquid particles fall through a fluid such as air or water.

Solution of Stokes problem solved by application of Navier-Stokes equation for steady flow, by assuming the inertial term is negligible and the fluid is incompressible. So, Stock's law relates the force resisting motion on sphere which is exerted by fluid, generally referred to as drag force F_D, to the diameter of particle, its velocity V and of such physical properties of the surrounding fluid as its density and viscosity μ. The equation takes a form [24].

$$F_D=3\pi D_p \mu V \qquad (2\text{-}28)$$

2.13 Previous works

In 1948, Pettyjhon and Christiansen [25] conducted an experimental work on falling particles freely in a fluid under the effect of gravity, using isomeric particles and different Newtonian fluids having different densities. They concluded that the sphericity is a satisfactory criterion for the effect of particle shape on the resistance to motion of particles. As sphericity decreased, the drag coefficient was increased, also they stated that for viscous flow (Re_p<0.05) Stock's law can be applied, while for highly turbulent flow (Re_p=200 to 20000) Newton's law can be applied, and for intermediate range (Re_p=0.05 to 200) the plot of Cd=f (Re_p, Ψ) can be used.

In 1959, Becker [26] stated that the drag on oriented bodies (or particles) in motion through an infinite fluid is composed of a viscous drag and inertial drag, in which a quadratic drag formula was adopted. This drag is related to the fluid velocity and to properties of the fluid and the particle. According to that drag is related to particle Reynolds' number. Application of their drag formulation showed that inertial drag decreases with minimum dependence on shape and Reynolds' number.

The first attempt is made by Slattery and Bird in 1961[27], to understand the behavior of non-Newtonian fluid around particles. They used Ellis rheological model in their study, and they measured the drag coefficient of spheres moving through CMC solutions. Two dimensionless correlations for drag coefficient in terms of a modified particle Reynolds' number based on Ellis parameters have been adopted.

The first study on a modified Cd-Re_p relationship for sphere in Bingham plastic non-Newtonian fluid is conducted by Valentik and Whitmore [28] in 1965. They used a flocculated aqueous clay suspensions follow six densities with different flow parameters. These suspensions follow Bingham plastic model. Spheres of different diameters and densities were dropped into these suspensions in order to measure their settling velocities. Reynolds' number was modified to take into account the flow parameters of Bingham plastic model. They stated that the drag forces of a particle moving in a Bingham fluid are composed of force of falling in Newtonian fluid and a force to overcome the yield stress of Bingham plastic.

Plessis and Ansely in 1967[29] studied experimentally the settling characteristics of solid particles settling in clay suspensions of different concentrations, which followed Bingham plastic models. According to that, sand and glass particles with different sizes were used; in which diameters were determined using sieving analysis. They compared the Cd-Re_p relationship for these clay suspensions with that for water. They concluded that the settling characteristics of particles in clay-water slurries is different from those in water, and that the drag coefficient for particles is functional of a plasticity number in terms of the rheological properties.

Hottovy and Sylvestor in 1979[3], used different irregularly shaped particles of different diameters with density of 0.88gm/cc. These particles were dropped through a column of liquid of 0.5gm/cc density. They concluded from a plot of Cd-Re_p curve that for $Re_p<100$, the drag force acting on the irregularly shaped particles is similar to that acting on a sphere of comparable size, while for $Re_p>100$, the Cd-Re_p curve was deviated from standard trend. This deviation was due to the drag force on these particles is higher than on sphere of comparable size. They repeated their runs for three different temperatures, and concluded that the settling rate increased with increasing temperature. They stated that this increasing was due to the fact that the temperature reduced the viscosity of liquid and increased density difference between liquid and solid.

Dietrich in 1982[10] suggested a more straight forward way of estimating settling velocities. He noted that many of investigators are particularly concerned with natural particles and restricted the data used in his analysis to settling velocities for spheres and natural-like sediment particles. He proposed using other non dimensional numbers (W_*, which includes settling velocity, and D_*, which included grain diameter) for mapping out the relationship between settling velocity and grain size. The non-dimensional settling velocity, W_* is the ratio of the particle Reynolds number to the drag coefficient; $W_*= [(4/3)*(R_D/C_D)]$, while the non-dimensional grain size $D_*=3/4\ C_D R_D^2$. Dietrich replotted available data, and noted that natural sediments tend to vary more with respect to sphericity than they do with angularity.

Torrest in 1983[30] studied the settling behavior of different sizes of solid particles in non-Newtonian polymer solutions. He concluded that as polymer concentrations increased, velocity will decrease. Also, as particle size increases, settling velocity will increase.

Flemmer and Banks in 1986[31], presented a mathematical approximation to experimental data of the drag coefficient and particle Reynolds' number of a sphere in Newtonian fluids. A Newton-Raphson technique was used, in order to determine settling velocity from the knowledge of $Cd\text{-}Re_p^2$ and of particle diameter from knowledge of Re_p/Cd.

In the same year, Meyer [32] presented a modified correlation for particle Reynolds’ number and the drag coefficient for laminar, transition, and turbulent flow. He stated that the wall and concentrations effects have a large impact on hindering particle settling.

Also in the same year, Concha and Barrientos [33] developed empirical equations for describing settling velocity and drag coefficient of isomeric particles. These equations were developed by using corrections to the available equations of sphere. They consider different assumptions, base pressure, thickness, and the angle of separation of the boundary layer, depend on the shape of the particle and on the densities of the particle and the fluid.

In 1987 Dedegil [34] stated that the forces due to yield stress must be considered in calculating drag force on particles settled in non-Newtonian fluids, which obey Bingham-plastic flow model. He stated that the Reynolds' number must be calculated by means of the fully representive shear stress including the yield stress which can be traced back to that of Newtonian fluids.

Reynolds and Jones in 1989[2] studied the settling behavior of spherical and of irregularly shaped particles of different diameters in non-Newtonian polymer fluids. These follow not correctly a power law model, but depended on its concentration. They concluded that settling of particles through the polymer fluids generate a localized shear rate in the fluid surrounding it. It is difficult to obtain a polymer fluid which behaved as a polymer power law fluid at shear rate generated by the falling spheres for Reynolds' number below 0.1. The other conclusion they made that settling velocities of particles of irregularly shaped could be approximated by that of a sphere of equivalent volume and density.

Chien in 1994[9] studied the settling velocity of irregularly shaped particles in non-Newtonian and Newtonian fluids for all types of slip regimes. He derived new settling velocity correlations for irregularly shaped particles. Also, these correlations consider size, surface condition, and velocity of the fluid and cover all types of fluids a net slip regime with particle Reynolds' numbers from 0.001 to 10000. He used an effective viscosity in determining the settling velocity, which is based on settling shear rate for Bingham plastic power law, Casson and divided by particle size. He concluded that the fluid rheology plays a minor role in turbulent-slip regime and the settling velocity is essentially determined by the fluid density and particle density and surface characteristics.

In 1997, Cheng N.S. [35] developed a new and simplified formula for predicting settling velocity of natural sediment particle. The formula proposes an explicit relationship between the particle Reynolds number and a dimensionless particle parameter. It is applicable to a wide range of Reynolds numbers from the Stokes flow to the turbulent regime. The proposed formula which has the highest degree of prediction accuracy when compared with other published formulas. It also agrees well with the widely used diagrams and tables proposed by the U. S. Inter-Agency Committee (1957).

In 1998 Muhannad A.R. [5] studied the factors that affect $Cd\text{-}Re_p$ relationships such as shape and size of particles, fluid type, and the rheological properties of non-Newtonian fluids. He used several solid particles, some of them were spherical with (Ψ=1) and the rest were irregularly shaped particles with various Ψ. He used different models which discuss the rheological behavior of fluids such as power law, modified power law, Casson model, Bingham model, Robertson-stiff model, modified stiff model and compared the results with each other and selected the best one of them.

In 1999, Ataid et. al., [36] studied the wall effects on terminal velocity. They used thirty spherical particles of several sizes ranging from 6.92 to 35mm and made of Teflon, glass, PVC, brass, steel, ceramics, and porcelain. They dropped each of them in five vertical different diameters cylindrical tubes in Newtonian and non Newtonian liquids. They presented a correlation firstly, for estimating Reynolds' number as a function of drag coefficient and particle diameter to tube diameter ratio (β). Secondly a correlation for estimating the wall factor in Newtonian liquid and thirdly expression for prediction the characteristics shear rate associated with the physical situation of falling spheres in non-Newtonian liquids, which accoutered for not only the particle and tube diameters, but also particle and fluid density.

In 2003, Kelessidis [4] measured the terminal velocity of solid spheres through stagnant Newtonian and shear thinning non-Newtonian fluids. They proposed an equation for predicting the terminal velocity in both types of fluids.

In 2004, Kelessidis [37] established an explicit equation which predicted the terminal velocity of solid spheres falling through stagnant Pseudo-plastic fluid from the knowledge of physical properties of spheres surrounding fluid. The equation is a generalization of the equation proposed for Newtonian liquids. He derived dimensionless velocity U* as a function of Reynolds number and a dimensionless diameter D* as a function of Archimedes number, r.

In 2006, Dina A.E.H [6] studied the factors that affect drag coefficient and settling velocity and also studied the Cd-Re_p relationship and the effect of rheological properties on this relationship. She measured the settling velocity of two types of solid particles: sphere and irregular particles with different diameters in order to compare results of other. The settling velocity was measured in non-Newtonian fluids which represented by power-law fluid. Two types of polymers were used, CMC and HEC and compared with Newtonian fluid which was water. Also she prepared new graphs to show factors that affect drag coefficient and settling velocity.

In the same year, Benoit Camenen [20] presented a simple, robust and general formula for the settling velocity of a particle, taking into account the shape and roudness of the particles. It was based on the two asymptotic behavior of the drag coefficient for low and high Reynolds numbers, respectively.

In 2008, Naslund E. and Thaning L. [38] used a general drag coefficient in the equation of motion for solid spherical particles. The time constants stopping times, and settling velocities in a still atmosphere were computed for a wide range of Reynolds numbers. The settling times were compared with the times calculated when a particle falling in a fluctuating atmosphere. It was found that such particles will get significantly longer settling times owning to an enhancement in the drag coefficient caused by an increase of the relative velocity between the particle and the fluid. Surprisingly, this enhancement is present for a horizontal wind field due to a coupling between particle motions in different directions, but it was also present for a vertical field. The effect was most pronounced in the intermediate Reynolds number region, slightly above the Stokes range, where the increase in settling time can be more than 10% for certain fluctuation frequencies and amplitudes. This indicated that such particles must be carefully treated when they are falling in a non stationary medium. Table 2.1 shows the summary of some Cd-Re relationship as shown beLow:

Table 2.1 Summary of some Cd-Re relationships

Authors	Functions	Range of applicability
Perry and Chilton	Cd= 24/Re Cd= $18.5/Re^{0.6}$ Cd= 0.44	Re < 0.3 0.3<Re<1000 Re>1000
Oseen	Cd= (24/Re) [1+3/16 Re]	Re<1
Massey	Cd=$(24/Re)[1+3/16Re]^{0.5}$	Re<100
Zeidler	Cd= $29.44/Re^{0.9751}$ Cd= $29.44/Re^{0.807}$ Cd= $21.95/Re^{0.6578}$ Cd= $6.78/Re^{0.3217}$ Cd= 1.4	 1<Re≤7.6 7.6<Re≤33 33<Re≤100 Re>100
Haider and Levenspiel	Cd= 24/Re $[1+0.1806Re^{0.6459}]$ + [0.4251/ (1+6880.95/Re)]	For sphere
Haider and Levenspiel	Cd= 24/Re [1+ (8.171exp	For irregular

	(−4.0655Ψ))] Re0.0964$^{+0.5565Ψ}$ + [73.69Re exp(−5.748Ψ)/Re+5.378exp(6.2122Ψ)	particles
Mpandelis and Kelessidis	Cd= 24/Re (1+0.1407 Re$^{0.6018}$) + (0.2118/ (1+ (0.4215/Re))	
Chien	Cd= (30/Re) + (67.289/exp (5.03Ψ))	0.001<Re<10000
Muhannad A.R.	Cd= 59/Re $^{1.33}$ Cd = (29.67/Re) + 1.21 Cd= 0.98	Re<5 5≤Re≤200 Re>200
Pettyjhon and Christiansen	Ψ4.88Cd=5.3	2000≤Re≤17000
Banks and Flemmer	Cd=24/Re 10^{E}	Re<3*10^{5}
Meyer	Cd=24x/Re Cd=18.2/Re$^{0.6}$ Cd=0.44	Re<0.1 100<Re<500 Re<500
Concha and Barrientos	CD_M=0.527+(22/Re_M)+(3.73/Re$^{1/2M}$_(0.0281$Re_M^{1/2}$)/(1+1.06*10^{-4}$Re_M^{3/2}$)	

Dedegil	Cd=24/Re Cd=22/Re+0.25 Cd=0.4	Re<8 8<Re<150 Re>150
Muhannad A.R.	$Cd = 59/Re^{1.33}$ Cd = (29.67/Re) + 1.21 Cd= 0.98	Re<5 5≤Re≤200 Re>200

Chapter Three

Theoretical Analysis

3.1 Introduction

This chapter deals with developing empirical equations for calculating drag coefficient and settling velocity.

An experimental data was collected from literatures and handled by statistical fitting. The empirical equations are achieved for all types of fluids; Newtonian and non- Newtonian fluids, for spherical and irregular shaped particles for wide ranges of Reynolds number from 0.001 to 10000.

3.2 Empirical Equations

There are two types of empirical equations; the first for drag coefficient and the second for settling velocity .The drag coefficient and settling velocity equations are general and applicable for all flow slip regimes. They are as follows:

3.2.1 Drag Coefficient Equation

This equation presents drag coefficient- Reynolds number relationship. An experimental data was from Pettyjhon and Christiansen and Barker[2], Moore [18], Zeidler [21], Walker and Mayes [39], , and finally Hopkins[40] to establish the relationship between the drag coefficient and Reynolds number. Walker's and some of zeidler's collected data were for particles settling in non- Newtonian fluids. Moore's data and some of zeidler's data were for settling of particles in Newtonian fluids. Pettyjhon's and Christiansen's and Barker's data were for isometric particles in non-Newtonian fluids. Hopkins used glass and rock particles of various shapes and sizes settling in water and twelve fluids. Most of his data were in the transitional and turbulent- slip regimes.

Three guidelines were followed to obtain a relationship between the drag coefficient and Reynolds number for the settling process:

1. In terms of drag coefficient, this means a lower value of drag for a given Reynolds number. In other words, the correlation is one that fits most experimental data close to the lower boundary of the experimental data.
2. Laminar- slip is likely to occur where Rep <10. As far as fluid rheology is concerned, the main interest is the drag coefficient for Rep<10.

3. For Rep>100, turbulent slip prevails and the surface condition of the particle has a dominate effect onC_d. Sphericity, ψ, is used to characterize particle surface condition.

As defined before $\psi=A_s/A_p$. A smooth sphere has a ψ value of 1. Most drill cuttings, sand particles, and other frequently occurring irregular particles have a ψ value close to 0.8(.7924)[20].With the laminar-slip data of Zeidler, Moore, Walker and Mayes and Pettyjohn and Christiansen the drag coefficient for the laminar- slip regime is correlated as

$$(C_d)L= 24.00987/Rep \qquad (3\text{-}1)$$

With turbulent- slip data of Hopkins, Walker and Mayes Petty john and Christiansen, the drag coefficient for the turbulent- slip regime and is correlated as

$$(C_d)\ t=54.4537/\exp(4.629538*\psi) \qquad (3\text{-}2)$$

Smooth spheres will have a drag coefficient of 0.53 in the turbulent- slip regime. For frequently occurring irregular particles, Cd for turbulent regime is (C_d) t=1.34 (3-3)

The relationship between Cd and Rep with these guidelines can be shown in equ. (3.4), which is:

C_d=24.00987/Rep+54.4537/exp(4.629538*ψ),for 0.2<ψ<1 (3-4)

The relationship in equ.(3.4) is valid for spherical and irregular shaped particles, in either Newtonian or non-Newtonian fluids and for Rep from0.001 to 10000.

3.2.2 Settling Velocity Equation

To obtain settling velocity equation, we follow two methods; the first by dimensionless analysis and the second by derivation the correlation of drag coefficient.

3.2.2.1 Dimensionless analysis derivation

Dimensionless analysis was used to predict settling velocity in this study. There are several factors that affect settling velocity such as particle diameter, density difference between the particle and the fluid and viscosity of fluid.

$$Vs=const\ [(d_p)^{n1}(\rho_p-\rho_f)^{n2}(\mu)^{n3}] \quad (3\text{-}5)$$

$$Vs=L.T^{-1} \quad (3\text{-}6)$$

$$D_p=L \quad (3\text{-}7)$$

$$\mu=M.L^{-1}T^{-1} \quad (3\text{-}8)$$

$Vs=a\,[\mu/ d_p (\rho_p-\rho_f)]^{b}$ (3-9)

Equ.(3.9) handled by statistical fitting and the result is as follows

$\text{Log}\ v_s= a1+ b\ \text{Log}\ [\mu/ d_p (\rho_p-\rho_f)]$ (3-10)

From Dina's data and Reynolds and Jones, for different types of fluids, Newtonian and non- Newtonian fluids, and for spherical and irregular shaped particles, we obtained the constants a1,b in equ(3-10) which were; a_1=2.025, b=-0.5553

$\text{Log}\ v_s= 2.025- 0.5553[\mu/dp (\rho_p-\rho_f)]$

the above equation has very high results and un ecceptable so that this equ. is careless and we will depend on the other equ. of settling velocity.

3.3.2 Settling Velocity Equation Derivation from C_d-Re_p Relationship

From the formula which was obtained from experimental data that collected from literatures which is

$C_d=(24.00987/Re_p)+(54.4537/\exp(4.629538\psi))$ (3-4)

We shall derive anew correlation for predicting settling velocity by introduction the above equation to the C_d-Re_p relationship. The drag coefficient and particle Reynolds number in the settling process are defined as

$C_d= 1308.7d_p (\rho_p-\rho_f)/v_s^2\, \rho_f$ (3-12)

And $Re_p= \rho_f\, d_p\, v_s/10\ \mu$ (3-13)

By introducing equ (4.15), (4.16) into equ (4.14) and rearranging;

V_s^2+[4.41(μ_{eq}/ ρ_f d_p) exp (4.629538 ψ) Vs]-[24.03 exp (4.629538ψ) d_p (ρ_p-ρ_f) / ρ_f]= 0 (3-14)

For frequently occurring irregular particles, equ (3-14) can be simplified to

V_s^2+ [172.8 (μ_{eq}/ ρ_f d_p) V_s]–[941.758 d_p (ρ_p-ρ_f)/ ρ_f] =0 (3-15)

The value of settling velocity can be solved from equ (3-15) by quadratic formula and choosing the positive root of V_s:

V_s= 86.4(μ_{eq}/ ρ_f d_p)[(1+0.126 d_p ($\rho_{p\_-\_}\rho_f$)/ ρ_f (d_p ρ_f / μ_{eq})^2)$^{0.5}$_1] (3-16)

Turbulent – slip drag coefficient of equ (3.2 for those interested only in the settling velocity in the turbulent- slip regime, settling velocity can be obtained by substituting the turbulent- slip regime the) in equ (3-16)

1308.7 d_p ($\rho_{p\_-\_}\rho_f$)/ $\rho_{f\,Vs2}$ =54.4537/ exp (4.629538ψ)

V_s (t) = 4.9 exp (2.315 ψ)[d_p (ρ_p-ρ_f)/ ρ_f]$^{0.5}$ (3-17)

For frequently occurring irregular shaped particles (ψ=0.7924) [9]

V_s (t) =30.7[dp (ρ_p-ρ_f)/ ρ_f] $^{0.5}$ (3-18)

For spherical particles (ψ= 1)

V_s^2+[24.00987(μ_{eq}/ ρ_f d_p) V_s]− [2467.5 d_p (ρ_p-ρ_f)/ ρ_f] = 0 (3-19)

Equ (3.19) can be solved by quadratic formula and choosing the positive root of V_s:

V_s= 12.0049(μ_{eq}/ ρ_f d_p) [1+17.12($\rho_{p\_-\_}\rho_f$)/ $\rho_{f\,(}d_p$ ρ_f / μ_{eq}) ^{2}]$^{0.5}$_1] (3-20)

For turbulent slip regime

$$V_s\,(t) = 49.7[dp\,(\rho_{p\,-}\,\rho_f)/\,\rho_f]^{\,0.5} \qquad (3\text{-}21)$$

The equs. (3-16) and (3-20) are solved by trial and error by using Newton Raphson method.

Chapter Four

Results and Discussion

4.1 Introduction

This chapter is divided into two parts, the first deals with the results of the empirical equations for drag coefficient and for settling velocity. These results depend on values of different parameters such as particle diameter, fluid and particle densities and rheological properties of fluid such as viscosity and fluid flow index(n) and consistency index(k) taken from experimental data. The second part deals with the discussion of these results and the factors which affect these results.

4.2 Drag Coefficient Results

The results of drag coefficient are included the results of spherical particles and the results of irregular particles which can be shown together in fig.4.1, these results depend on equation (3-4) for different values of sphercities. The results can be shown in appendix A.

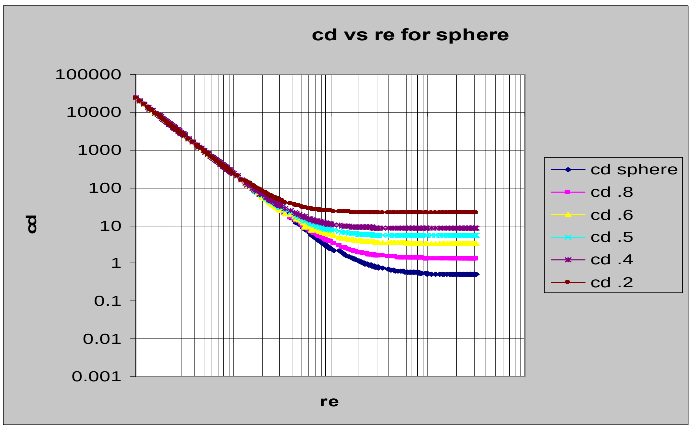

Fig. (4.1) the results of drag coefficient equation.

4.3 Results of settling velocity equation

Results of equation of settling velocity which found by derivation of C_d-Re_p relationship and results can be divided into the results of spherical particles and the second for irregular particles.

4.3.1 The results of settling velocity for spherical particles

Results of spherical particles can be divided into two parts, the first for spherical particles in Newtonian fluids and the second of spheres in non-Newtonian fluids.

4.3.1.1 Spherical particles in Newtonian fluids

Equations 3-20 and 3-21 are used to obtain the results of spherical particles in Newtonian fluids. The results of spherical particles can be shown in table 4.1. The parameters ρ_p, ρ_f, μ and d_p are obtained from Dina's data [6].

Table 4.1 The results of spherical particles in Newtonian fluids

d_p, cm	v_s, cm/s	Re	c_d
0.22	28.97557	568.3966	0.572241
0.3	32.33557	829.3966	0.558949
0.4	38.29158	1403.448	0.547108
0.6	49.3909	2775	0.538652
0.8	54.76931	4143.448	0.535795
1	61.93585	6003.448	0.533999
1.43	72.74118	10206.01	0.532353
2	88.57204	17594.83	0.531365

4.3.1.2 The results of settling velocity for spherical particles in non- Newtonian fluids

The results of settling velocity in non- Newtonian fluids can be divided according to the flow behavior index (n). Equations 3-20 and 3-21 are also used to obtain the results of settling velocity and the experimental parameters ρ_p, $\rho_{f,}$ μ and d_p are also taken from Dina's data.

a-n= 0.73

Table 4.2 The results of settling velocity of spherical particles n=0.73

d_p, cm	v_s, cm/s	Re	C_d
0.22	27.247	270.5753	0.618736
0.3	31.04	400.4091	0.589963
0.4	37.88	636.1351	0.567743
0.6	48.88	1182.257	0.550309
0.8	54.18	1662.249	0.544444
1	61.28	2287.493	0.540496
1.43	71.96	3642.218	0.536592
2	87.64	5976.526	0.534017

b-n= 0.71

Table 4.3 the results of settling velocity of spherical particles n=0.71

d_p, cm	v_s, cm/s	Re	C_d
0.22	23.46	38.73917	1.149783
0.3	27.7	60.44207	0.927238
0.4	48.01	135.9213	0.706645
0.6	54.2	221.4764	0.638408

0.8	62	324.9306	0.603892
1	75	479.5817	0.580064
1.43	75.03527	653.9894	0.566713
2	91.30297	1076.849	0.552296

d- n=0.63

Table 4.4 The results of settling velocity of spherical particles n=0.63

d_p, cm	v_s, cm/s	Re	c_d
0.22	18.1555	12.35337	2.473589
0.3	23.99935	21.65872	1.638554
0.4	32.13043	37.22726	1.174954
0.6	46.22464	76.2018	0.845083
0.8	53.39774	110.6745	0.746941
1	61.96701	155.7277	0.684179
1.43	74.81282	254.4279	0.624368
2	92.54705	421.3187	0.586987

e- n=0.61

Table 4.5 The results of settling velocity of spherical particles n=0.61

d_p, cm	v_s, cm/s	Re	c_d
0.22	25.26993	3.898466	6.688799
0.3	28.28682	6.320787	4.328557
0.4	33.43896	9.66878	3.013237
0.6	42.98869	19.06542	1.789342
0.8	47.79245	32.93166	1.259082
1	54.00596	50.18615	1.008416
1.43	63.50284	100.7022	0.768424
2	77.17692	189.7928	0.656506

4.3.2 The results of irregular shaped particles

Results of irregular particles can be divided into two parts, the first for irregular particles in Newtonian fluids and the second of irregular in non-Newtonian fluids.

4.3.2.1 **irregular particles in Newtonian fluids**

Equations 3-16 and 3-18 are used to obtain the results of irregular particles in Newtonian fluids. The results of irregular particles can be shown in table 4.6. The parameters ρ_p, ρ_f, μ and d_p are obtained from Dina's data [6].

Table 4.6 Results of settling velocity for irregular particle n=1

d_p, cm	v_s, cm/s	Re	c_d
0.984	29.53	2504.959	1.349585
1.101	35.58	3377.033	1.34711
1.152	37.96	3769.821	1.346369
1.198	36.05	3723.095	1.346449
1.24	45.05	4815.69	1.344986
1.388	39.2	4690.483	1.345119
1.42	32.99	4038.431	1.345945
1.563	45.44	6122.648	1.343921
1.789	55.04	8488.497	1.342829
1.823	45.78	7194.564	1.343337
1.847	51.66	8225.519	1.342919
2.121	47.49	8683.301	1.342765

4.3.2.2 The results of settling velocity for irregular particles in non- Newtonian fluids

The results of settling velocity in non- Newtonian fluids can be divided according to the flow behavior index (n). Equations 3-16 and 3-18 are also used to obtain the results of settling velocity and the experimental parameters ρ_p, $\rho_{f,}$ μ and d_p are also taken from Dina's data.

b-Settling of irregular in non-Newtonian fluids n=0.73

Table 4.7 result of settling velocity for irregular particle n=0.73

d_p, cm	v_s, cm/s	Re	c_d
0.984	29.13	1681.944	1.354275
1.101	35.16	2318.46	1.350356
1.152	37.53	2603.358	1.349223
1.198	35.85	2527.479	1.3495
1.24	44.59	3419.339	1.347022
1.388	38.7	3101.345	1.347742
1.42	32.5	2526.243	1.349504
1.563	44.79	4071.953	1.345896
1.789	54.49	5764.222	1.344165
1.823	45.25	4615.542	1.345202
1.847	51.11	5439.213	1.344414
2.121	46.9	5394.82	1.344451

C-Settling of irregular particles n=0.71

Table 4.8results of settling velocity for irregular particle n=0.71

d_p, cm	v_s, cm/s	Re	c_d
0.984	17.47	83.1881	1.628621
1.101	24.3	134.1895	1.518925
1.152	27.13	159.2453	1.490773
1.198	25.82	159.2934	1.490727
1.24	34.72	221.4735	1.44841
1.388	29.95	208.5025	1.455154
1.42	24.41	176.6122	1.475947
1.563	36.53	270.8563	1.428644
1.789	33.45	287.4209	1.423536
1.823	38.56	341.8887	1.410227
1.847	43.73	405.1086	1.399268
2.121	49.2	512.715	1.386829

d-Settling in non-Newtonian fluids n=0.63

Table 4.9 results of settling velocity for irregular particle n=0.63

d_p, cm	v_s, cm/s	Re	c_d
0.984	9.41	23.64222	2.355551
1.101	14.66	46.58145	1.855438
1.152	17.28	60.039	1.739905
1.198	16.49	57.71698	1.755993
1.24	23.94	98.29677	1.584259
1.388 / 1.42	[illegible]	[illegible]	[illegible]

1.563	27.05	134.4457	1.518584
1.789	36.7	222.3415	1.447986
1.823	29.45	166.4278	1.484266
1.847	35.56	217.2577	1.450513
2.121	33.68	220.0411	1.449115

e- Settling in non – Newtonian fluids n=0.61

Table 4.10 results of settling velocity for irregular particle n=0.61

d_p, cm	v_s, cm/s	Re	c_d
0.984	6.12261	5.273529	5.892904
1.101	12.06788	12.15509	3.315293
1.152	15.55427	17.37866	2.721572
1.198	13.49893	15.90685	2.849404
1.24	25.46418	30.91027	2.11676
1.388	17.88736	24.86378	2.305656
1.42	10.62395	14.98738	2.942006
1.56	25.79804	37.93484	1.972924
1.789	39.27598	61.13003	1.732767
1.823	26.38884	43.73252	1.889016
1.847	35.79395	63.13937	1.720268
2.121	29.85506	59.62922	1.742653

f- Settling in non – Newtonian fluids n=0.58

Table 4.11 result of settling velocity for irregular particle n=0. 58

d_p, cm	v_s, cm/s	Re	c_d
0.984	2.649113	1.321054	18.17478
1.101	3.458187	2.027819	11.84024

1.152	4.067361	2.699854	8.893025
1.198	4.554559	3.259465	7.366199
1.24	4.82654	3.58989	6.688191
1.388	6.034628	5.005688	4.796517
1.42	6.422363	5.496483	4.368224
1.56	7.415133	6.87923	3.490197
1.789	8.60563	8.338176	2.879511
1.823	9.437632	9.668856	2.483217
1.847	9.796113	10.54405	2.277101
2.121	12.3306	14.59425	1.64516

g- Settling in non – Newtonian fluids n=0.51

Table 14.12results of settling velocity for irregular particle n=0. 51

d_p, cm	v_s, cm/s	Re	c_d
0.984	4.025419	3.284631	7.309761
1.101	6.907095	7.946319	3.021508
1.152	7.782891	9.455074	2.539364
1.198	7.122657	8.969388	2.676868
1.24	10.77353	14.70754	1.632487
1.388	8.530486	13.02178	1.843823
1.42	6.581168	10.49712	2.287283
1.56	10.9317	19.06549	1.259337
1.789	14.41269	27.80481	0.863515
1.823	10.82389	21.44094	1.119814
1.847	13.6362	29.05426	0.82638
2.121	11.60342	27.49914	0.873114

4.4 Discussion

This section contains the discussion of the developed equations results, and the comparisons between these results and experimental results taken from literatures. Comparisons were also made between all these results and similar results by using Muhannad [5] and Haider equations for C_d- Re_p relation ships, and comparisons were made for settling velocity equations with other equations such as Newton, Allen and Stoke equations. Graphs were plotted to show the factors that affect settling velocity and drag coefficient.

4.4.1 C_d- Re_p relation ships

It is clear from figure 4.1 that the values of drag coefficient are high at low values of Reynolds number, and as Reynolds number increased the drag coefficient will decrease, due to the fact that the viscous forces are dominated in laminar- slip regime. When the transition- slip regime is started, the effect of Reynolds number on drag coefficient is decreased until the turbulent- slip regime is reached, the drag coefficient will be constant value due to the fact that the inertial forces are dominated in this region and viscous forces will have a little effect. For this reason, the Reynolds number has no effect on drag coefficient.

4.4.2 The factors that affect C_d- Re_p relation ships

4.4.2.1 Settling velocity

Settling velocity is a major factor which affects C_d – Re_p relationship because it enters in calculating both C_d and Re_p. It is clear from figs. 4.2 to 4.6 that drag coefficient decreases as settling velocity increased because any increase in settling velocity causes an increase in Reynolds number which decrease drag coefficient especially in laminar and turbulent regions. Drag coefficient of irregular shaped particles is higher than spherical particles, since the settling velocities of these particles are lower than spherical particles because that they have different orientations during settling.

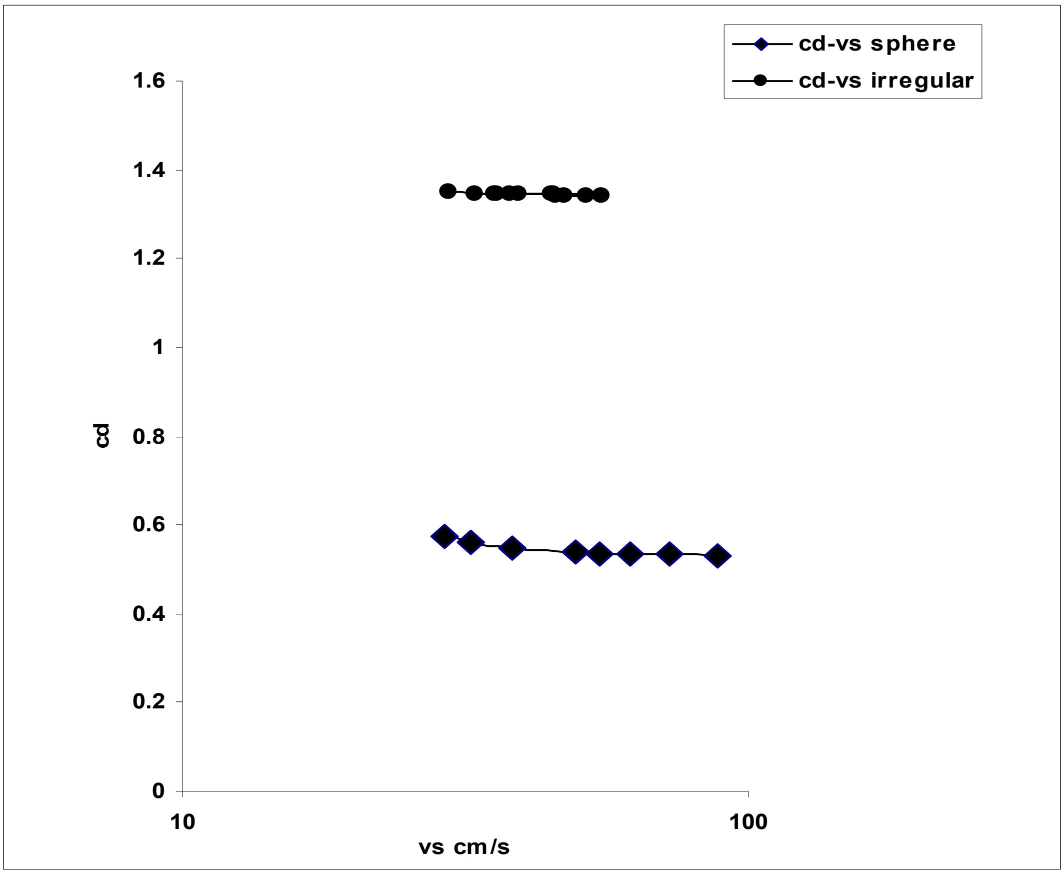

Fig 4.2 the effect of settling velocity on drag coefficient for Newtonian fluid

In the above figure the drag coefficient decrease with increasing settling velocity in Newtonian fluids which is presented by water, the decrease of drag coefficient is little because the most particles fall with high settling velocity and the flow regime is turbulent and drag coefficient has values closed to each other.

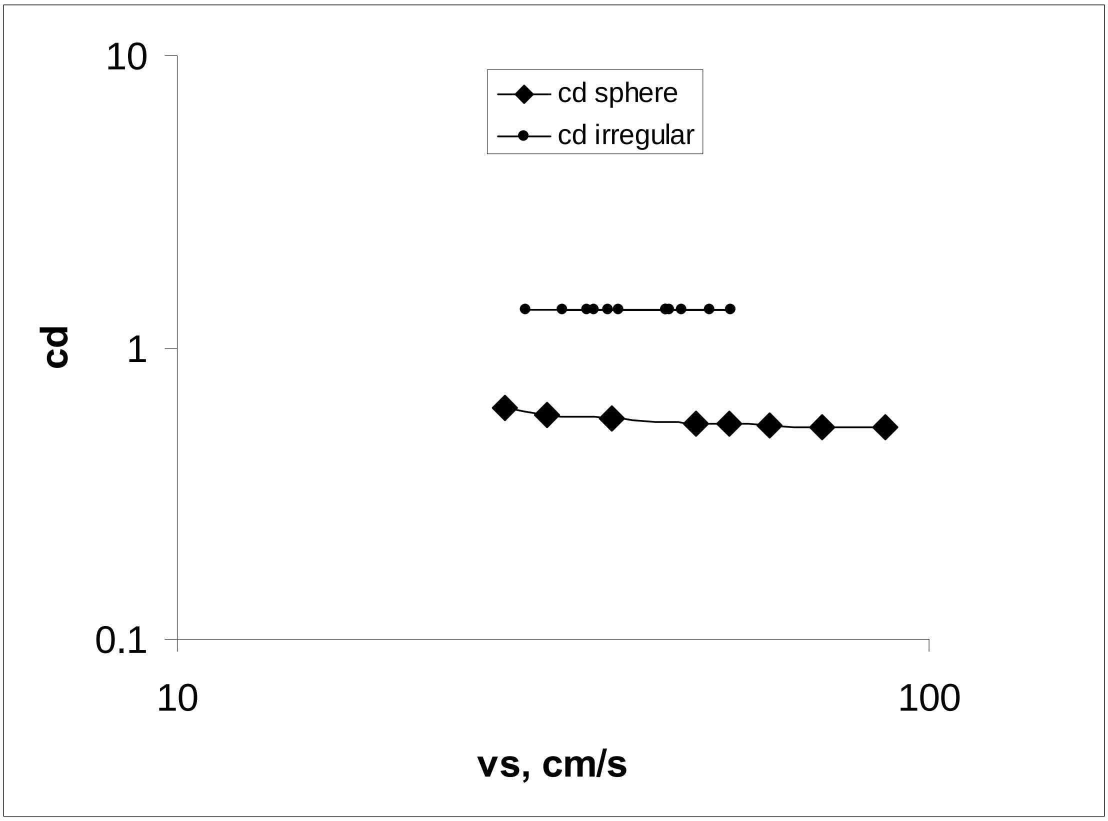

Fig 4.3 the effect of settling velocity on drag coefficient at n= 0.73

In the above figure the drag coefficient decrease with increasing settling velocity in non-Newtonian fluids with flow behaviour index= 0.73, the decrease of drag coefficient is little because the most particles fall with high settling velocity and the flow regime is turbulent and drag coefficient has values closed to each other.

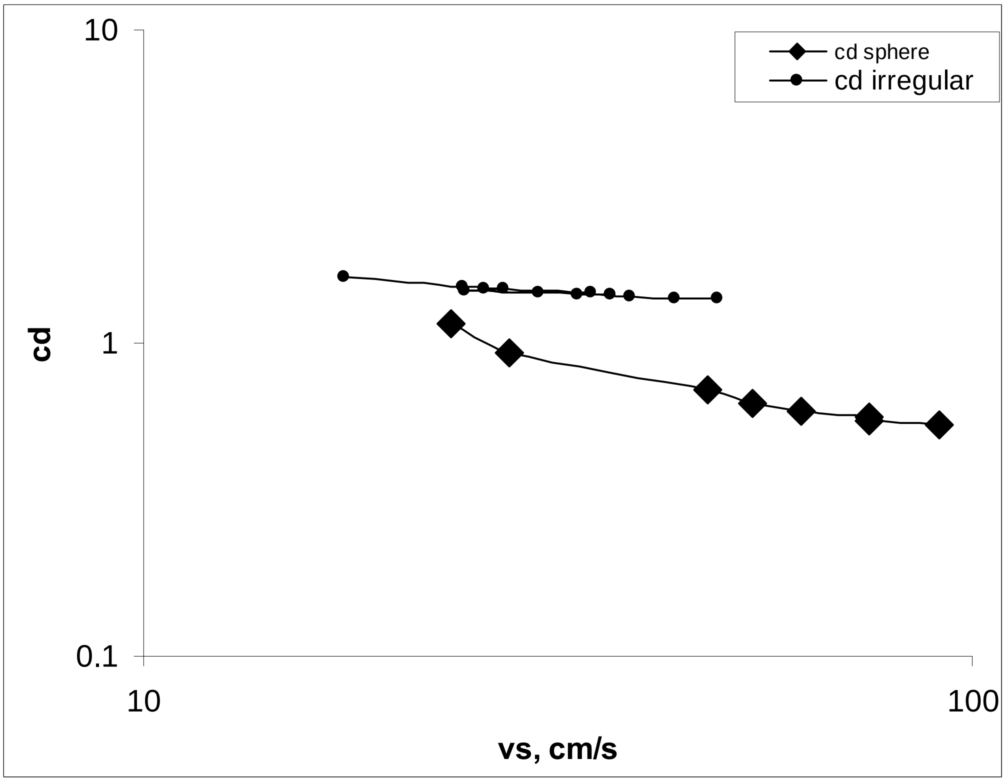

Fig 4.4 the effect of settling velocity on drag coefficient at n= 0.71

In the above figure the drag coefficient decrease with increasing settling velocity in non-Newtonian fluids with flow behaviour index= 0.71. in this fig. the effect of settling velocity on drag coefficient began to be more clear than the fig. 4.2 and 4.3 because that the particle falled in different velocities and the flow was turbulent and transition regions.

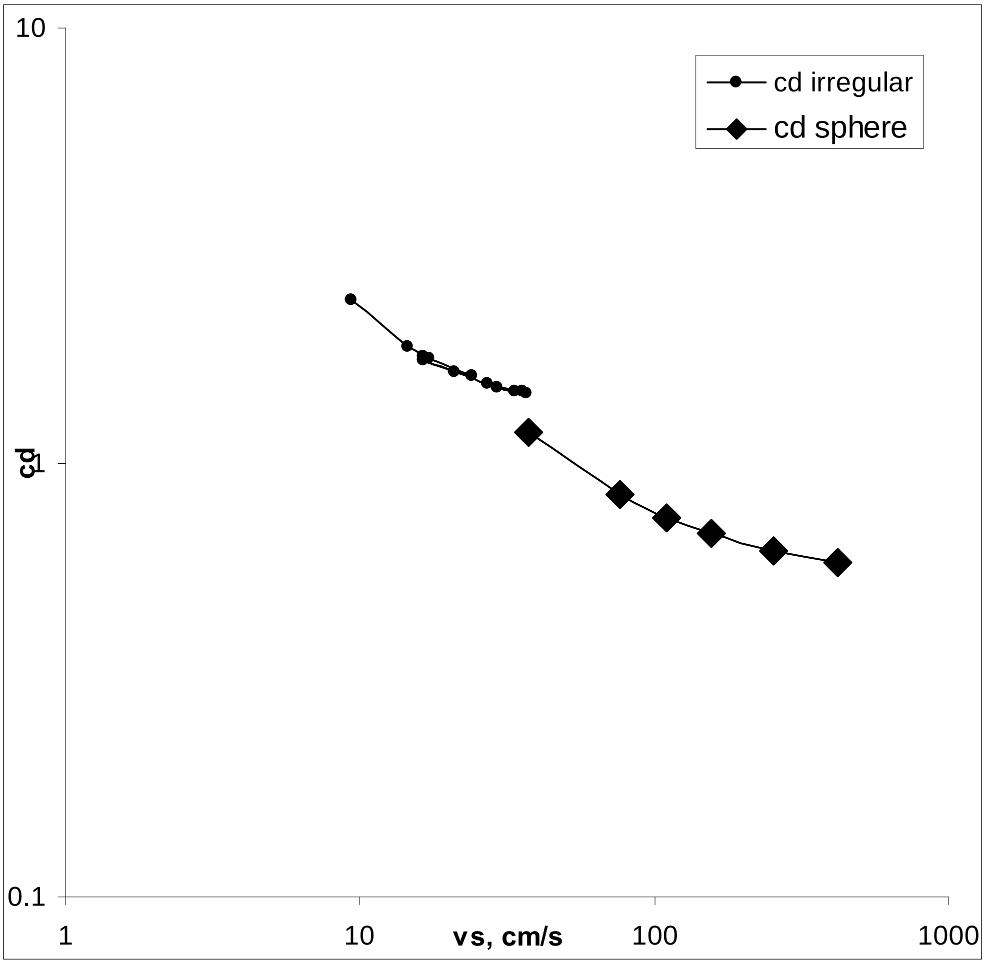

Fig 4.5 the effect of settling velocity on drag coefficient at n= 0.63

In the above figure the drag coefficient decreases with increasing settling velocity in non-Newtonian fluids with flow behaviour index= 0.63 in this fig. the effect of settling velocity on drag coefficient can be shown well because that the particles falled in different velocities and the flow was turbulent and transition regions.

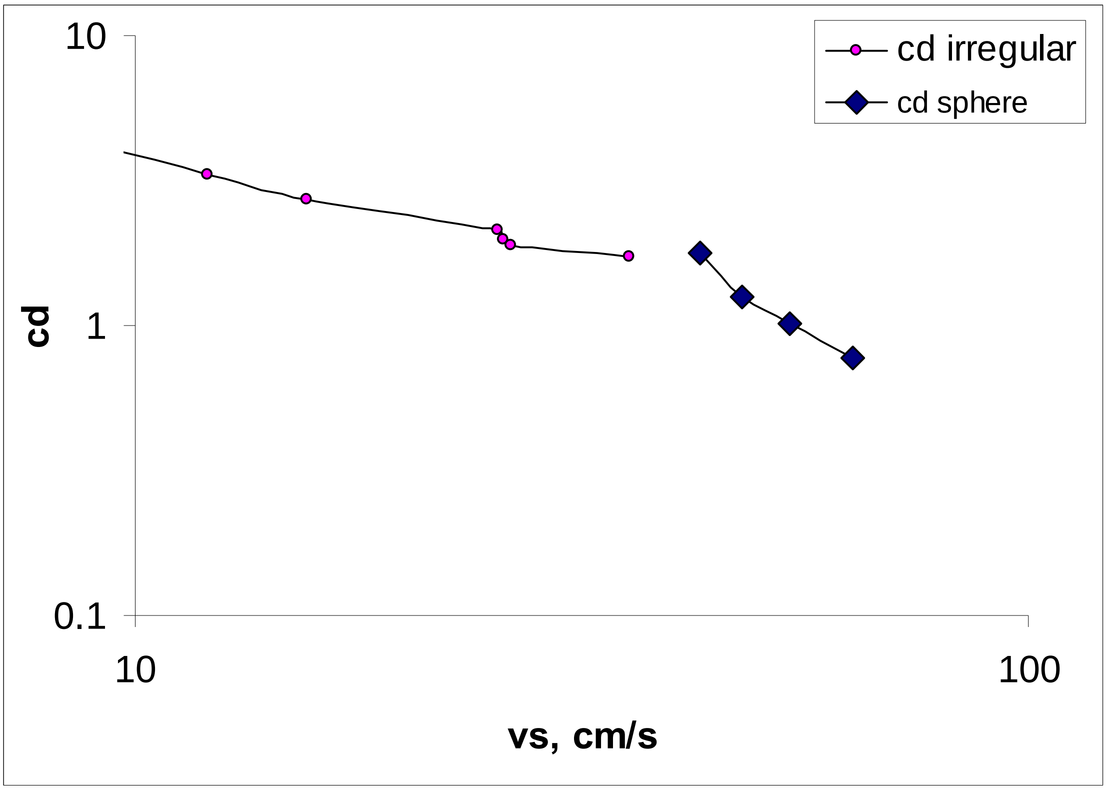

Fig 4.6the effect of settling velocity on drag coefficient at n= 0.61

In the above figure the drag coefficient decreases with increasing settling velocity in non-Newtonian fluids with flow behaviour index= 0.61 in this fig. the effect of settling velocity on drag coefficient can be shown well because that the particles falled in different velocities and the flow was turbulent, transition and laminar regions.

4.4.2.2 Particle diameter effect

It is clear from figs. 4.7 and 4.8 that the particle diameter has an important effect on C_d – Re relationship because it enters in calculating both C_d and Re. We can see from these figs. that as the diameter of particle increase C_d will decrease because settling velocity will increase, because the drag force exerted on particle will be decreased, so the drag coefficient will decrease.

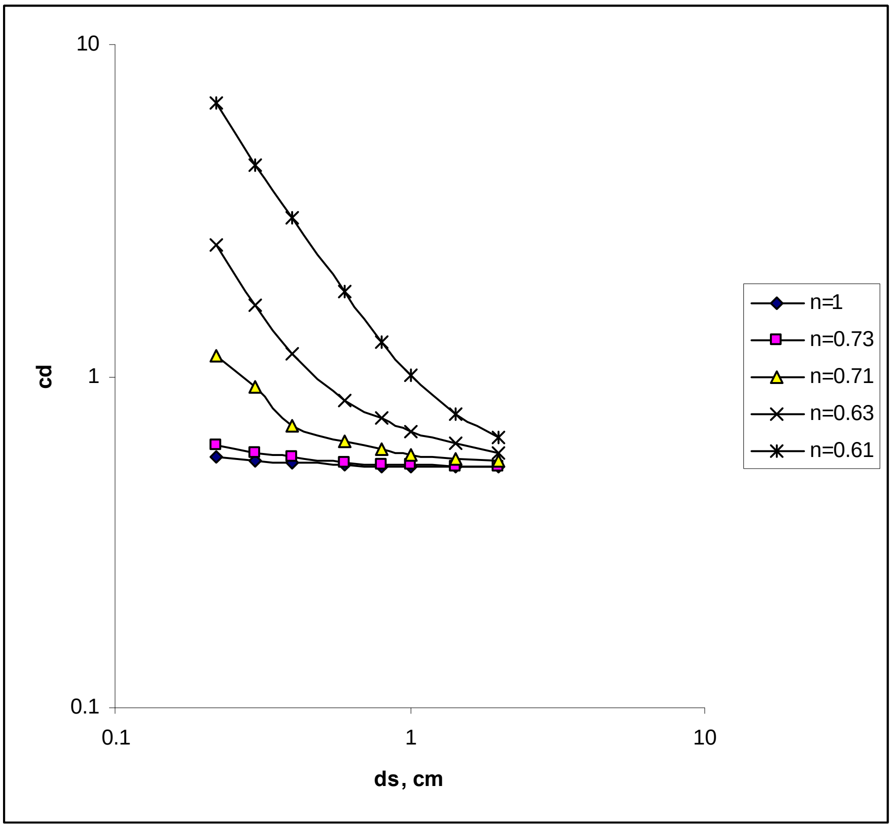

Fig.4.7 the effect of particle diameter on drag coefficient for spherical particles

In the above figure the drag coefficient decreased with increasing particle diameter for spherical particles for Newtonian and non-Newtonian fluids, the decrease of drag coefficient with increasing particle diameter become more clear as the fluid become non-Newtonian fluids and far from Newtonian behaviour.

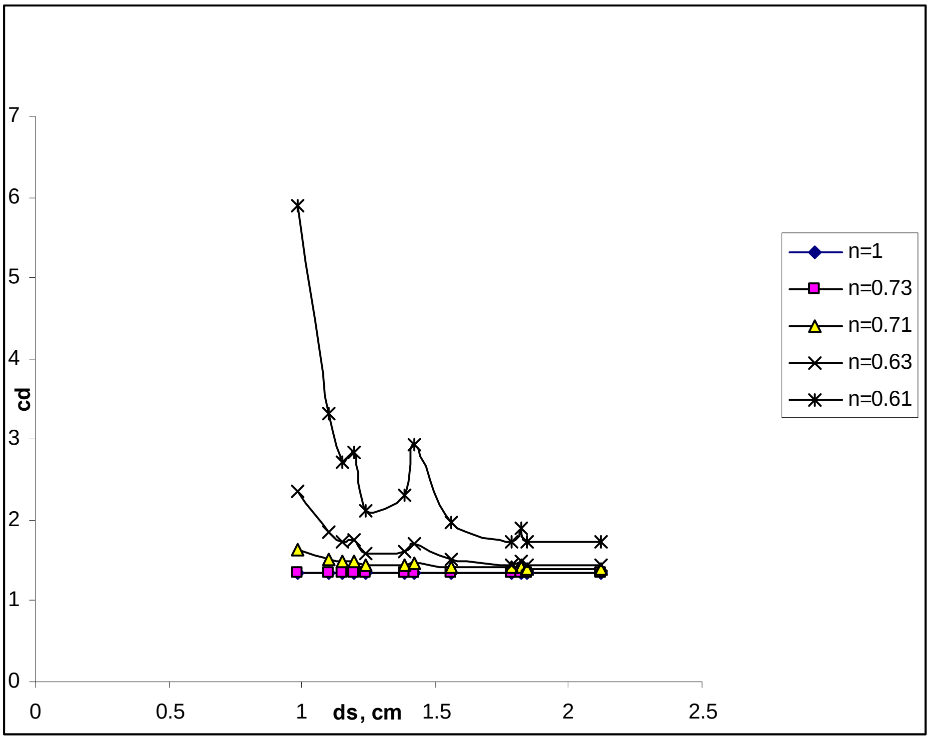

Fig.4.8 the effect of particle diameter on drag coefficient for irregular particles

In the above figure the drag coefficient decreased with increasing particle diameter for irregular particles for Newtonian and non-Newtonian fluids, the decrease of drag coefficient with increasing particle diameter become more clear as the fluid become non-Newtonian fluids and far from Newtonian behaviour.

4.4.2.3 Rheological properties effect

In this research a Power-law model was used, so we study the effects of flow behavior index (n) and consistency index (k) on C_d -Re_p relationship. The importance of the n and k parameters n, k comes from their entrances in the determination of the equivalent viscosity of non- Newtonian fluids which affect the Reynolds number. As n increases and approaches unity, drag coefficient will decrease. Figs. 4.7 and 4.8 show the effect of rheological properties on C_d -Re_p relationship for spherical and irregular particles.

4.4.2.4 Shape factor effect

Sphericity was used to distinguish the shape of particles. Sphericity has a minor effect on C_d –Re relationship in laminar region but at transition region the effect increases until it reaches the turbulent region. The shape factor has an important effect on C_d –Re relationship. When sphericity becomes smaller ($\psi<1$); i.e. far from sphere value the drag coefficient will increase because the settling velocity will increase due to the difference in orientation for irregular particles. Fig. 4.1 showed the effect of sphericity on C_d –Re relationship.

4.5 Factors affect settling velocity

There are many factors which affect settling velocity such as:

4.5.1 Particle diameter

4.5.1.1 Spherical particles

It is clear from fig. 4.9 that as particle diameter increases settling velocity will increase, because the drag force exerted on particle will be decreased. Settling velocity of irregular particles is less than that of spherical particles because the first has different orientation in settling.

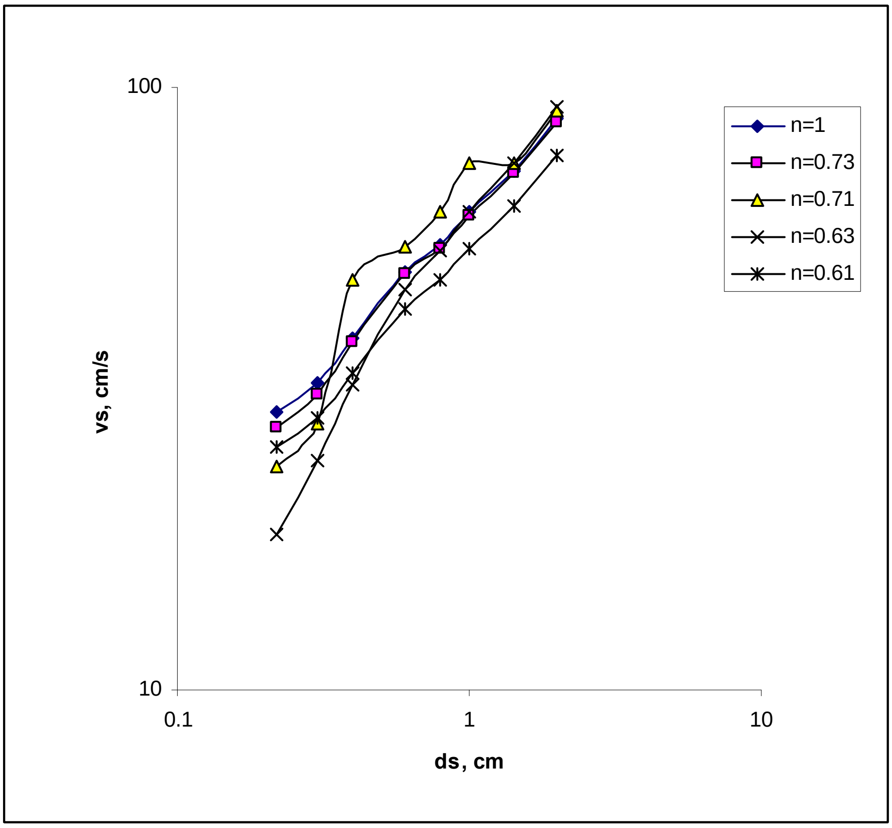

Fig. 4.9 the effect of particle diameter on settling velocity for spherical particles

4.5.1.2 Irregular particles

For irregular shaped particles the effect of particle volume on settling velocity would be studied. It can be seen from fig. 4.10 that as particle volume increases, settling velocity will increase with different flow behavior index.

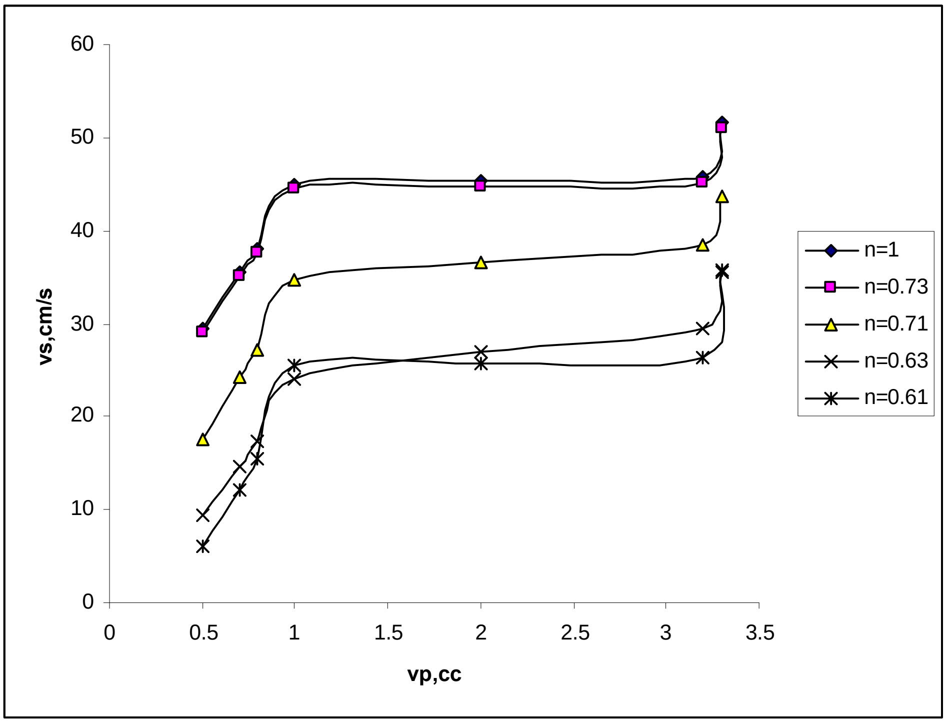

Fig 4.10 the effect of particle volume on settling velocity for irregular particles

4.6 Comparisons of the empirical equations with other investigators

Drag coefficient of the proposed equation was compared with Muhammad's drag correlation [5], Haider correlation and with the experimental data for various values of sphericity as follows:

4.6.1 Comparisons of spherical particles

Comparison was achieved for spherical particles by two methods, the first by absolute error, and the second by graph. Detailed comparison is presented in appendix A.2 with their absolute errors, a short comparison is presented in table (4.15).

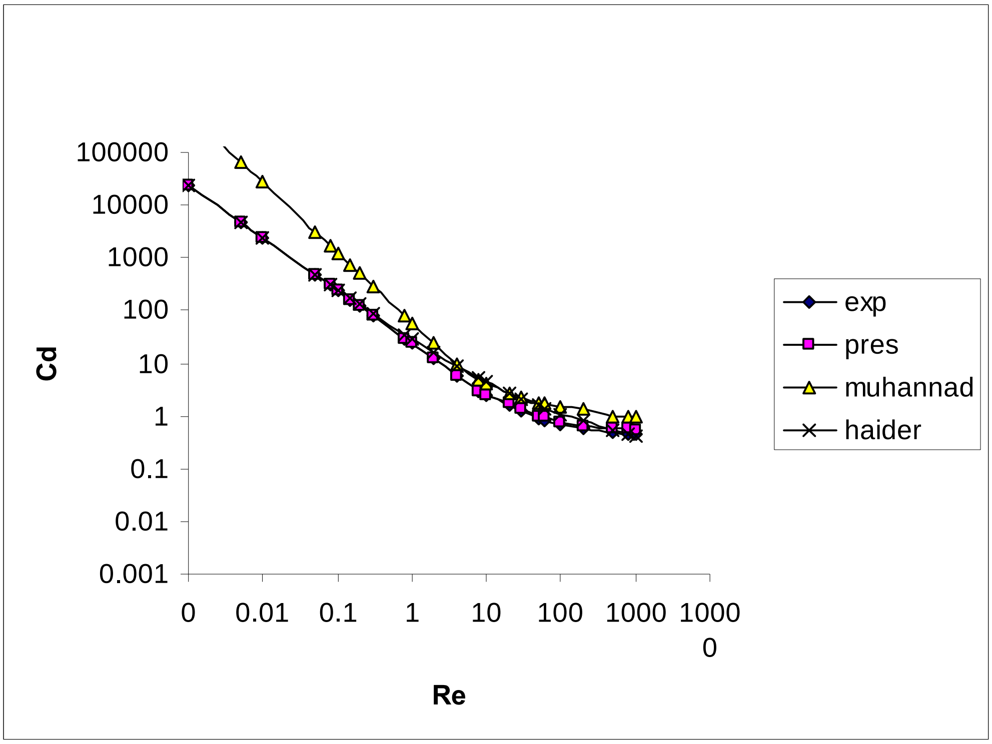

Fig 4.11 comparison results of drag coefficient for spherical particles

It is clear from fig 4.11 and from table 4.13 that the present equation is the nearest to the experimental work and gives more acceptable results than others. It is clear that in the region of Re<5, the other equations are far from experimental data while the present equation is near from it, for Re>5 the equations become near and the error become less. Fig 4.1 is presented to show this comparison.

Table 4.13 comparison of semi – empirical equation with other investigator for spherical $\psi = 1$

Re_p	C_{dexp}	C_d ψ=0.8	C_d Muhannad	C_d Haider	Error pres	error Muhannad	error Haider
0.001	24000	24009.87	576570	24050.03	9.87	552570	50.03131
0.005	4800	4801.974	67798.78	4828.297	1.974	62998.78	28.2974
0.01	2400	2400.987	26968.2	2422.139	0.987	24568.2	22.139

0.05	480	480.1974	3171.187	492.522	0.1974	2691.187	12.52204
0.08	300	300.1234	1697.236	310.6024	0.123375	1397.236	10.60243
0.1	240	240.0987	1261.398	249.797	0.0987	1021.398	9.797036
0.15	160	160.0658	735.6151	168.4869	0.0658	575.6151	8.486936
0.2	120	120.0494	501.7439	127.6651	0.04935	381.7439	7.665108
0.3	80	80.0329	292.6043	86.64014	0.0329	212.6043	6.640141
0.8	30	30.01234	79.38571	34.69226	0.012337	49.38571	4.69226
1	24	24.00987	59	28.33588	0.00987	35	4.335882
2	12	12.00494	23.46833	15.39253	0.004935	11.46833	3.392533
4	6	6.002468	9.334955	8.654496	0.002467	3.334955	2.654496
8	3	3.001234	4.91875	5.077087	0.001234	1.91875	2.077087
10	2.4	2.400987	4.177	4.319394	0.000987	1.777	1.919394
20	1.64	1.730494	2.6935	2.701972	0.090494	1.0535	1.061972
30	1.24	1.330329	2.199	2.101288	0.090329	0.959	0.861288
50	0.92	1.010197	1.8034	1.566207	0.090197	0.8834	0.646207
60	0.84	0.930165	1.7045	1.418387	0.090165	0.8645	0.578387
100	0.68	0.770099	1.5067	1.090112	0.090099	0.8267	0.410112
200	0.56	0.650049	1.35835	0.785389	0.090049	0.79835	0.225389
500	0.488	0.57802	0.98	0.529363	0.09002	0.492	0.041363
800	0.47	0.560012	0.98	0.43772	0.090012	0.51	0.03228
1000	0.44	0.53	0.98	0.400811	0.09	0.54	0.039189
					0.593405	26938.61	7.464552

4.6.2Comparison for irregular shaped particles

Comparison of drag coefficient for irregular shaped particles can be shown in figs. 4.12 to 4. 16 and tables 4.14 to 4.18 as follows

a- For ψ=0.8

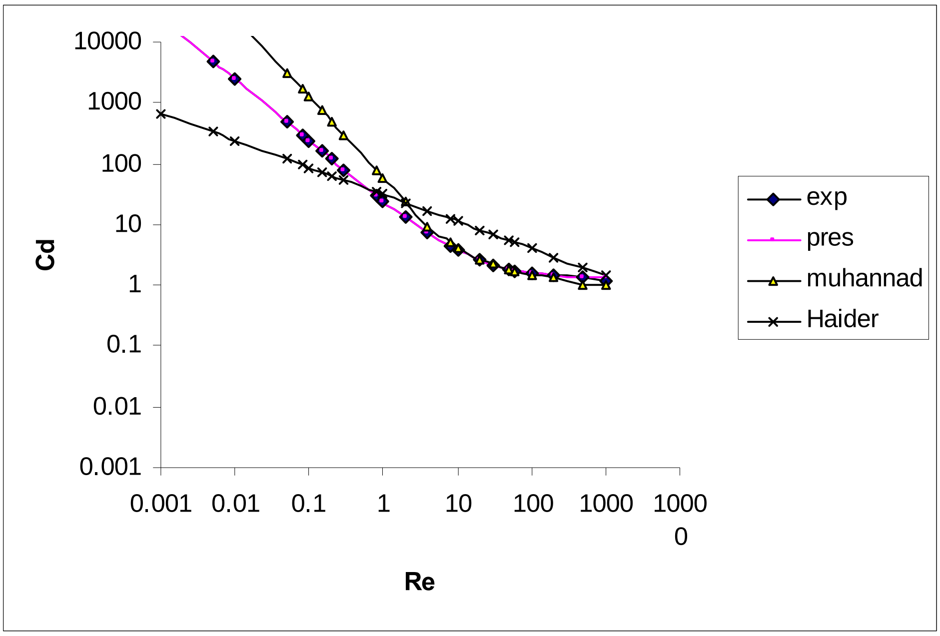

Fig. 4.12comparison results of drag coefficient for irregular particles for ψ=0.8

Table 4.14 comparison of semi – empirical equation with other investigator for irregular shaped particles ψ=0.8

Re_p	C_{dexp}	C_d ψ=0.8	C_d Muhannad	C_d Haider	Error pres	error Muhannad	Error Haider
0.001	24000	24010	576570	678.9105	10	552570	23321.09
0.005	4800	4802	67798.78	330.6626	2	62998.78	4469.337
0.01	2400	2401	26968.2	242.5679	1	24568.2	2157.432
0.05	480	480.2	3171.187	118.1484	0.2	2691.187	361.8516
0.08	300	300.1	1697.236	95.76386	0.1	1397.236	204.2361
0.1	240	240.1	1261.398	86.67453	0.1	1021.398	153.3255
0.15	160	160.1	735.6151	72.30972	0.1	575.6151	87.69028
0.2	120	120.1	501.7439	63.58635	0.1	381.7439	56.41365
0.3	80	80	292.6043	53.04881	0	212.6043	26.95119
0.8	30	30	79.38571	34.22542	0	49.38571	4.22542
1	24	24	59	30.97805	0	35	6.97805
2	13.34	13.34494	23.46833	22.72925	0.004935	10.12833	9.38925
4	7.34	7.342468	9.334955	16.67818	0.002467	1.994955	9.33818

8	4.34	4.341234	4.91875	12.23927	0.001234	0.57875	7.89927
10	3.74	3.740987	4.177	11.07903	0.000987	0.437	7.33903
20	2.54	2.540494	2.6935	8.131766	0.000494	0.1535	5.591766
30	2.14	2.140329	2.199	6.786518	0.000329	0.059	4.646518
50	1.82	1.820197	1.8034	5.404269	0.000197	0.0166	3.584269
60	1.74	1.740165	1.7045	4.982458	0.000164	0.0355	3.242458
100	1.58	1.580099	1.5067	3.968071	9.87E-05	0.0733	2.388071
200	1.46	1.460049	1.35835	2.913659	4.94E-05	0.10165	1.453659
500	1.388	1.38802	0.98	1.936197	1.97E-05	0.408	0.548197
1000	1.2	1.34	0.98	1.42033	0.14	0.22	0.22033
					0.597868	28109.36	1343.703

b- For ψ=0.6

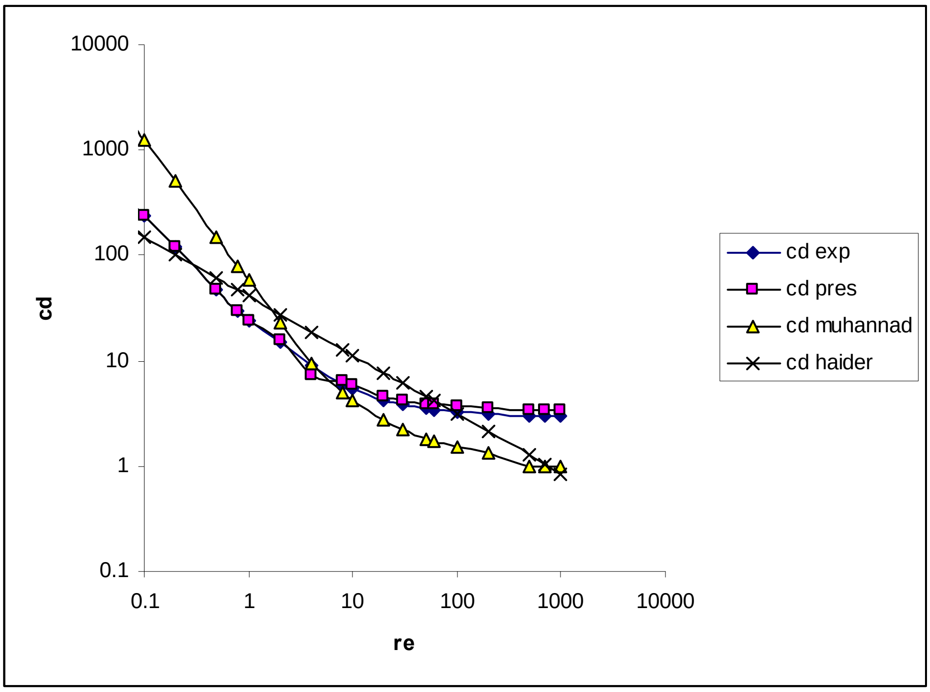

Fig. 4.13 comparison results of drag coefficient for irregular particles for ψ=0.6

Table 4.15comparison of semi – empirical equation with other investigator for irregular shaped particles ψ=0.6

Re_p	C_{dexp}	C_d ψ=0.6	C_d Muhannad	C_d Haider	error pres	error Muhannad	error Haider
0.001	24000	24010	576570	2092.726	10	552570	21907.27
0.005	4800	4802	67798.78	836.3301	2	62999	3963.67
0.01	2400	2401	26968.2	563.4479	1	24568	1836.552
0.05	480	480.2	3171.187	225.2847	0.2	2691.2	254.7153
0.08	300	300.1	1697.236	172.3978	0.1	1397.2	127.6022
0.1	240	240.1	1261.398	151.8374	0.1	1021.4	88.16258
0.2	120	120.1	501.7439	102.3621	0.1	381.74	17.63794
0.5	48	48	148.3276	60.81916	0	100.33	12.81916
0.8	30	30	79.38571	46.58417	0	49.386	16.58417
1	24	24	59	41.05004	0	35	17.05004
2	15	15.4	23.46833	27.7325	0.4	8.4683	12.7325
4	9	7.34	9.334955	18.76014	1.66	0.335	9.760139
8	6	6.4	4.91875	12.71342	0.4	1.0813	6.713418
10	5.4	5.8	4.177	11.22187	0.4	1.223	5.821873
20	4.2	4.6	2.6935	7.627839	0.4	1.5065	3.427839
30	3.8	4.2	2.199	6.092302	0.4	1.601	2.292302
50	3.48	3.88	1.8034	4.593068	0.4	1.6766	1.113068
60	3.4	3.8	1.7045	4.152729	0.4	1.6955	0.752729
100	3.24	3.64	1.5067	3.129246	0.4	1.7333	0.110754
200	3.12	3.52	1.35835	2.123586	0.4	1.7617	0.996414
500	3	3.4	0.98	1.258605	0.4	2.02	1.741395
700	3	3.4	0.98	1.035693	0.4	2.02	1.964307
1000	3	3.4	0.98	0.841434	0.4	2.02	2.158566

b- For ψ=0.5

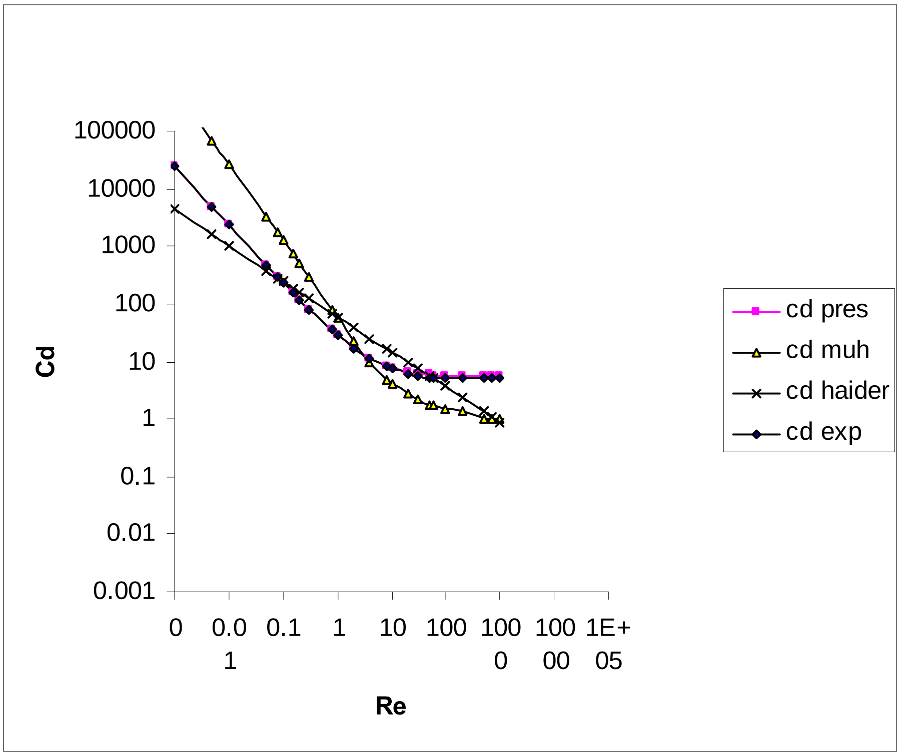

Fig. 4.14comparison results of drag coefficient for irregular particles for ψ=0.5

Table 4.16 comparison of semi – empirical equation with other investigator for irregular shaped particles ψ=0.5

Re_p	C_{dexp}	C_d ψ=0.8	C_d Muhannad	$C_{d\ Haider}$	error pres	error Muhannad	error Haider
0.001	24000	24010	576570	2092.726	10	552570	21907.27
0.005	4800	4802	67798.78	836.3301	2	62999	3963.67
0.01	2400	2401	26968.2	563.4479	1	24568	1836.552
0.05	480	480.2	3171.187	225.2847	0.2	2691.2	254.7153
0.08	300	300.1	1697.236	172.3978	0.1	1397.2	127.6022
0.1	240	240.1	1261.398	151.8374	0.1	1021.4	88.16258
0.2	120	120.1	501.7439	102.3621	0.1	381.74	17.63794
0.5	48	48	148.3276	60.81916	0	100.33	12.81916
0.8	30	30	79.38571	46.58417	0	49.386	16.58417
1	24	24	59	41.05004	0	35	17.05004
2	15	15.4	23.46833	27.7325	0.4	8.4683	12.7325

4	9	7.34	9.334955	18.76014	1.66	0.335	9.760139
8	6	6.4	4.91875	12.71342	0.4	1.0813	6.713418
10	5.4	5.8	4.177	11.22187	0.4	1.223	5.821873
20	4.2	4.6	2.6935	7.627839	0.4	1.5065	3.427839
30	3.8	4.2	2.199	6.092302	0.4	1.601	2.292302
50	3.48	3.88	1.8034	4.593068	0.4	1.6766	1.113068
60	3.4	3.8	1.7045	4.152729	0.4	1.6955	0.752729
100	3.24	3.64	1.5067	3.129246	0.4	1.7333	0.110754
200	3.12	3.52	1.35835	2.123586	0.4	1.7617	0.996414
500	3	3.4	0.98	1.258605	0.4	2.02	1.741395
700	3	3.4	0.98	1.035693	0.4	2.02	1.964307
1000	3	3.4	0.98	0.841434	0.4	2.02	2.158566

d- For $\psi = 0.4$

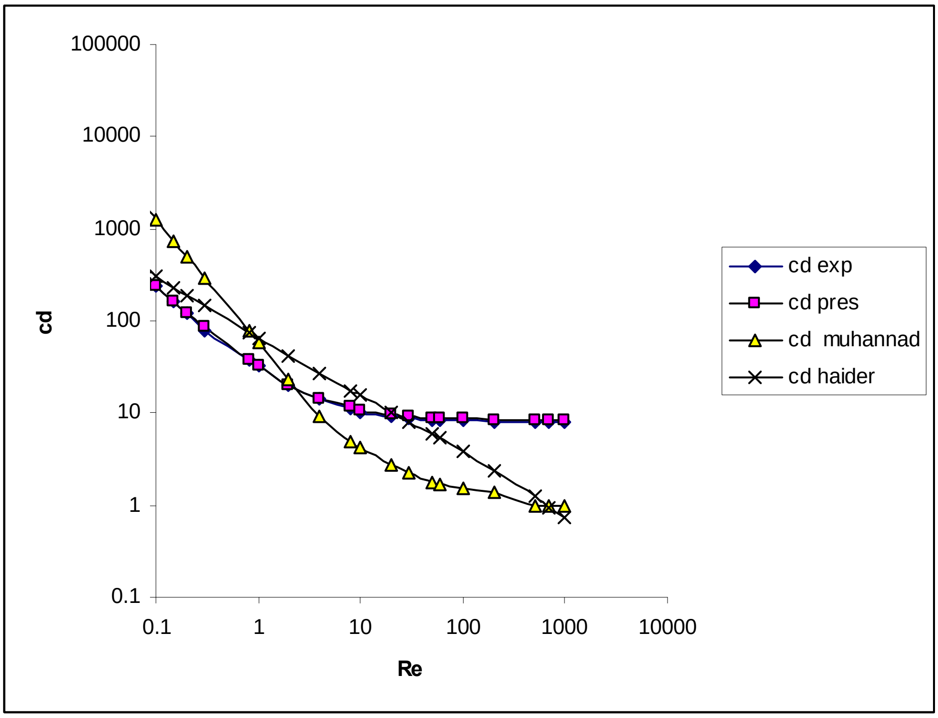

Fig. 4.15 comparison results of drag coefficient for irregular particles for ψ=0.4

Table 4.17 comparison of semi – empirical equation with other investigator for irregular shaped particles ψ=0.4

Re_p	C_{dexp}	C_d ψ=0.4	C_d Muhannad	C_d Haider	error pres	error Muhannad	error Haider
0.001	24000	24010	576570	6844.777	10	552570	528560
0.005	4800	4802	67798.78	2293.014	2	62998.78	58196.78
0.01	2400	2401	26968.2	1432.254	1	24568.2	22167.2
0.05	480	480.2	3171.187	481.2547	0.2	2691.187	2210.987
0.08	300	300.1	1697.236	350.353	0.1	1397.236	1097.136
0.1	240	240.1	1261.398	301.4144	0.1	1021.398	781.2976
0.15	160	160.1	735.6151	229.4424	0.1	575.6151	415.5151
0.2	120	120.1	501.7439	189.1614	0.1	381.7439	261.6439
0.3	80	88.5	292.6043	144.2361	8.5	212.6043	124.1043
0.8	38	38.5	79.38571	75.3443	0.5	41.38571	2.885709
1	32	32.5	59	65.11145	0.5	27	5.5
2	20	20.5	23.46833	41.61859	0.5	3.468326	17.03167
4	14	14.5	9.334955	26.90264	0.5	4.665045	9.834955
8	11	11.5	4.91875	17.62307	0.5	6.08125	5.41875
10	10.4	10.9	4.177	15.4211	0.5	6.223	4.677
20	9.2	9.7	2.6935	10.23915	0.5	6.5065	3.1935
30	8.8	9.3	2.199	8.055861	0.5	6.601	2.699
50	8.48	8.98	1.8034	5.915094	0.5	6.6766	2.3034
60	8.4	8.9	1.7045	5.28166	0.5	6.6955	2.2045
100	8.24	8.74	1.5067	3.803494	0.5	6.7333	2.0067
200	8	8.5	1.35835	2.371594	0.5	6.64165	1.85835
500	8	8.5	0.98	1.226384	0.5	7.02	1.48
700	8	8.5	0.98	0.957609	0.5	7.02	1.48
1000	8	8.5	0.98	0.735934	0.5	7.02	1.48
					1.233333	26940.27	25578.28

e-Forψ =0.2

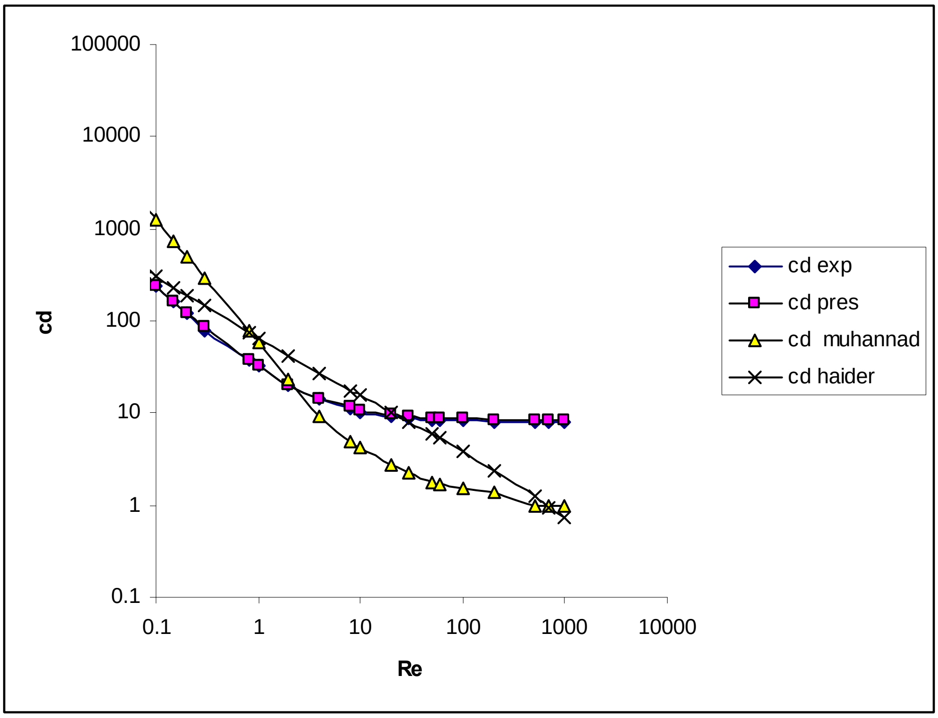

Fig. 4.16 comparison results of drag coefficient for irregular particles for ψ=0.2

Table 4.18 comparison of semi – empirical equation with other investigator for irregular shaped particles ψ=0.2

Re_p	C_{dexp}	C_d ψ=0.2	C_d Muhannad	$C_{d\ Haider}$	error pres	error Muhannad	error Haider
0.001	24000	24010	576570	7616.193	10	552570	16383.81
0.005	4800	4802	67798.78	2510.615	2	62998.78	2289.385
0.01	2400	2401	26968.2	1557.3	1	24568.2	842.6996
0.05	480	480.2	3171.187	514.8734	0.2	2691.187	34.87344
0.1	240	240.1	1261.398	320.2293	0.1	1021.398	80.22929
0.15	160	160.1	735.6151	242.7734	0.1	575.6151	82.77344
0.2	140	141.7	501.7439	199.5753	1.7	361.7439	59.57526
0.3	100	101.6	292.6043	151.5608	1.6	192.6043	51.56085
0.8	50	51.6	79.38571	78.40759	1.6	29.38571	28.40759
1	44	45.8	59	67.6125	1.8	15	23.6125
2	32	33.6	23.46833	42.93576	1.6	8.531674	10.93576
4	26	27.6	9.334955	27.58395	1.6	16.66505	1.583945
8	23	24.6	4.91875	17.96795	1.6	18.08125	5.032053
10	22.4	24	4.177	15.69622	1.6	18.223	6.703785
20	21.2	22.8	2.6935	10.36963	1.6	18.5065	10.83037
30	20.8	22.4	2.199	8.135051	1.6	18.601	12.66495
50	20.48	22.08	1.8034	5.950905	1.6	18.6766	14.52909

60	20.4	22	1.7045	5.30624	1.6	18.6955	15.09376
80	20.3	21.9	1.580875	4.409701	1.6	18.71913	15.8903
100	20	21.6	1.5067	3.805608	1.6	18.4933	16.19439
200	20	21.6	1.35835	2.35883	1.6	18.64165	17.64117
500	20	21.6	0.98	1.209502	1.6	19.02	18.7905
700	20	21.6	0.98	0.941346	1.6	19.02	19.05865
1000	20	21.6	0.98	0.72087	1.6	19.02	19.27913
					1.770833	26886.37	835.8815

It is clear from the above figs. and tables that the present equation gives good results and near from experimental data and it can be also seen that Muhannad's equation is far from present equation and experimental data in the laminar region but it begins to be near at the transition and turbulent region. This can be shown very well when ψ=0.8 and 0.6, but it returns to be far in the other values of ψ , while Haider's equation becomes near from present equation as sphericity becomes far from the value of sphere, and this occur especially in laminar and transition region.

4.7 Comparison of settling velocity correlation with others'

In order to know the accuracy of present equs, comparison of the present correlations with others' were found.

4.7.1 spherical particles

a- Newtonian fluids

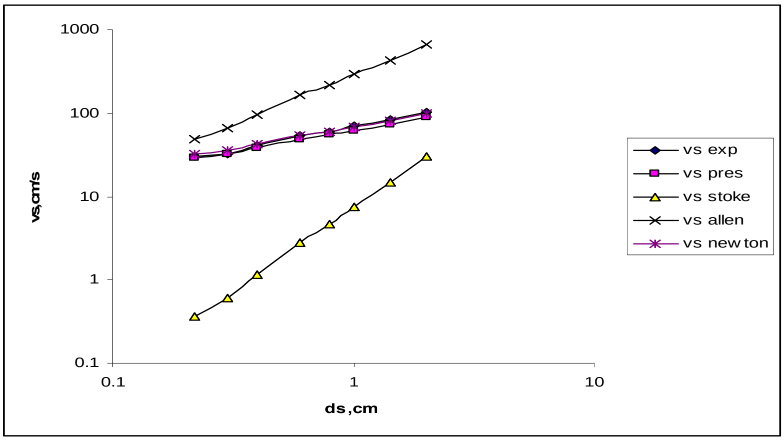

Fig 4.17 settling comparison n=1

Table (4.19) Comparison of settling velocity equation with other investigators atn=1

D_p,cm	$v_{s\ exp}$	$v_{s\ present}$	$v_{s\ Stoke}$	$v_{s\ Allen}$	$v_{s\ Newton}$
0.22	29.97	28.97557	0.358132	48.50583	31.93939
0.3	32.07	32.33557	0.60819	65.51929	35.64308
0.4	40.7	38.29158	1.137165	95.40472	42.20832
0.6	53.65	49.3909	2.837931	165.865	54.44295
0.8	60.08	54.76931	4.652874	219.7195	60.3715
1	69.64	61.93585	7.437739	290.6365	68.27109
1.43	82.79	72.74118	14.67079	431.8886	80.18165
2	102.05	88.57204	30.42146	669.1669	97.63181

b- Non- Newtonian fluids

a- n=0.71

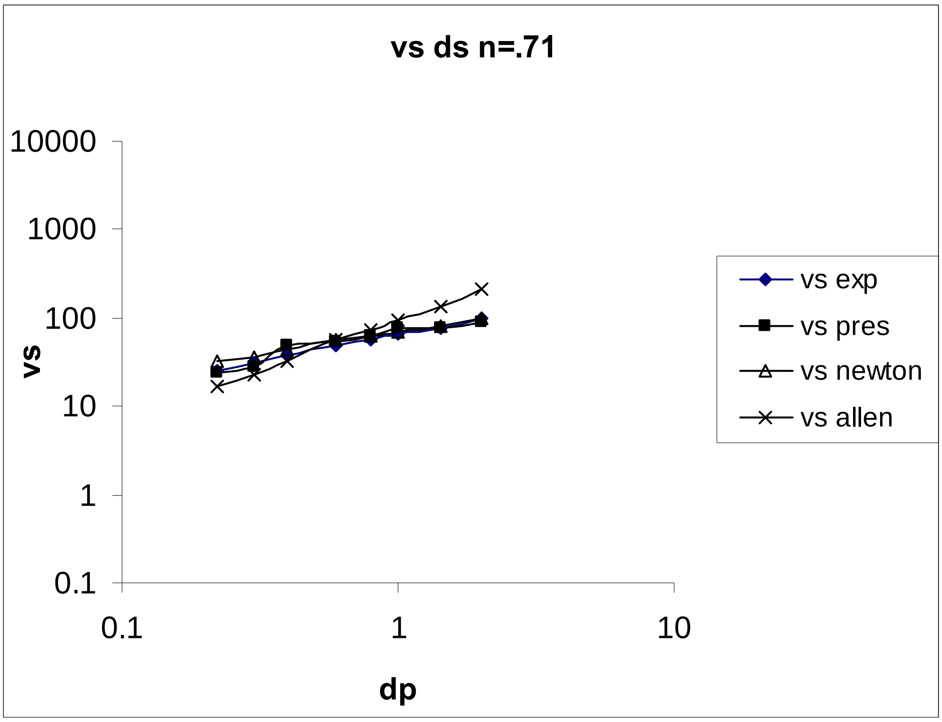

Fig 4.18 settling comparison n=0.71

Table (4.20) Comparison of settling velocity equation with other investigators **at n=0.71**

Dp	vs exp	vs pres	Vs newton	vs allen
0.22	24.84	23.46	32.9	16.90126
0.3	30.36	27.7	36.7945	22.5445
0.4	36.89	48.01	43.54442	32.39804
0.6	48.44	54.2	56.09907	55.26249
0.8	56.33	62	62.26559	72.03324
1	65.02	75	70.39414	94.21463
1.43	78.77	75.03527	82.71041	137.0984

b- n=0.63

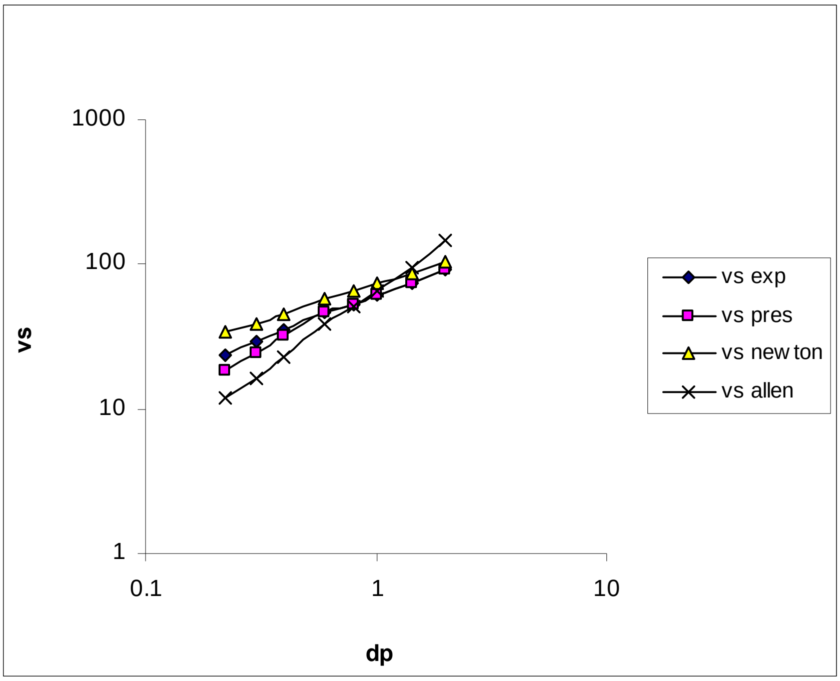

Fig 4.19 Comparison of settling n=0.63

Table (4.21) Comparison of settling velocity equation with other investigators at n=0.63

Dp	vs exp	vs pres	Vs newton	vs allen
0.22	23.42	18.1555	34.35478	12.05176
0.3	29.63	23.99935	38.43547	16.13563
0.4	35.66	32.13043	45.44999	23.05152
0.6	46.38	46.22464	58.46428	38.98077
0.8	52.76	53.39774	64.96781	50.44379
1	60.74	61.96701	73.42393	65.75879
1.43	74.82	74.81282	86.3174	95.45641
2	92.97	92.54705	104.9393	144.6865

c- n=0.61

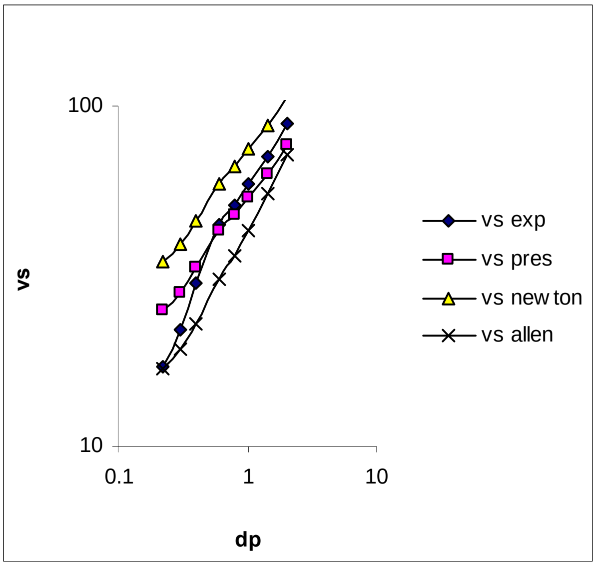

Fig 4.20 Comparison of settling n=0.61

Table (4.22) Comparison of settling velocity equation with other investigators at n=0.61

Dp	vs exp	vs pres	Vs newton	vs allen
0.22	17.23	25.26993	34.94956	16.92519
0.3	22.12	28.28682	39.12207	19.29285
0.4	30.39	33.43896	46.24773	22.9389
0.6	44.58	42.98869	59.45549	30.81445
0.8	51.01	47.79245	66.09933	36.41488
1	59.1	54.00596	74.69291	43.07661
1.43	70.82	63.50284	87.82758	55.04712
2	89.33	77.17692	106.7395	71.67074

4.7.2 Irregular particles

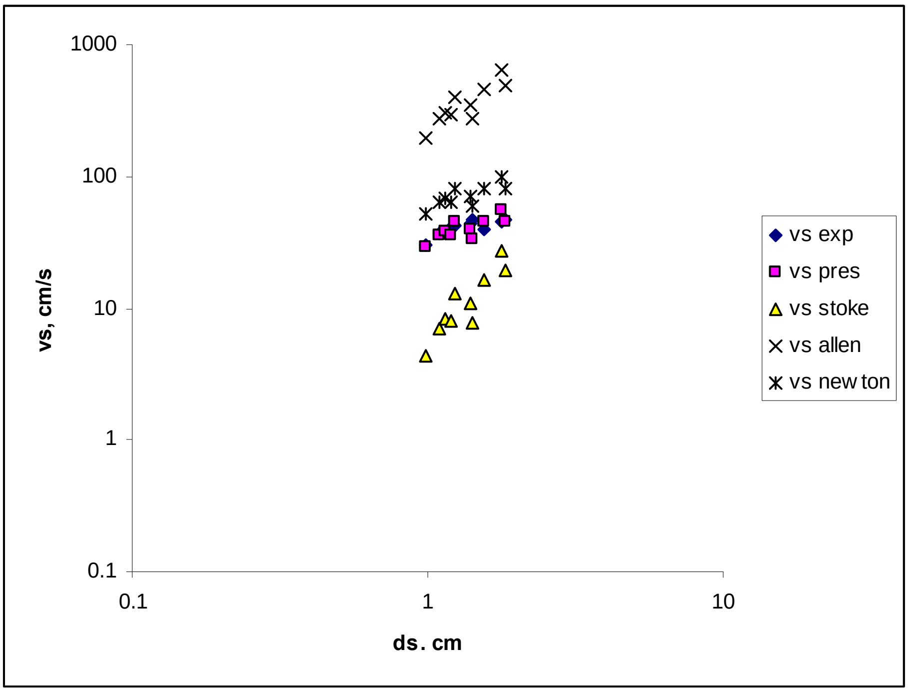

Fig 4.21 Comparison of settling n= **1**For irregular particles

Table (4.23) Comparison of settling velocity equation with other investigatorsat n=1 for irregular particles

Dp,cm	vs exp	vs pres	Vs stoke	vs allen	Vs Newton
0.984	30.01	29.52568	4.359007	198.652	52.6881
1.102	36.91	35.59665	7.095656	273.9429	63.52165
1.152	37.13	37.95772	8.434229	306.6816	67.73495
1.198	41.05	36.04993	7.911493	289.9078	64.33053
1.241	42.13	45.04784	12.79713	406.1187	80.38717
1.388	44.42	39.17257	10.82297	349.6398	69.90284
1.42	47.72	32.98586	7.851213	275.8494	58.86275
1.563	39.8	45.33201	16.32156	455.6923	80.89425
1.789	45.39	55.04496	27.54471	641.3004	98.22687
1.823	47.66	45.78378	19.41792	496.1512	81.70043

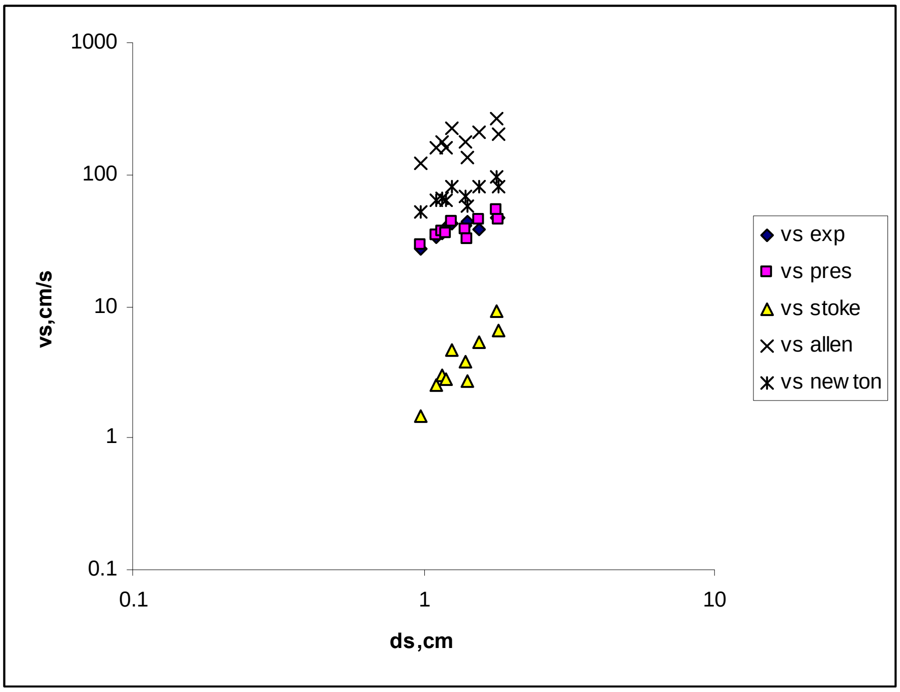

Fig 4.22 Comparison of settling n=0.73

Table (4.24) Comparison of settling velocity equation with other investigators for n=0.73 for irregular particles

Dp,cm	Vs exp	vs pres	vs stoke	vs allen	Vs Newton
0.984	27.52	29.13	1.487099	122.8344	51.98569
1.102	33.53	35.16	2.484404	159.3737	62.77559
1.152	36.35	37.53	2.984819	174.2323	66.96846
1.198	39.31	35.85	2.825283	161.1598	63.55444
1.241	41.69	44.59	4.616795	221.8969	79.5699
1.388	40.39	38.7	3.742479	174.1223	69.06692
1.42	43.52	32.5	2.739482	135.7255	58.01436
1.563	38.51	44.79	5.405496	206.1642	79.99814
1.789	46.78	54.49	9.289678	268.1233	97.2409
1.823	47	45.25	6.501421	204.0588	80.74085

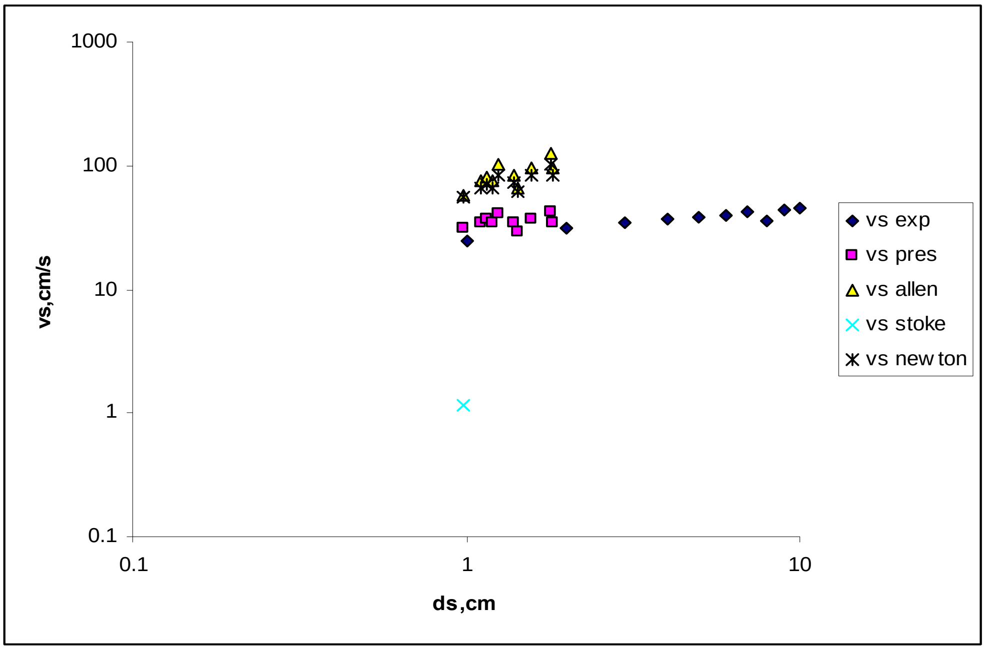

Fig 4.23 Comparison of settling n=0.71

Table (4.25) Comparison of settling velocity equation with other investigators for n=0.71 for irregular particles

dp,cm	vs exp	vs pres	Vs allen	vs stoke	Vs Newton
0.984	24.4	30.92235	58.0174	0.263965	54.73722
1.102	30.76	35.07461	75.02095	0.440827	65.70469
1.152	34.01	36.53696	82.06122	0.531313	69.97959
1.198	37.09	34.09836	76.32434	0.507398	66.60005
1.241	38.08	41.64445	103.5895	0.810032	82.78577
1.388	39.27	34.41263	82.64996	0.675827	72.34781
1.42	41.92	28.84262	65.23555	0.503061	61.33263
1.563	36.01	37.43663	96.75157	0.955569	83.5197
1.789	43.34	42.36673	125.0309	1.626849	101.1213
1.823	46.01	35.07461	96.72838	1.17164	84.50821

Chapter Five

Conclusions and Recommendations

(5.1)Conclusions

1- An empirical equation was found to relate c_d and Re for wide ranges of Rep from 0.001 to 10000 and for spherical and irregular shaped particles for Newtonian and non-Newtonian fluids.

2-Results of c_d-rep showed that as particle shape become far from a sphere shape, c_d will be increased, especially at transition and turbulent –slip regime. While at laminar –slip regime, the shape of particles has minor effect.

3- Results of c_d-Rep relationships showed that as Rep increased cd would be decreased especially in laminar-slip region.

4- Graphs were plotted to show cd-rep relationships for various value of ψ.

5- Settling velocity correlations were found for spherical and irregular shaped particles and for Newtonian and non- Newtonian fluids. These correlations consider size, surface condition, density of fluid, and density of particle

6- Graphs were plotted to show the effect of the factors which affect on settling velocity such as diameter of particles.

5.2 Recommendations

1 - Study the hindered settling velocity effect on C_{d}-Re_{p} relationship.

2- Study drag coefficient- Reynolds number relationship and settling velocity by using another models such as Bingham-plastic model, Robertson- Stiff model and else and compare between them to find the. best.

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Appendix A

Drag coefficient results

A.1 The overall results of spherical and irregular shaped particles

Table A.1 results of drag coefficient for different sphercities

re	C_d sphere	C_d ψ=0.8	C_d ψ =0.6	C_d ψ=0.5	C_d ψ=	cd .2
0.001	24010	24010	24010	24010	24010	24010
0.0012	20008	20008	20008	20008	20008	20008
0.0015	16007	16007	16007	16007	16007	16007
0.0018	13339	13339	13339	13339	13339	13339
0.002	12005	12005	12005	12005	12005	12005
0.0022	10914	10914	10914	10914	10914	10914
0.0025	9604	9604	9604	9604	9604	9604
0.0028	8575	8575	8575	8575	8575	8575
0.003	8003.3	8003.3	8003.3	8003.3	8003.3	8003.3
0.0032	7503	7503	7503	7503	7503	7503
0.0035	6860	6860	6860	6860	6860	6860
0.0038	6318.4	6318.4	6318.4	6318.4	6318.4	6318.4
0.004	6002.5	6002.5	6002.5	6002.5	6002.5	6002.5
0.0042	5717	5717	5717	5717	5717	5717
0.0045	5335.5	5335.5	5335.5	5335.5	5335.5	5335.5
0.0048	5002	5002	5002	5002	5002	5002
0.005	4802	4802	4802	4802	4802	4802
0.0052	4617.3	4617.3	4617.3	4617.3	4617.3	4617.3
0.0055	4365.4	4365.4	4365.4	4365.4	4365.4	4365.4
0.0058	4140	4140	4140	4140	4140	4140
0.006	4001.6	4001.6	4001.6	4001.6	4001.6	4001.6
0.0062	3872.6	3872.6	3872.6	3872.6	3872.6	3872.6
0.0065	3694	3694	3694	3694	3694	3694
0.0068	3531	3531	3531	3531	3531	3531
0.007	3430	3430	3430	3430	3430	3430
0.0072	3334.7	3334.7	3334.7	3334.7	3334.7	3334.7
0.0075	3201.3	3201.3	3201.3	3201.3	3201.3	3201.3
0.0078	3078.2	3078.2	3078.2	3078.2	3078.2	3078.2
0.008	3001.2	3001.2	3001.2	3001.2	3001.2	3001.2
0.0082	2928	2928	2928	2928	2928	2928
0.0085	2824.7	2824.7	2824.7	2824.7	2824.7	2824.7
0.0088	2728.4	2728.4	2728.4	2728.4	2728.4	2728.4
0.009	2667.8	2667.8	2667.8	2667.8	2667.8	2667.8
0.0092	2609.8	2609.8	2609.8	2609.8	2609.8	2609.8
0.0095	2527.4	2527.4	2527.4	2527.4	2527.4	2527.4
0.0098	2450	2450	2450	2450	2450	2450

0.01	2401	2401	2401	2401	2401	2401
0.012	2008	2008	2008	2008	2008	2008
0.015	1600.7	1600.7	1600.7	1600.7	1600.7	1600.7
0.018	1333.9	1333.9	1333.9	1333.9	1333.9	1333.9
0.02	1200.5	1200.5	1200.5	1200.5	1200.5	1200.5
0.022	1091.4	1091.4	1091.4	1091.4	1091.4	1091.4
0.025	960.4	960.4	960.4	960.4	960.4	960.4
0.028	857.5	857.5	857.5	857.5	857.5	857.5
0.03	800.3	800.3	800.3	800.3	800.3	800.3
0.032	750.3	750.3	750.3	750.3	750.3	750.3
0.035	686	686	686	686	686	686
0.038	631.8	631.8	631.8	631.8	631.8	631.8
0.04	600.3	600.3	600.3	600.3	600.3	600.3
0.042	571.7	571.7	571.7	571.7	571.7	571.7
0.045	533.6	533.6	533.6	533.6	533.6	533.6
0.048	500.2	500.2	500.2	500.2	500.2	500.2
0.05	480.2	480.2	480.2	480.2	480.2	480.2
0.052	461.7	461.7	461.7	461.7	461.7	461.7
0.055	436.5	436.5	436.5	436.5	436.5	436.5
0.058	414	414	414	414	414	414
0.06	400.2	400.2	400.2	400.2	400.2	400.2
0.062	387.3	387.3	387.3	387.3	387.3	387.3
0.065	369.4	369.4	369.4	369.4	369.4	369.4
0.068	353.1	353.1	353.1	353.1	353.1	353.1
0.07	343	343	343	343	343	343
0.072	333.5	333.5	333.5	333.5	333.5	333.5
0.075	320.1	320.1	320.1	320.1	320.1	320.1
0.078	307.8	307.8	307.8	307.8	307.8	307.8
0.08	300.1	300.1	300.1	300.1	300.1	300.1
0.082	292.8	292.8	292.8	292.8	292.8	292.8
0.085	282.5	282.5	282.5	282.5	282.5	282.5
0.088	272.8	272.8	272.8	272.8	272.8	272.8
0.09	266.8	266.8	266.8	266.8	266.8	266.8
0.092	261	261	261	261	261	261
0.095	252.7	252.7	252.7	252.7	252.7	252.7
0.098	245	245	245	245	245	245
0.1	240.1	240.1	240.1	240.1	240.1	240.1
0.12	200.8	200.8	200.8	200.8	200.8	200.8
0.15	160.1	160.1	160.1	160.1	160.1	160.1
0.18	133.4	133.4	133.4	133.4	133.4	155
0.2	120.1	120.1	120.1	120.1	120.1	141.7
0.22	109.1	109.1	109.1	109.1	109.1	130.7
0.25	96	96	96	96	96	117.6
0.28	85.8	85.8	85.8	85.8	94.3	107.4
0.3	80	80	80	80	88.5	101.6
0.32	75	75	75	75	83.5	96.6
0.35	68.6	68.6	68.6	68.6	68.6	90.2
0.38	63.2	63.2	63.2	68.58	71.7	84.8
0.4	60	60	60	65.38	68.5	81.6
0.42	57.2	57.2	57.2	62.58	65.7	78.8

0.45	53.4	53.4	53.4	58.78	61.5	75
0.48	50	50	50	55.38	58.5	71.6
0.5	48	48	48	53.38	56.5	69.6
0.52	46.2	46.2	46.2	51.58	54.7	67.8
0.55	43.7	43.7	43.7	49.08	52.2	64.6
0.58	41.4	41.4	41.4	46.78	49.9	63
0.6	40	40	40	45.38	48.5	61.6
0.62	38.7	38.7	38.7	44.08	47.2	60.3
0.65	36.9	36.9	36.9	42.28	45.4	58.5
0.68	35.3	35.3	35.3	40.68	43.8	56.9
0.7	34.3	34.3	34.3	39.68	42.8	55.9
0.72	33.4	33.4	33.4	38.78	41.9	55
0.75	32	32	32	37.38	40.5	53.6
0.78	30.8	30.8	30.8	36.18	39.3	52.4
0.8	30	30	30	35.38	38.5	51.6
0.82	29.3	29.3	29.3	34.68	37.8	50.9
0.85	28.3	28.3	28.3	33.68	36.8	49.9
0.88	27.3	27.3	27.3	32.68	35.8	48.9
0.9	26.7	26.7	26.7	32.08	35.2	48.3
0.92	26.1	26.1	26.1	31.48	34.6	47.7
0.95	25.3	25.3	25.3	30.68	33.8	46.9
0.98	24.5	24.5	24.5	29.88	33	46.1
1	24	24	24	29.38	32.5	45.8
1.2	20.1	20.1	20.1	25.48	28.6	41.7
1.5	16	16	19.4	21.38	24.5	37.6
1.8	13.3	14.64	16.7	18.68	21.8	34.9
2	12	13.34	15.4	17.38	20.5	33.6
2.2	10.9	14.3	14.3	16.28	19.4	32.9
2.5	9.6	10.94	13	14.98	18.1	31.2
2.8	8.6	9.94	12	13.98	17.1	30.2
3	8	9.34	9.34	13.38	16.5	29.6
3.2	7.5	8.84	10.9	12.88	16	29.1
3.5	6.9	8.24	10.3	12.28	15.4	28.9
3.8	6.3	7.64	9.72	11.68	14.8	27.9
4	6	7.34	7.34	11.38	14.5	27.6
4.2	5.7	7.04	9.1	11.08	14.2	27.3
4.5	5.3	6.64	8.7	10.68	13.8	26.9
4.8	5	6.34	8.4	10.38	13.5	26.6
5	4.8	6.14	8.2	10.18	13.3	26.4
5.2	4.6	5.94	8	9.98	13.1	26.2
5.5	4.4	5.74	7.8	9.78	12.9	26
5.8	4.14	5.48	7.54	9.52	12.64	25.74
6	4	5.34	7.4	9.38	12.5	25.6
6.2	3.9	5.24	7.3	9.28	12.4	25.5
6.5	3.7	5.04	7.1	9.08	12.2	25.3
6.8	3.5	4.84	6.5	8.88	12	25.1
7	3.4	4.74	6.8	8.78	11.9	25
7.2	3.3	4.64	6.7	8.68	11.8	24.9
7.5	3.2	4.54	6.6	8.58	11.7	24.8
7.8	3.1	4.44	6.5	8.48	11.6	24.7

8	3	4.34	6.4	8.38	11.5	24.6
8.2	2.9	4.24	6.3	8.28	11.4	24.5
8.5	2.8	4.14	6.2	8.18	11.3	24.4
8.8	2.73	4.07	6.13	8.11	11.23	24.33
9	2.67	4.01	6.07	8.09	11.17	24.27
9.2	2.61	3.95	6.01	7.99	11.11	24.21
9.5	2.53	3.87	5.93	7.91	11.03	24.13
9.8	2.45	3.79	5.85	7.83	10.95	24.05
10	2.4	3.74	5.8	7.78	10.9	24
12	2.15	3.34	5.4	7.38	10.5	23.6
15	2.13	2.94	5	6.98	10.1	23.2
18	1.86	2.67	4.7	6.7	9.8	22.9
20	1.73	2.54	4.6	6.6	9.7	22.8
22	1.62	2.43	4.5	6.5	9.6	22.7
25	1.49	2.3	4.36	6.3	9.46	22.6
28	1.39	2.2	4.3	6.2	9.36	22.5
30	1.33	2.14	4.2	6.18	9.3	22.4
32	1.28	2.09	4.15	6.13	9.25	22.35
35	1.22	2.03	4.1	6.1	9.2	22.29
38	1.16	1.97	4.03	6.01	9.13	22.23
40	1.13	1.94	4	5.98	9.1	22.2
42	1.1	1.91	3.97	5.95	9.07	22.17
45	1.06	1.87	3.93	5.91	9.03	22.13
48	1.03	1.84	3.9	5.88	9	22.1
50	1.01	1.82	3.88	5.86	8.98	22.08
52	0.99	1.8	3.86	5.84	8.96	22.06
55	0.97	1.78	3.84	5.81	8.94	22.04
58	0.944	1.75	3.81	5.79	8.91	22.01
60	0.93	1.74	3.8	5.78	8.9	22
62	0.92	1.73	3.79	5.77	8.89	21.98
65	0.9	1.71	3.77	5.75	8.87	21.97
68	0.88	1.69	3.75	5.73	8.85	21.95
70	0.87	1.68	3.74	5.72	8.84	21.94
72	0.86	1.67	3.73	5.71	8.83	21.93
75	0.85	1.66	3.72	5.7	8.82	21.92
78	0.84	1.65	3.71	5.69	8.81	21.91
80	0.83	1.64	3.7	5.68	8.8	21.9
82	0.82	1.63	3.69	5.67	8.79	21.89
85	0.81	1.62	3.68	5.66	8.78	21.88
88	0.803	1.61	3.67	5.65	8.77	21.87
90	0.797	1.606	3.666	5.647	8.767	21.867
92	0.791	1.6	3.66	5.64	8.761	21.861
95	0.783	1.593	3.65	5.63	8.753	21.85
98	0.775	1.595	3.645	5.625	8.745	21.84
100	0.77	1.58	3.64	5.62	8.74	21.6
120	0.73	1.54	3.6	5.58	8.7	21.6
150	0.69	1.5	3.56	5.54	8.66	21.6
180	0.66	1.47	3.53	5.51	8.5	21.6
200	0.65	1.46	3.52	5.5	8.5	21.6
220	0.64	1.45	3.51	5.49	8.5	21.6

250	0.63	1.44	3.5	5.38	8.5	21.6
280	0.62	1.43	3.49	5.38	8.5	21.6
300	0.61	1.42	3.48	5.38	8.5	21.6
320	0.605	1.415	3.475	5.38	8.5	21.6
350	0.6	1.41	3.47	5.38	8.5	21.6
380	0.593	1.403	3.46	5.38	8.5	21.6
400	0.59	1.4	3.4	5.38	8.5	21.6
420	0.587	1.397	3.4	5.38	8.5	21.6
450	0.583	1.393	3.4	5.38	8.5	21.6
480	0.58	1.39	3.4	5.38	8.5	21.6
500	0.578	1.388	3.4	5.38	8.5	21.6
520	0.576	1.386	3.4	5.38	8.5	21.6
550	0.574	1.384	3.4	5.38	8.5	21.6
580	0.571	1.381	3.4	5.38	8.5	21.6
600	0.57	1.38	3.4	5.38	8.5	21.6
620	0.569	1.379	3.4	5.38	8.5	21.6
650	0.567	1.377	3.4	5.38	8.5	21.6
680	0.565	1.375	3.4	5.38	8.5	21.6
700	0.564	1.374	3.4	5.38	8.5	21.6
720	0.563	1.373	3.4	5.38	8.5	21.6
750	0.562	1.372	3.4	5.38	8.5	21.6
780	0.561	1.371	3.4	5.38	8.5	21.6
900	0.557	1.34	3.4	5.38	8.5	21.6
920	0.556	1.34	3.4	5.38	8.5	21.6
950	0.555	1.34	3.4	5.38	8.5	21.6
980	0.554	1.34	3.4	5.38	8.5	21.6
1000	0.53	1.34	3.4	5.38	8.5	21.6
1200	0.53	1.34	3.4	5.38	8.5	21.6
1500	0.53	1.34	3.4	5.38	8.5	21.6
1800	0.53	1.34	3.4	5.38	8.5	21.6
2000	0.53	1.34	3.4	5.38	8.5	21.6
2200	0.53	1.34	3.4	5.38	8.5	21.6
2500	0.53	1.34	3.4	5.38	8.5	21.6
2800	0.53	1.34	3.4	5.38	8.5	21.6
3000	0.53	1.34	3.4	5.38	8.5	21.6
3200	0.53	1.34	3.4	5.38	8.5	21.6
3500	0.53	1.34	3.4	5.38	8.5	21.6
3800	0.53	1.34	3.4	5.38	8.5	21.6
4000	0.53	1.34	3.4	5.38	8.5	21.6
4200	0.53	1.34	3.4	5.38	8.5	21.6
4500	0.53	1.34	3.4	5.38	8.5	21.6
4800	0.53	1.34	3.4	5.38	8.5	21.6
5000	0.53	1.34	3.4	5.38	8.5	21.6
5200	0.53	1.34	3.4	5.38	8.5	21.6
5500	0.53	1.34	3.4	5.38	8.5	21.6
5800	0.53	1.34	3.4	5.38	8.5	21.6
6000	0.53	1.34	3.4	5.38	8.5	21.6
6200	0.53	1.34	3.4	5.38	8.5	21.6
6500	0.53	1.34	3.4	5.38	8.5	21.6
6800	0.53	1.34	3.4	5.38	8.5	21.6

7000	0.53	1.34	3.4	5.38	8.5	21.6
7200	0.53	1.34	3.4	5.38	8.5	21.6
7500	0.53	1.34	3.4	5.38	8.5	21.6
7800	0.53	1.34	3.4	5.38	8.5	21.6
8000	0.53	1.34	3.4	5.38	8.5	21.6
8200	0.53	1.34	3.4	5.38	8.5	21.6
8500	0.53	1.34	3.4	5.38	8.5	21.6
8800	0.53	1.34	3.4	5.38	8.5	21.6
9000	0.53	1.34	3.4	5.38	8.5	21.6
9200	0.53	1.34	3.4	5.38	8.5	21.6
9500	0.53	1.34	3.4	5.38	8.5	21.6
9800	0.53	1.34	3.4	5.38	8.5	21.6
10000	0.53	1.34	3.4	5.38	8.5	21.6

A.2 Comparison results of drag coefficient with other investigator

Tab le (A.2) comparison results for spherical particles

Re_p	C_{dexp}	$C_{d\ sphere}$	$C_{d\ Muhannad}$	$C_{d\ Haider}$	s.d 1	s.d2	s.d 3
0.001	24000	24010	576570	24050.03	10	552570	50.03131
0.0012	20000	20008	452419.1	20046.9	8	432419.1	46.90343
0.0015	16000	16007	336241	16043.34	7	320241	43.3401
0.0018	13333	13339	263839.4	13373.96	6	250506.4	40.9639
0.002	12000	12005	229341.2	12039.14	5	217341.2	39.14269
0.0022	10909	10914	202036.5	10946.93	5	191127.5	37.93466
0.0025	9600	9604	170448	9636.169	4	160848	36.16898
0.0028	8570	8575	146599.3	8606.175	5	138029.3	36.1749
0.003	8000	8003.3	133746	8033.908	3.3	125746	33.90778
0.0032	7500	7503	122744.6	7533.142	3	115244.6	33.1417
0.0035	6857	6860	108953.6	6889.249	3	102096.6	32.24947
0.0038	6315.8	6318.4	97665.2	6346.975	2.6	91349.4	31.17465
0.004	6000	6002.5	91224.65	6030.624	2.5	85224.65	30.62391
0.0042	5714.3	5717	85492.98	5744.385	2.7	79778.68	30.08511
0.0045	5333.3	5335.5	77997.26	5362.706	2.2	72663.96	29.40634
0.0048	5000	5002	71581.56	5028.709	2	66581.56	28.70939
0.005	4800	4802	67798.78	4828.297	2	62998.78	28.2974
0.0052	4615.4	4617.3	64352.82	4643.292	1.9	59737.42	27.89175
0.0055	4363.6	4365.4	59726.85	4390.995	1.8	55363.25	27.39473
0.0058	4137.9	4140	55653.54	4164.78	2.1	51515.64	26.87973
0.006	4000	4001.6	53199.9	4026.528	1.6	49199.9	26.52833
0.0062	3871	3872.6	50929.69	3897.19	1.6	47058.69	26.18985
0.0065	3692.3	3694	47827.45	3718.095	1.7	44135.15	25.79472
0.0068	3529.4	3531	45041.74	3554.79	1.6	41512.34	25.39009
0.007	3428.6	3430	43338.27	3453.691	1.4	39909.67	25.09061
0.0072	3333.3	3334.7	41744.55	3358.203	1.4	38411.25	24.9032
0.0075	3200	3201.3	39538.53	3224.513	1.3	36338.53	24.51298
0.0078	3076.9	3078.2	37528.93	3101.098	1.3	34452.03	24.19799
0.008	3000	3001.2	36286.27	3023.959	1.2	33286.27	23.95917
0.0082	2926.8	2928	35113.94	2950.58	1.2	32187.14	23.77987
0.0085	2823.5	2824.7	33475.33	2846.98	1.2	30651.83	23.47976
0.0088	2727.3	2728.4	31966.13	2750.437	1.1	29238.83	23.13683
0.009	2666.7	2667.8	31024.83	2689.647	1.1	28358.13	22.94718
0.0092	2608.7	2609.8	30131.05	2631.498	1.1	27522.35	22.79802
0.0095	2526.3	2527.4	28872.18	2548.861	1.1	26345.88	22.56055
0.0098	2449	2450	27702.65	2471.278	1	25253.65	22.27753
0.01	2400	2401	26968.2	2422.139	1	24568.2	22.139
0.012	2000	2008	21161.23	2020.755	8	19161.23	20.75496
0.015	1600	1600.7	15727.17	1619.178	0.7	14127.17	19.17824
0.018	1333.3	1333.9	12340.69	1351.313	0.6	11007.39	18.01263
0.02	1200	1200.5	10727.09	1217.321	0.5	9527.095	17.32094
0.022	1090.9	1091.4	9449.957	1107.655	0.5	8359.057	16.75526
0.025	960	960.4	7972.45	976.0051	0.4	7012.45	16.00511
0.028	857	857.5	6856.963	872.5185	0.5	5999.963	15.51846

0.03	800	800.3	6255.77	815.0046	0.3	5455.77	15.00456
0.032	750	750.3	5741.198	764.6656	0.3	4991.198	14.66558
0.035	685.7	686	5096.142	699.9219	0.3	4410.442	14.22186
0.038	631.6	631.8	4568.144	645.3788	0.2	3936.544	13.77879
0.04	600	600.3	4266.898	613.5515	0.3	3666.898	13.55149
0.042	571.4	571.7	3998.807	584.748	0.3	3427.407	13.34797
0.045	533.3	533.6	3648.206	546.3313	0.3	3114.906	13.03132
0.048	500	500.2	3348.121	512.7043	0.2	2848.121	12.70434
0.05	480	480.2	3171.187	492.522	0.2	2691.187	12.52204
0.052	461.5	461.7	3010.007	473.8878	0.2	2548.507	12.38782
0.055	436.4	436.5	2793.635	448.4702	0.1	2357.235	12.07017
0.058	413.8	414	2603.112	425.6741	0.2	2189.312	11.87411
0.06	400	400.2	2488.346	411.7393	0.2	2088.346	11.73925
0.062	387.1	387.3	2382.161	398.7005	0.2	1995.061	11.60053
0.065	369.2	369.4	2237.058	380.642	0.2	1867.858	11.44201
0.068	352.9	353.1	2106.76	364.1716	0.2	1753.86	11.27157
0.07	342.9	343	2027.083	353.9729	0.1	1684.183	11.07287
0.072	333.3	333.5	1952.539	344.3387	0.2	1619.239	11.03874
0.075	320	320.1	1849.356	330.8475	0.1	1529.356	10.84749
0.078	307.7	307.8	1755.36	318.3902	0.1	1447.66	10.69021
0.08	300	300.1	1697.236	310.6024	0.1	1397.236	10.60243
0.082	292.7	292.8	1642.402	303.1931	0.1	1349.702	10.49307
0.085	282.4	282.5	1565.759	292.7302	0.1	1283.359	10.33023
0.088	272.7	272.8	1495.168	282.9779	0.1	1222.468	10.2779
0.09	266.7	266.8	1451.141	276.8361	0.1	1184.441	10.13606
0.092	260.9	261	1409.335	270.9601	0.1	1148.435	10.06013
0.095	252.6	252.7	1350.453	262.6082	0.1	1097.853	10.00816
0.098	244.9	245	1295.75	254.7653	0.1	1050.85	9.865322
0.1	240	240.1	1261.398	249.797	0.1	1021.398	9.797036
0.12	200	200.8	989.7852	209.1846	0.8	789.7852	9.184616
0.15	160	160.1	735.6151	168.4869	0.1	575.6151	8.486936
0.18	133.3	133.4	577.2176	141.2898	0.1	443.9176	7.989758
0.2	120	120.1	501.7439	127.6651	0.1	381.7439	7.665108
0.22	109.09	109.1	442.0077	116.5017	0.01	332.9177	7.411691
0.25	96	96	372.8995	103.0829	0	276.8995	7.082871
0.28	85.7	85.8	320.7243	92.51861	0.1	235.0243	6.818609
0.3	80	80	292.6043	86.64014	0	212.6043	6.640141
0.32	75	75	268.536	81.49015	0	193.536	6.490148
0.35	68.57	68.6	238.3645	74.85891	0.03	169.7945	6.288913
0.38	63.16	63.2	213.6682	69.26497	0.04	150.5082	6.104967
0.4	60	60	199.5778	65.99718	0	139.5778	5.997178
0.42	57.14	57.2	187.0383	63.03734	0.06	129.8983	5.897339
0.45	53.3	53.4	170.6394	59.08559	0.1	117.3394	5.785592
0.48	50	50	156.6034	55.62233	0	106.6034	5.622326
0.5	48	48	148.3276	53.54166	0	100.3276	5.54166
0.52	46.15	46.2	140.7886	51.6191	0.05	94.63862	5.469096
0.55	43.64	43.7	130.6681	48.99417	0.06	87.02812	5.354167
0.58	41.38	41.4	121.7567	46.63732	0.02	80.37668	5.257323
0.6	40	40	116.3887	45.19529	0	76.38871	5.195287
0.62	38.71	38.7	111.422	43.84501	0.01	72.71203	5.135009
0.65	36.92	36.9	104.6351	41.97322	0.02	67.71506	5.053222
0.68	35.29	35.3	98.54059	40.26424	0.01	63.25059	4.974241

0.7	34.29	34.3	94.81381	39.2051	0.01	60.52381	4.915098
0.72	33.33	33.4	91.32713	38.2039	0.07	57.99713	4.873903
0.75	32	32	86.50088	36.80069	0	54.50088	4.800693
0.78	30.77	30.8	82.10435	35.50373	0.03	51.33435	4.733733
0.8	30	30	79.38571	34.69226	0	49.38571	4.69226
0.82	29.27	29.3	76.82093	33.91972	0.03	47.55093	4.649717
0.85	28.24	28.3	73.23604	32.82793	0.06	44.99604	4.587929
0.88	27.27	27.3	69.93427	31.80932	0.03	42.66427	4.539318
0.9	26.67	26.7	67.87494	31.16731	0.03	41.20494	4.497312
0.92	26.09	26.1	65.91955	30.55272	0.01	39.82955	4.462722
0.95	25.26	25.3	63.16545	29.67849	0.04	37.90545	4.418485
0.98	24.49	24.5	60.6068	28.8568	0.01	36.1168	4.366797
1	24	24	59	28.33588	0	35	4.335882
1.2	20	20.1	46.29573	24.06489	0.1	26.29573	4.064895
1.5	16	16	34.4073	19.75618	0	18.4073	3.756181
1.8	13.33	13.3	26.9985	16.85477	0.03	13.6685	3.52477
2	12	12	23.46833	15.39253	0	11.46833	3.392533
2.2	10.91	10.9	20.67425	14.18909	0.01	9.764254	3.279088
2.5	9.6	9.6	17.44182	12.7349	0	7.84182	3.134901
2.8	8.57	8.6	15.0014	11.58308	0.03	6.431401	3.013076
3	8	8	13.68613	10.939	0	5.686133	2.938999
3.2	7.5	7.5	12.56037	10.37263	0	5.060373	2.872629
3.5	6.86	6.9	11.14914	9.640096	0.04	4.289144	2.780096
3.8	6.316	6.3	9.994011	9.018912	0.016	3.678011	2.702912
4	6	6	9.334955	8.654496	0	3.334955	2.654496
4.2	5.71	5.7	8.748437	8.32334	0.01	3.038437	2.61334
4.5	5.33	5.3	7.981405	7.879456	0.03	2.651405	2.549456
4.8	5	5	7.324891	7.488629	0	2.324891	2.488629
5	4.8	4.8	7.144	7.252935	0	2.344	2.452935
5.2	4.615	4.6	6.915769	7.034509	0.015	2.300769	2.419509
5.5	4.36	4.4	6.604545	6.735217	0.04	2.244545	2.375217
5.8	4.13	4.14	6.325517	6.465356	0.01	2.195517	2.335356
6	4	4	6.155	6.29967	0	2.155	2.29967
6.2	3.87	3.9	5.995484	6.144108	0.03	2.125484	2.274108
6.5	3.69	3.7	5.774615	5.927754	0.01	2.084615	2.237754
6.8	3.529	3.5	5.573235	5.729449	0.029	2.044235	2.200449
7	3.429	3.4	5.448571	5.606157	0.029	2.019571	2.177157
7.2	3.33	3.3	5.330833	5.48932	0.03	2.000833	2.15932
7.5	3.2	3.2	5.166	5.325067	0	1.966	2.125067
7.8	3.08	3.1	5.013846	5.172701	0.02	1.933846	2.092701
8	3	3	4.91875	5.077087	0	1.91875	2.077087
8.2	2.927	2.9	4.828293	4.985846	0.027	1.901293	2.058846
8.5	2.82	2.8	4.700588	4.856533	0.02	1.880588	2.036533
8.8	2.727	2.73	4.581591	4.735478	0.003	1.854591	2.008478
9	2.667	2.67	4.506667	4.658966	0.003	1.839667	1.991966
9.2	2.6	2.61	4.435	4.585561	0.01	1.835	1.985561
9.5	2.526	2.53	4.333158	4.480863	0.004	1.807158	1.954863
9.8	2.449	2.45	4.237551	4.382143	0.001	1.788551	1.933143
10	2.4	2.4	4.177	4.319394	0	1.777	1.919394
12	2	2.15	3.6825	3.799485	0.15	1.6825	1.799485
15	2.04	2.13	3.188	3.262883	0.09	1.148	1.222883
18	1.77	1.86	2.858333	2.892344	0.09	1.088333	1.122344

20	1.64	1.73	2.6935	2.701972	0.09	1.0535	1.061972
22	1.53	1.62	2.558636	2.543085	0.09	1.028636	1.013085
25	1.4	1.49	2.3968	2.347973	0.09	0.9968	0.947973
28	1.3	1.39	2.269643	2.190577	0.09	0.969643	0.890577
30	1.24	1.33	2.199	2.101288	0.09	0.959	0.861288
32	1.19	1.28	2.137188	2.021919	0.09	0.947188	0.831919
35	1.13	1.22	2.057714	1.917953	0.09	0.927714	0.787953
38	1.07	1.16	1.990789	1.828493	0.09	0.920789	0.758493
40	1.04	1.13	1.95175	1.775397	0.09	0.91175	0.735397
42	1.01	1.1	1.916429	1.726718	0.09	0.906429	0.716718
45	0.97	1.06	1.869333	1.660775	0.09	0.899333	0.690775
48	0.94	1.03	1.828125	1.602001	0.09	0.888125	0.662001
50	0.92	1.01	1.8034	1.566207	0.09	0.8834	0.646207
52	0.9	0.99	1.780577	1.532784	0.09	0.880577	0.632784
55	0.88	0.97	1.749455	1.486572	0.09	0.869455	0.606572
58	0.85	0.944	1.721552	1.444462	0.094	0.871552	0.594462
60	0.64	0.93	1.7045	1.418387	0.29	1.0645	0.778387
62	0.83	0.92	1.688548	1.393745	0.09	0.858548	0.563745
65	0.81	0.9	1.666462	1.359199	0.09	0.856462	0.549199
68	0.79	0.88	1.646324	1.327241	0.09	0.856324	0.537241
70	0.78	0.87	1.633857	1.307222	0.09	0.853857	0.527222
72	0.77	0.86	1.622083	1.28814	0.09	0.852083	0.51814
75	0.76	0.85	1.6056	1.261125	0.09	0.8456	0.501125
78	0.75	0.84	1.590385	1.235857	0.09	0.840385	0.485857
80	0.74	0.83	1.580875	1.219893	0.09	0.840875	0.479893
82	0.73	0.82	1.571829	1.20458	0.09	0.841829	0.47458
85	0.72	0.81	1.559059	1.182739	0.09	0.839059	0.462739
88	0.71	0.803	1.547159	1.16214	0.093	0.837159	0.45214
90	0.707	0.797	1.539667	1.149041	0.09	0.832667	0.442041
92	0.7	0.791	1.5325	1.136414	0.091	0.8325	0.436414
95	0.69	0.783	1.522316	1.1183	0.093	0.832316	0.4283
98	0.685	0.775	1.512755	1.101103	0.09	0.827755	0.416103
100	0.68	0.77	1.5067	1.090112	0.09	0.8267	0.410112
120	0.64	0.73	1.45725	0.99705	0.09	0.81725	0.35705
150	0.6	0.69	1.4078	0.8966	0.09	0.8078	0.2966
180	0.57	0.66	1.374833	0.823965	0.09	0.804833	0.253965
200	0.56	0.65	1.35835	0.785389	0.09	0.79835	0.225389
220	0.55	0.64	0.98	0.752442	0.09	0.43	0.202442
250	0.54	0.63	0.98	0.710936	0.09	0.44	0.170936
280	0.53	0.62	0.98	0.676512	0.09	0.45	0.146512
300	0.52	0.61	0.98	0.65657	0.09	0.46	0.13657
320	0.515	0.605	0.98	0.638571	0.09	0.465	0.123571
350	0.51	0.6	0.98	0.614578	0.09	0.47	0.104578
380	0.503	0.593	0.98	0.593529	0.09	0.477	0.090529
400	0.5	0.59	0.98	0.580846	0.09	0.48	0.080846
420	0.497	0.587	0.98	0.569088	0.09	0.483	0.072088
450	0.493	0.583	0.98	0.552951	0.09	0.487	0.059951
480	0.49	0.58	0.98	0.538355	0.09	0.49	0.048355
500	0.488	0.578	0.98	0.529363	0.09	0.492	0.041363
520	0.486	0.576	0.98	0.520893	0.09	0.494	0.034893
550	0.484	0.574	0.98	0.509061	0.09	0.496	0.025061
580	0.481	0.571	0.98	0.498153	0.09	0.499	0.017153

600	0.48	0.57	0.98	0.491336	0.09	0.5	0.011336
620	0.479	0.569	0.98	0.484848	0.09	0.501	0.005848
650	0.477	0.567	0.98	0.475675	0.09	0.503	0.001325
680	0.475	0.565	0.98	0.467108	0.09	0.505	0.007892
700	0.474	0.564	0.98	0.4617	0.09	0.506	0.0123
720	0.473	0.563	0.98	0.456515	0.09	0.507	0.016485
750	0.472	0.562	0.98	0.449123	0.09	0.508	0.022877
780	0.471	0.561	0.98	0.442152	0.09	0.509	0.028848
900	0.467	0.557	0.98	0.417769	0.09	0.513	0.049231
920	0.466	0.556	0.98	0.414164	0.09	0.514	0.051836
950	0.465	0.555	0.98	0.408965	0.09	0.515	0.056035
980	0.464	0.554	0.98	0.404	0.09	0.516	0.06
1000	0.44	0.53	0.98	0.400811	0.09	0.54	0.039189
1200	0.44	0.53	0.98	0.373302	0.09	0.54	0.066698
1500	0.44	0.53	0.98	0.342511	0.09	0.54	0.097489
1800	0.44	0.53	0.98	0.319464	0.09	0.54	0.120536
2000	0.44	0.53	0.98	0.306937	0.09	0.54	0.133063
2200	0.44	0.53	0.98	0.296071	0.09	0.54	0.143929
2500	0.44	0.53	0.98	0.282155	0.09	0.54	0.157845
2800	0.44	0.53	0.98	0.270415	0.09	0.54	0.169585
3000	0.44	0.53	0.98	0.263528	0.09	0.54	0.176472
3200	0.44	0.53	0.98	0.257258	0.09	0.54	0.182742
3500	0.44	0.53	0.98	0.248816	0.09	0.54	0.191184
3800	0.44	0.53	0.98	0.241331	0.09	0.54	0.198669
4000	0.44	0.53	0.98	0.236785	0.09	0.54	0.203215
4200	0.44	0.53	0.98	0.232545	0.09	0.54	0.207455
4500	0.44	0.53	0.98	0.226688	0.09	0.54	0.213312
4800	0.44	0.53	0.98	0.221351	0.09	0.54	0.218649
5000	0.44	0.53	0.98	0.218044	0.09	0.54	0.221956
5200	0.44	0.53	0.98	0.214916	0.09	0.54	0.225084
5500	0.44	0.53	0.98	0.210524	0.09	0.54	0.229476
5800	0.44	0.53	0.98	0.206454	0.09	0.54	0.233546
6000	0.44	0.53	0.98	0.203899	0.09	0.54	0.236101
6200	0.44	0.53	0.98	0.201459	0.09	0.54	0.238541
6500	0.44	0.53	0.98	0.197997	0.09	0.54	0.242003
6800	0.44	0.53	0.98	0.19475	0.09	0.54	0.24525
7000	0.44	0.53	0.98	0.192693	0.09	0.54	0.247307
7200	0.44	0.53	0.98	0.190716	0.09	0.54	0.249284
7500	0.44	0.53	0.98	0.187889	0.09	0.54	0.252111
7800	0.44	0.53	0.98	0.185214	0.09	0.54	0.254786
8000	0.44	0.53	0.98	0.183508	0.09	0.54	0.256492
8200	0.44	0.53	0.98	0.181861	0.09	0.54	0.258139
8500	0.44	0.53	0.98	0.179491	0.09	0.54	0.260509
8800	0.44	0.53	0.98	0.177233	0.09	0.54	0.262767
9000	0.44	0.53	0.98	0.175786	0.09	0.54	0.264214
9200	0.44	0.53	0.98	0.174383	0.09	0.54	0.265617
9500	0.44	0.53	0.98	0.172356	0.09	0.54	0.267644
9800	0.44	0.53	0.98	0.170414	0.09	0.54	0.269586
10000	0.44	0.53	0.98	0.169165	0.09	0.54	0.270835
					0.519711	15932.2	7.614118

A.3 Comparison results for irregular shaped particles

Table (A.3) Comparsion results for ψ=0.8

Re_p	C_{dexp}	C_d ψ=0.8	$C_{d\ Muhannad}$	$C_{d\ Haider}$	s.d pres	s.d muh	s.d haider
0.001	24000	24010	576570	678.9105	10	552570	23321.09
0.0012	20000	20008	452419.1	625.7768	8	432419.1	19374.22
0.0015	16000	16007	336241	566.3723	7	320241	15433.63
0.0018	13333	13339	263839.4	522.0465	6	250506.4	12810.95
0.002	12000	12005	229341.2	498.0309	5	217341.2	11501.97
0.0022	10909	10914	202036.5	477.2594	5	191127.5	10431.74
0.0025	9600	9604	170448	450.7536	4	160848	9149.246
0.0028	8570	8575	146599.3	428.4889	5	138029.3	8141.511
0.003	8000	8003.3	133746	415.4767	3.3	125746	7584.523
0.0032	7500	7503	122744.6	403.6624	3	115244.6	7096.338
0.0035	6857	6860	108953.6	387.8134	3	102096.6	6469.187
0.0038	6315.8	6318.4	97665.2	373.8167	2.6	91349.4	5941.983
0.004	6000	6002.5	91224.65	365.3437	2.5	85224.65	5634.656
0.0042	5714.3	5717	85492.98	357.4625	2.7	79778.68	5356.838
0.0045	5333.3	5335.5	77997.26	346.6072	2.2	72663.96	4986.693
0.0048	5000	5002	71581.56	336.7514	2	66581.56	4663.249
0.005	4800	4802	67798.78	330.6626	2	62998.78	4469.337
0.0052	4615.4	4617.3	64352.82	324.9164	1.9	59737.42	4290.484
0.0055	4363.6	4365.4	59726.85	316.8718	1.8	55363.25	4046.728
0.0058	4137.9	4140	55653.54	309.4382	2.1	51515.64	3828.462
0.006	4000	4001.6	53199.9	304.7846	1.6	49199.9	3695.215
0.0062	3871	3872.6	50929.69	300.3502	1.6	47058.69	3570.65
0.0065	3692.3	3694	47827.45	294.0731	1.7	44135.15	3398.227
0.0068	3529.4	3531	45041.74	288.2018	1.6	41512.34	3241.198
0.007	3428.6	3430	43338.27	284.4918	1.4	39909.67	3144.108
0.0072	3333.3	3334.7	41744.55	280.932	1.4	38411.25	3052.368
0.0075	3200	3201.3	39538.53	275.8526	1.3	36338.53	2924.147
0.0078	3076.9	3078.2	37528.93	271.0589	1.3	34452.03	2805.841
0.008	3000	3001.2	36286.27	268.0088	1.2	33286.27	2731.991
0.0082	2926.8	2928	35113.94	265.0671	1.2	32187.14	2661.733
0.0085	2823.5	2824.7	33475.33	260.844	1.2	30651.83	2562.656
0.0088	2727.3	2728.4	31966.13	256.8312	1.1	29238.83	2470.469
0.009	2666.7	2667.8	31024.83	254.2643	1.1	28358.13	2412.436
0.0092	2608.7	2609.8	30131.05	251.7787	1.1	27522.35	2356.921
0.0095	2526.3	2527.4	28872.18	248.1933	1.1	26345.88	2278.107
0.0098	2449	2450	27702.65	244.7682	1	25253.65	2204.232
0.01	2400	2401	26968.2	242.5679	1	24568.2	2157.432
0.012	2000	2008	21161.23	223.5847	8	19161.23	1776.415
0.015	1600	1600.7	15727.17	202.3611	0.7	14127.17	1397.639
0.018	1333.3	1333.9	12340.69	186.5246	0.6	11007.39	1146.775
0.02	1200	1200.5	10727.09	177.9445	0.5	9527.095	1022.055
0.022	1090.9	1091.4	9449.957	170.5234	0.5	8359.057	920.3766
0.025	960	960.4	7972.45	161.0536	0.4	7012.45	798.9464
0.028	857	857.5	6856.963	153.0991	0.5	5999.963	703.9009

0.03	800	800.3	6255.77	148.4501	0.3	5455.77	651.5499
0.032	750	750.3	5741.198	144.2292	0.3	4991.198	605.7708
0.035	685.7	686	5096.142	138.5668	0.3	4410.442	547.1332
0.038	631.6	631.8	4568.144	133.5662	0.2	3936.544	498.0338
0.04	600	600.3	4266.898	130.539	0.3	3666.898	469.461
0.042	571.4	571.7	3998.807	127.7232	0.3	3427.407	443.6768
0.045	533.3	533.6	3648.206	123.845	0.3	3114.906	409.455
0.048	500	500.2	3348.121	120.3237	0.2	2848.121	379.6763
0.05	480	480.2	3171.187	118.1484	0.2	2691.187	361.8516
0.052	461.5	461.7	3010.007	116.0954	0.2	2548.507	345.4046
0.055	436.4	436.5	2793.635	113.2213	0.1	2357.235	323.1787
0.058	413.8	414	2603.112	110.5655	0.2	2189.312	303.2345
0.06	400	400.2	2488.346	108.9029	0.2	2088.346	291.0971
0.062	387.1	387.3	2382.161	107.3186	0.2	1995.061	279.7814
0.065	369.2	369.4	2237.058	105.076	0.2	1867.858	264.124
0.068	352.9	353.1	2106.76	102.9783	0.2	1753.86	249.9217
0.07	342.9	343	2027.083	101.6528	0.1	1684.183	241.2472
0.072	333.3	333.5	1952.539	100.381	0.2	1619.239	232.919
0.075	320	320.1	1849.356	98.56623	0.1	1529.356	221.4338
0.078	307.7	307.8	1755.36	96.85358	0.1	1447.66	210.8464
0.08	300	300.1	1697.236	95.76386	0.1	1397.236	204.2361
0.082	292.7	292.8	1642.402	94.71287	0.1	1349.702	197.9871
0.085	282.4	282.5	1565.759	93.20407	0.1	1283.359	189.1959
0.088	272.7	272.8	1495.168	91.77041	0.1	1222.468	180.9296
0.09	266.7	266.8	1451.141	90.85334	0.1	1184.441	175.8467
0.092	260.9	261	1409.335	89.96529	0.1	1148.435	170.9347
0.095	252.6	252.7	1350.453	88.68435	0.1	1097.853	163.9157
0.098	244.9	245	1295.75	87.46063	0.1	1050.85	157.4394
0.1	240	240.1	1261.398	86.67453	0.1	1021.398	153.3255
0.12	200	200.8	989.7852	79.89234	0.8	789.7852	120.1077
0.15	160	160.1	735.6151	72.30972	0.1	575.6151	87.69028
0.18	133.3	133.4	577.2176	66.65179	0.1	443.9176	66.64821
0.2	120	120.1	501.7439	63.58635	0.1	381.7439	56.41365
0.22	109.09	109.1	442.0077	60.93499	0.01	332.9177	48.15501
0.25	96	96	372.8995	57.55169	0	276.8995	38.44831
0.28	85.7	85.8	320.7243	54.70975	0.1	235.0243	30.99025
0.3	80	80	292.6043	53.04881	0	212.6043	26.95119
0.32	75	75	268.536	51.54079	0	193.536	23.45921
0.35	68.57	68.6	238.3645	49.51775	0.03	169.7945	19.05225
0.38	63.16	63.2	213.6682	47.73116	0.04	150.5082	15.42884
0.4	60	60	199.5778	46.64964	0	139.5778	13.35036
0.42	57.14	57.2	187.0383	45.64365	0.06	129.8983	11.49635
0.45	53.3	53.4	170.6394	44.25804	0.1	117.3394	9.04196
0.48	50	50	156.6034	43.00001	0	106.6034	6.999994
0.5	48	48	148.3276	42.22281	0	100.3276	5.777192
0.52	46.15	46.2	140.7886	41.48934	0.05	94.63862	4.660663
0.55	43.64	43.7	130.6681	40.46249	0.06	87.02812	3.177505
0.58	41.38	41.4	121.7567	39.51365	0.02	80.37668	1.866353
0.6	40	40	116.3887	38.91964	0	76.38871	1.080357
0.62	38.71	38.7	111.422	38.35362	0.01	72.71203	0.356382
0.65	36.92	36.9	104.6351	37.55238	0.02	67.71506	0.632383
0.68	35.29	35.3	98.54059	36.80294	0.01	63.25059	1.512942

0.7	34.29	34.3	94.81381	36.32938	0.01	60.52381	2.039379
0.72	33.33	33.4	91.32713	35.875	0.07	57.99713	2.544999
0.75	32	32	86.50088	35.22663	0	54.50088	3.226635
0.78	30.77	30.8	82.10435	34.61475	0.03	51.33435	3.844749
0.8	30	30	79.38571	34.22542	0	49.38571	4.225423
0.82	29.27	29.3	76.82093	33.84993	0.03	47.55093	4.579931
0.85	28.24	28.3	73.23604	33.31088	0.06	44.99604	5.070877
0.88	27.27	27.3	69.93427	32.79867	0.03	42.66427	5.52867
0.9	26.67	26.7	67.87494	32.47102	0.03	41.20494	5.801025
0.92	26.09	26.1	65.91955	32.15375	0.01	39.82955	6.063749
0.95	25.26	25.3	63.16545	31.6961	0.04	37.90545	6.436101
0.98	24.49	24.5	60.6068	31.2589	0.01	36.1168	6.768899
1	24	24	59	30.97805	0	35	6.978045
1.2	20	20.1	46.29573	28.55495	0.1	26.29573	8.55495
1.5	16	16	34.4073	25.84588	0	18.4073	9.845879
1.8	14.53333	14.67882	26.9985	23.82445	0.145483	12.46516	9.291118
2	13.34	13.34494	23.46833	22.72925	0.004935	10.12833	9.389249
2.2	12.24909	12.25358	20.67425	21.78199	0.004486	8.425163	9.532894
2.5	10.94	10.94395	17.44182	20.57322	0.003948	6.50182	9.633217
2.8	9.911429	9.914954	15.0014	19.55786	0.003525	5.089973	9.646434
3	9.34	9.34329	13.68613	18.96445	0.00329	4.346133	9.62445
3.2	8.84	8.843084	12.56037	18.42567	0.003084	3.720373	9.585674
3.5	8.197143	8.199963	11.14914	17.70289	0.00282	2.952001	9.505748
3.8	7.655789	7.658387	9.994011	17.06458	0.002597	2.338222	9.408795
4	7.34	7.342468	9.334955	16.67818	0.002467	1.994955	9.338181
4.2	7.054286	7.056636	8.748437	16.31876	0.00235	1.694151	9.264479
4.5	6.673333	6.675527	7.981405	15.82372	0.002193	1.308072	9.150385
4.8	6.34	6.342056	7.324891	15.37425	0.002056	0.984891	9.034251
5	6.14	6.141974	7.144	15.09658	0.001974	1.004	8.956575
5.2	5.955385	5.957283	6.915769	14.83452	0.001898	0.960385	8.879137
5.5	5.703636	5.705431	6.604545	14.46765	0.001795	0.900909	8.764017
5.8	5.477931	5.479633	6.325517	14.12865	0.001702	0.847586	8.650719
6	5.34	5.341645	6.155	13.91642	0.001645	0.815	8.576424
6.2	5.210968	5.21256	5.995484	13.71419	0.001592	0.784516	8.503227
6.5	5.032308	5.033826	5.774615	13.42793	0.001518	0.742308	8.395622
6.8	4.869412	4.870863	5.573235	13.16017	0.001451	0.703824	8.290757
7	4.768571	4.769981	5.448571	12.99097	0.00141	0.68	8.222402
7.2	4.673333	4.674704	5.330833	12.82863	0.001371	0.6575	8.155299
7.5	4.54	4.541316	5.166	12.59698	0.001316	0.626	8.056983
7.8	4.416923	4.418188	5.013846	12.37837	0.001265	0.596923	7.961444
8	4.34	4.341234	4.91875	12.23927	0.001234	0.57875	7.899268
8.2	4.266829	4.268033	4.828293	12.10511	0.001204	0.561463	7.838282
8.5	4.163529	4.164691	4.700588	11.91252	0.001161	0.537059	7.748986
8.8	4.067273	4.068394	4.581591	11.72951	0.001122	0.514318	7.66224
9	4.006667	4.007763	4.506667	11.61245	0.001097	0.5	7.605783
9.2	3.948696	3.949768	4.435	11.49909	0.001073	0.486304	7.550396
9.5	3.866316	3.867355	4.333158	11.33558	0.001039	0.466842	7.469265
9.8	3.78898	3.789987	4.237551	11.17937	0.001007	0.448571	7.390395
10	3.74	3.740987	4.177	11.07903	0.000987	0.437	7.339029
12	3.34	3.340823	3.6825	10.21328	0.000822	0.3425	6.873284
15	2.94	2.940658	3.188	9.245349	0.000658	0.248	6.305349
18	2.673333	2.673882	2.858333	8.523091	0.000548	0.185	5.849757

20	2.54	2.540494	2.6935	8.131766	0.000494	0.1535	5.591766
22	2.430909	2.431358	2.558636	7.793297	0.000449	0.127727	5.362388
25	2.3	2.300395	2.3968	7.361381	0.000395	0.0968	5.061381
28	2.197143	2.197495	2.269643	6.998566	0.000352	0.0725	4.801423
30	2.14	2.140329	2.199	6.786518	0.000329	0.059	4.646518
32	2.09	2.090308	2.137188	6.593991	0.000308	0.047188	4.503991
35	2.025714	2.025996	2.057714	6.335706	0.000282	0.032	4.309991
38	1.971579	1.971839	1.990789	6.107601	0.00026	0.019211	4.136022
40	1.94	1.940247	1.95175	5.969513	0.000247	0.01175	4.029513
42	1.911429	1.911664	1.916429	5.841068	0.000235	0.005	3.929639
45	1.873333	1.873553	1.869333	5.664147	0.000219	0.004	3.790814
48	1.84	1.840206	1.828125	5.503511	0.000206	0.011875	3.663511
50	1.82	1.820197	1.8034	5.404269	0.000197	0.0166	3.584269
52	1.801538	1.801728	1.780577	5.31061	0.00019	0.020962	3.509071
55	1.776364	1.776543	1.749455	5.179485	0.000179	0.026909	3.403121
58	1.753793	1.753963	1.721552	5.058316	0.00017	0.032241	3.304522
60	1.74	1.740165	1.7045	4.982458	0.000164	0.0355	3.242458
62	1.727097	1.727256	1.688548	4.910173	0.000159	0.038548	3.183076
65	1.709231	1.709383	1.666462	4.807847	0.000152	0.042769	3.098616
68	1.692941	1.693086	1.646324	4.712132	0.000145	0.046618	3.01919
70	1.682857	1.682998	1.633857	4.651649	0.000141	0.049	2.968792
72	1.673333	1.67347	1.622083	4.593615	0.000137	0.05125	2.920281
75	1.66	1.660132	1.6056	4.510803	0.000132	0.0544	2.850803
78	1.647692	1.647819	1.590385	4.432647	0.000127	0.057308	2.784954
80	1.64	1.640123	1.580875	4.382917	0.000123	0.059125	2.742917
82	1.632683	1.632803	1.571829	4.334953	0.00012	0.060854	2.70227
85	1.622353	1.622469	1.559059	4.266095	0.000116	0.063294	2.643742
88	1.612727	1.612839	1.547159	4.200664	0.000112	0.065568	2.587937
90	1.606667	1.606776	1.539667	4.158808	0.00011	0.067	2.552141
92	1.60087	1.600977	1.5325	4.118276	0.000107	0.06837	2.517406
95	1.592632	1.592735	1.522316	4.059809	0.000104	0.070316	2.467178
98	1.584898	1.584999	1.512755	4.003953	0.000101	0.072143	2.419055
100	1.58	1.580099	1.5067	3.968071	9.87E-05	0.0733	2.388071
120	1.54	1.540082	1.45725	3.658453	8.22E-05	0.08275	2.118453
150	1.5	1.500066	1.4078	3.312193	6.58E-05	0.0922	1.812193
180	1.473333	1.473388	1.374833	3.053733	5.48E-05	0.0985	1.5804
200	1.46	1.460049	1.35835	2.913659	4.94E-05	0.10165	1.453659
220	1.449091	1.449136	0.98	2.792479	4.49E-05	0.469091	1.343388
250	1.436	1.436039	0.98	2.637803	3.95E-05	0.456	1.201803
280	1.425714	1.42575	0.98	2.507836	3.53E-05	0.445714	1.082121
300	1.42	1.420033	0.98	2.431858	3.29E-05	0.44	1.011858
320	1.415	1.415031	0.98	2.362862	3.08E-05	0.435	0.947862
350	1.408571	1.4086	0.98	2.270281	2.82E-05	0.428571	0.861709
380	1.403158	1.403184	0.98	2.188497	2.6E-05	0.423158	0.785339
400	1.4	1.400025	0.98	2.138978	2.47E-05	0.42	0.738978
420	1.397143	1.397166	0.98	2.09291	2.35E-05	0.417143	0.695767
450	1.393333	1.393355	0.98	2.029445	2.19E-05	0.413333	0.636112
480	1.39	1.390021	0.98	1.97181	2.06E-05	0.41	0.58181
500	1.388	1.38802	0.98	1.936197	1.97E-05	0.408	0.548197
520	1.386154	1.386173	0.98	1.902584	1.9E-05	0.406154	0.51643
550	1.383636	1.383654	0.98	1.855518	1.79E-05	0.403636	0.471881
580	1.381379	1.381396	0.98	1.812018	1.7E-05	0.401379	0.430639

600	1.38	1.380016	0.98	1.784782	1.64E-05	0.4	0.404782
620	1.37871	1.378726	0.98	1.758826	1.59E-05	0.39871	0.380116
650	1.376923	1.376938	0.98	1.722079	1.52E-05	0.396923	0.345156
680	1.375294	1.375309	0.98	1.687702	1.45E-05	0.395294	0.312407
700	1.374286	1.3743	0.98	1.665976	1.41E-05	0.394286	0.291691
720	1.373333	1.373347	0.98	1.645129	1.37E-05	0.393333	0.271796
750	1.372	1.372013	0.98	1.615378	1.32E-05	0.392	0.243378
780	1.370769	1.370782	0.98	1.587298	1.27E-05	0.390769	0.216529
900	1.2	1.34	0.98	1.48889	0.14	0.22	0.28889
920	1.2	1.34	0.98	1.474322	0.14	0.22	0.274322
950	1.2	1.34	0.98	1.453307	0.14	0.22	0.253307
980	1.2	1.34	0.98	1.433229	0.14	0.22	0.233229
1000	1.2	1.34	0.98	1.42033	0.14	0.22	0.22033
1200	1.2	1.34	0.98	1.309016	0.14	0.22	0.109016
1500	1.2	1.34	0.98	1.184509	0.14	0.22	0.015491
1800	1.2	1.34	0.98	1.091578	0.14	0.22	0.108422
2000	1.2	1.34	0.98	1.041222	0.14	0.22	0.158778
2200	1.2	1.34	0.98	0.997665	0.14	0.22	0.202335
2500	1.2	1.34	0.98	0.942084	0.14	0.22	0.257916
2800	1.2	1.34	0.98	0.895398	0.14	0.22	0.304602
3000	1.2	1.34	0.98	0.868114	0.14	0.22	0.331886
3200	1.2	1.34	0.98	0.843343	0.14	0.22	0.356657
3500	1.2	1.34	0.98	0.810116	0.14	0.22	0.389884
3800	1.2	1.34	0.98	0.780775	0.14	0.22	0.419225
4000	1.2	1.34	0.98	0.763015	0.14	0.22	0.436985
4200	1.2	1.34	0.98	0.746496	0.14	0.22	0.453504
4500	1.2	1.34	0.98	0.723746	0.14	0.22	0.476254
4800	1.2	1.34	0.98	0.703094	0.14	0.22	0.496906
5000	1.2	1.34	0.98	0.690336	0.14	0.22	0.509664
5200	1.2	1.34	0.98	0.678297	0.14	0.22	0.521703
5500	1.2	1.34	0.98	0.661444	0.14	0.22	0.538556
5800	1.2	1.34	0.98	0.645873	0.14	0.22	0.554127
6000	1.2	1.34	0.98	0.636125	0.14	0.22	0.563875
6200	1.2	1.34	0.98	0.626838	0.14	0.22	0.573162
6500	1.2	1.34	0.98	0.613693	0.14	0.22	0.586307
6800	1.2	1.34	0.98	0.601398	0.14	0.22	0.598602
7000	1.2	1.34	0.98	0.59363	0.14	0.22	0.60637
7200	1.2	1.34	0.98	0.586177	0.14	0.22	0.613823
7500	1.2	1.34	0.98	0.575543	0.14	0.22	0.624457
7800	1.2	1.34	0.98	0.565508	0.14	0.22	0.634492
8000	1.2	1.34	0.98	0.559124	0.14	0.22	0.640876
8200	1.2	1.34	0.98	0.552967	0.14	0.22	0.647033
8500	1.2	1.34	0.98	0.544128	0.14	0.22	0.655872
8800	1.2	1.34	0.98	0.53573	0.14	0.22	0.66427
9000	1.2	1.34	0.98	0.530359	0.14	0.22	0.669641
9200	1.2	1.34	0.98	0.525158	0.14	0.22	0.674842
9500	1.2	1.34	0.98	0.517656	0.14	0.22	0.682344
9800	1.2	1.34	0.98	0.51049	0.14	0.22	0.68951
10000	1.2	1.34	0.98	0.505886	0.14	0.22	0.694114
					0.502938	66581.56	963.9806

Table(A.4) Comparison results of drag coefficient with other investigators for ψ=0.6

Re_p	C_{dexp}	C_d ψ=0.6	C_d Muhannad	$C_{d\ Haider}$	s.d pres	s.d muh	s.d haider
0.001	24000	24010	576570	2092.726	10	552570	21907.27
0.0012	20000	20008	452419.1	1886.187	8	432419	18113.81
0.0015	16000	16007	336241	1660.94	7	320241	14339.06
0.0018	13333	13339	263839.4	1497.02	6	250506	11835.98
0.002	12000	12005	229341.2	1409.777	5	217341	10590.22
0.0022	10909	10914	202036.5	1335.245	5	191128	9573.755
0.0025	9600	9604	170448	1241.43	4	160848	8358.57
0.0028	8570	8575	146599.3	1163.788	5	138029	7406.212
0.003	8000	8003.3	133746	1118.919	3.3	125746	6881.081
0.0032	7500	7503	122744.6	1078.514	3	115245	6421.486
0.0035	6857	6860	108953.6	1024.821	3	102097	5832.179
0.0038	6315.8	6318.4	97665.2	977.9014	2.6	91349	5337.899
0.004	6000	6002.5	91224.65	949.7315	2.5	85225	5050.269
0.0042	5714.3	5717	85492.98	923.6899	2.7	79779	4790.61
0.0045	5333.3	5335.5	77997.26	888.0796	2.2	72664	4445.22
0.0048	5000	5002	71581.56	856.0125	2	66582	4143.988
0.005	4800	4802	67798.78	836.3301	2	62999	3963.67
0.0052	4615.4	4617.3	64352.82	817.8462	1.9	59737	3797.554
0.0055	4363.6	4365.4	59726.85	792.1203	1.8	55363	3571.48
0.0058	4137.9	4140	55653.54	768.5076	2.1	51516	3369.392
0.006	4000	4001.6	53199.9	753.8044	1.6	49200	3246.196
0.0062	3871	3872.6	50929.69	739.8512	1.6	47059	3131.149
0.0065	3692.3	3694	47827.45	720.1965	1.7	44135	2972.103
0.0068	3529.4	3531	45041.74	701.9166	1.6	41512	2827.483
0.007	3428.6	3430	43338.27	690.4183	1.4	39910	2738.182
0.0072	3333.3	3334.7	41744.55	679.4245	1.4	38411	2653.875
0.0075	3200	3201.3	39538.53	663.8036	1.3	36339	2536.196
0.0078	3076.9	3078.2	37528.93	649.1339	1.3	34452	2427.766
0.008	3000	3001.2	36286.27	639.837	1.2	33286	2360.163
0.0082	2926.8	2928	35113.94	630.8981	1.2	32187	2295.902
0.0085	2823.5	2824.7	33475.33	618.113	1.2	30652	2205.387
0.0088	2727.3	2728.4	31966.13	606.0174	1.1	29239	2121.283
0.009	2666.7	2667.8	31024.83	598.3073	1.1	28358	2068.393
0.0092	2608.7	2609.8	30131.05	590.8616	1.1	27522	2017.838
0.0095	2526.3	2527.4	28872.18	580.1573	1.1	26346	1946.143
0.0098	2449	2450	27702.65	569.9709	1	25254	1879.029
0.01	2400	2401	26968.2	563.4479	1	24568	1836.552
0.012	2000	2008	21161.23	507.8573	8	19161	1492.143
0.015	1600	1600.7	15727.17	447.2312	0.7	14127	1152.769
0.018	1333.3	1333.9	12340.69	403.1118	0.6	11007	930.1882
0.02	1200	1200.5	10727.09	379.6299	0.5	9527.1	820.3701
0.022	1090.9	1091.4	9449.957	359.5694	0.5	8359.1	731.3306
0.025	960	960.4	7972.45	334.3187	0.4	7012.4	625.6813
0.028	857	857.5	6856.963	313.4211	0.5	6000	543.5789
0.03	800	800.3	6255.77	301.3444	0.3	5455.8	498.6556
0.032	750	750.3	5741.198	290.4693	0.3	4991.2	459.5307
0.035	685.7	686	5096.142	276.0175	0.3	4410.4	409.6825

0.038	631.6	631.8	4568.144	263.3891	0.2	3936.5	368.2109
0.04	600	600.3	4266.898	255.8071	0.3	3666.9	344.1929
0.042	571.4	571.7	3998.807	248.7979	0.3	3427.4	322.6021
0.045	533.3	533.6	3648.206	239.2132	0.3	3114.9	294.0868
0.048	500	500.2	3348.121	230.5822	0.2	2848.1	269.4178
0.05	480	480.2	3171.187	225.2847	0.2	2691.2	254.7153
0.052	461.5	461.7	3010.007	220.3097	0.2	2548.5	241.1903
0.055	436.4	436.5	2793.635	213.3854	0.1	2357.2	223.0146
0.058	413.8	414	2603.112	207.03	0.2	2189.3	206.77
0.06	400	400.2	2488.346	203.0726	0.2	2088.3	196.9274
0.062	387.1	387.3	2382.161	199.317	0.2	1995.1	187.783
0.065	369.2	369.4	2237.058	194.0269	0.2	1867.9	175.1731
0.068	352.9	353.1	2106.76	189.1068	0.2	1753.9	163.7932
0.07	342.9	343	2027.083	186.012	0.1	1684.2	156.888
0.072	333.3	333.5	1952.539	183.053	0.2	1619.2	150.247
0.075	320	320.1	1849.356	178.8485	0.1	1529.4	141.1515
0.078	307.7	307.8	1755.36	174.9001	0.1	1447.7	132.7999
0.08	300	300.1	1697.236	172.3978	0.1	1397.2	127.6022
0.082	292.7	292.8	1642.402	169.9919	0.1	1349.7	122.7081
0.085	282.4	282.5	1565.759	166.5507	0.1	1283.4	115.8493
0.088	272.7	272.8	1495.168	163.2951	0.1	1222.5	109.4049
0.09	266.7	266.8	1451.141	161.2199	0.1	1184.4	105.4801
0.092	260.9	261	1409.335	159.2159	0.1	1148.4	101.6841
0.095	252.6	252.7	1350.453	156.3348	0.1	1097.9	96.26519
0.098	244.9	245	1295.75	153.5931	0.1	1050.9	91.30688
0.1	240	240.1	1261.398	151.8374	0.1	1021.4	88.16258
0.12	200	200.8	989.7852	136.875	0.8	789.79	63.12502
0.15	160	160.1	735.6151	120.5572	0.1	575.62	39.44277
0.18	133.3	133.4	577.2176	108.6823	0.1	443.92	24.6177
0.2	120	120.1	501.7439	102.3621	0.1	381.74	17.63794
0.22	109.09	109.1	442.0077	96.96266	0.01	332.92	12.12734
0.25	96	96	372.8995	90.16634	0	276.9	5.83366
0.28	85.7	85.8	320.7243	84.54163	0.1	235.02	1.158369
0.3	80	80	292.6043	81.29113	0	212.6	1.291128
0.32	75	75	268.536	78.36405	0	193.54	3.364048
0.35	68.57	68.6	238.3645	74.47426	0.03	169.79	5.904261
0.38	63.16	63.2	213.6682	71.07524	0.04	150.51	7.915236
0.4	60	60	199.5778	69.03448	0	139.58	9.034483
0.42	57.14	57.2	187.0383	67.14791	0.06	129.9	10.00791
0.45	53.3	53.4	170.6394	64.56814	0.1	117.34	11.26814
0.48	50	50	156.6034	62.24505	0	106.6	12.24505
0.5	48	48	148.3276	60.81916	0	100.33	12.81916
0.52	46.15	46.2	140.7886	59.48011	0.05	94.639	13.33011
0.55	43.64	43.7	130.6681	57.6164	0.06	87.028	13.9764
0.58	41.38	41.4	121.7567	55.90577	0.02	80.377	14.52577
0.6	40	40	116.3887	54.8406	0	76.389	14.8406
0.62	38.71	38.7	111.422	53.82975	0.01	72.712	15.11975
0.65	36.92	36.9	104.6351	52.40587	0.02	67.715	15.48587
0.68	35.29	35.3	98.54059	51.08157	0.01	63.251	15.79157
0.7	34.29	34.3	94.81381	50.24857	0.01	60.524	15.95857
0.72	33.33	33.4	91.32713	49.45212	0.07	57.997	16.12212
0.75	32	32	86.50088	48.32045	0	54.501	16.32045

0.78	30.77	30.8	82.10435	47.25769	0.03	51.334	16.48769
0.8	30	30	79.38571	46.58417	0	49.386	16.58417
0.82	29.27	29.3	76.82093	45.93657	0.03	47.551	16.66657
0.85	28.24	28.3	73.23604	45.01035	0.06	44.996	16.77035
0.88	27.27	27.3	69.93427	44.13406	0.03	42.664	16.86406
0.9	26.67	26.7	67.87494	43.57549	0.03	41.205	16.90549
0.92	26.09	26.1	65.91955	43.03608	0.01	39.83	16.94608
0.95	25.26	25.3	63.16545	42.26059	0.04	37.905	17.00059
0.98	24.49	24.5	60.6068	41.52262	0.01	36.117	17.03262
1	24	24	59	41.05004	0	35	17.05004
1.2	20	20.1	46.29573	37.02264	0.1	26.296	17.02264
1.5	19	19.4	34.4073	32.63033	0.4	15.407	13.63033
1.8	16.33333	16.7	26.9985	29.43383	0.366667	10.665	13.1005
2	15	15.4	23.46833	27.7325	0.4	8.4683	12.7325
2.2	13.90909	14.3	20.67425	26.27902	0.390909	6.7652	12.36993
2.5	12.6	13	17.44182	24.44944	0.4	4.8418	11.84944
2.8	11.57143	12	15.0014	22.93521	0.428571	3.43	11.36379
3	11	9.34	13.68613	22.06012	1.66	2.6861	11.06012
3.2	10.5	10.9	12.56037	21.27207	0.4	2.0604	10.77207
3.5	9.857143	10.3	11.14914	20.2248	0.442857	1.292	10.36766
3.8	9.315789	9.72	9.994011	19.30963	0.404211	0.6782	9.993837
4	9	7.34	9.334955	18.76014	1.66	0.335	9.760139
4.2	8.714286	9.1	8.748437	18.25215	0.385714	0.0342	9.537866
4.5	8.333333	8.7	7.981405	17.55748	0.366667	0.3519	9.224148
4.8	8	8.4	7.324891	16.9319	0.4	0.6751	8.9319
5	7.8	8.2	7.144	16.54791	0.4	0.656	8.747908
5.2	7.615385	8	6.915769	16.18729	0.384615	0.6996	8.571903
5.5	7.363636	7.8	6.604545	15.68535	0.436364	0.7591	8.321715
5.8	7.137931	7.54	6.325517	15.22462	0.402069	0.8124	8.086688
6	7	7.4	6.155	14.93772	0.4	0.845	7.937717
6.2	6.870968	7.3	5.995484	14.66544	0.429032	0.8755	7.794469
6.5	6.692308	7.1	5.774615	14.28188	0.407692	0.9177	7.589576
6.8	6.529412	6.5	5.573235	13.92514	0.029412	0.9562	7.395724
7	6.428571	6.8	5.448571	13.70073	0.371429	0.98	7.272154
7.2	6.333333	6.7	5.330833	13.48615	0.366667	1.0025	7.152819
7.5	6.2	6.6	5.166	13.18125	0.4	1.034	6.981253
7.8	6.076923	6.5	5.013846	12.8949	0.423077	1.0631	6.817979
8	6	6.4	4.91875	12.71342	0.4	1.0813	6.713418
8.2	5.926829	6.3	4.828293	12.53891	0.373171	1.0985	6.612083
8.5	5.823529	6.2	4.700588	12.28931	0.376471	1.1229	6.465781
8.8	5.727273	6.13	4.581591	12.05315	0.402727	1.1457	6.325879
9	5.666667	6.07	4.506667	11.90261	0.403333	1.16	6.235943
9.2	5.608696	6.01	4.435	11.75722	0.401304	1.1737	6.148527
9.5	5.526316	5.93	4.333158	11.5482	0.403684	1.1932	6.021879
9.8	5.44898	5.85	4.237551	11.34927	0.40102	1.2114	5.900288
10	5.4	5.8	4.177	11.22187	0.4	1.223	5.821873
12	5	5.4	3.6825	10.1359	0.4	1.3175	5.135901
15	4.6	5	3.188	8.950789	0.4	1.412	4.350789
18	4.333333	4.7	2.858333	8.087607	0.366667	1.475	3.754273
20	4.2	4.6	2.6935	7.627839	0.4	1.5065	3.427839
22	4.090909	4.5	2.558636	7.234811	0.409091	1.5323	3.143902
25	3.96	4.36	2.3968	6.739704	0.4	1.5632	2.779704

28	3.857143	4.3	2.269643	6.329539	0.442857	1.5875	2.472396
30	3.8	4.2	2.199	6.092302	0.4	1.601	2.292302
32	3.75	4.15	2.137188	5.878523	0.4	1.6128	2.128523
35	3.685714	4.1	2.057714	5.594186	0.414286	1.628	1.908472
38	3.631579	4.03	1.990789	5.34546	0.398421	1.6408	1.713881
40	3.6	4	1.95175	5.195994	0.4	1.6483	1.595994
42	3.571429	3.97	1.916429	5.057721	0.398571	1.655	1.486292
45	3.533333	3.93	1.869333	4.868474	0.396667	1.664	1.33514
48	3.5	3.9	1.828125	4.697874	0.4	1.6719	1.197874
50	3.48	3.88	1.8034	4.593068	0.4	1.6766	1.113068
52	3.461538	3.86	1.780577	4.494573	0.398462	1.681	1.033035
55	3.436364	3.84	1.749455	4.357368	0.403636	1.6869	0.921004
58	3.413793	3.81	1.721552	4.231299	0.396207	1.6922	0.817506
60	3.4	3.8	1.7045	4.152729	0.4	1.6955	0.752729
62	3.387097	3.79	1.688548	4.078115	0.402903	1.6985	0.691018
65	3.369231	3.77	1.666462	3.972922	0.400769	1.7028	0.603691
68	3.352941	3.75	1.646324	3.874985	0.397059	1.7066	0.522044
70	3.342857	3.74	1.633857	3.813329	0.397143	1.709	0.470472
72	3.333333	3.73	1.622083	3.754339	0.396667	1.7113	0.421006
75	3.32	3.72	1.6056	3.67045	0.4	1.7144	0.35045
78	3.307692	3.71	1.590385	3.591592	0.402308	1.7173	0.2839
80	3.3	3.7	1.580875	3.541575	0.4	1.7191	0.241575
82	3.292683	3.69	1.571829	3.493452	0.397317	1.7209	0.200769
85	3.282353	3.68	1.559059	3.424568	0.397647	1.7233	0.142215
88	3.272727	3.67	1.547159	3.359337	0.397273	1.7256	0.08661
90	3.266667	3.666	1.539667	3.317725	0.399333	1.727	0.051058
92	3.26087	3.66	1.5325	3.277514	0.39913	1.7284	0.016645
95	3.252632	3.65	1.522316	3.219662	0.397368	1.7303	0.03297
98	3.244898	3.645	1.512755	3.164559	0.400102	1.7321	0.080339
100	3.24	3.64	1.5067	3.129246	0.4	1.7333	0.110754
120	3.2	3.6	1.45725	2.827401	0.4	1.7428	0.372599
150	3.16	3.56	1.4078	2.496106	0.4	1.7522	0.663894
180	3.133333	3.53	1.374833	2.25339	0.396667	1.7585	0.879943
200	3.12	3.52	1.35835	2.123586	0.4	1.7617	0.996414
220	3.109091	3.51	0.98	2.012329	0.400909	2.1291	1.096762
250	3.096	3.5	0.98	1.871793	0.404	2.116	1.224207
280	3.085714	3.49	0.98	1.755056	0.404286	2.1057	1.330658
300	3.08	3.48	0.98	1.687417	0.4	2.1	1.392583
320	3.075	3.475	0.98	1.626398	0.4	2.095	1.448602
350	3.068571	3.47	0.98	1.545151	0.401429	2.0886	1.523421
380	3.063158	3.46	0.98	1.474009	0.396842	2.0832	1.589148
400	3	3.4	0.98	1.431234	0.4	2.02	1.568766
420	3	3.4	0.98	1.391651	0.4	2.02	1.608349
450	3	3.4	0.98	1.337464	0.4	2.02	1.662536
480	3	3.4	0.98	1.288613	0.4	2.02	1.711387
500	3	3.4	0.98	1.258605	0.4	2.02	1.741395
520	3	3.4	0.98	1.230408	0.4	2.02	1.769592
550	3	3.4	0.98	1.191141	0.4	2.02	1.808859
580	3	3.4	0.98	1.155077	0.4	2.02	1.844923
600	3	3.4	0.98	1.132611	0.4	2.02	1.867389
620	3	3.4	0.98	1.111285	0.4	2.02	1.888715
650	3	3.4	0.98	1.081236	0.4	2.02	1.918764

680	3	3.4	0.98	1.05328	0.4	2.02	1.94672
700	3	3.4	0.98	1.035693	0.4	2.02	1.964307
720	3	3.4	0.98	1.018875	0.4	2.02	1.981125
750	3	3.4	0.98	0.994976	0.4	2.02	2.005024
780	3	3.4	0.98	0.97253	0.4	2.02	2.02747
900	3	3.4	0.98	0.894759	0.4	2.02	2.105241
920	3	3.4	0.98	0.883367	0.4	2.02	2.116633
950	3	3.4	0.98	0.866992	0.4	2.02	2.133008
980	3	3.4	0.98	0.851411	0.4	2.02	2.148589
1000	3	3.4	0.98	0.841434	0.4	2.02	2.158566
1200	3	3.4	0.98	0.756462	0.4	2.02	2.243538
1500	3	3.4	0.98	0.663959	0.4	2.02	2.336041
1800	3	3.4	0.98	0.596808	0.4	2.02	2.403192
2000	3	3.4	0.98	0.561143	0.4	2.02	2.438857
2200	3	3.4	0.98	0.530723	0.4	2.02	2.469277
2500	3	3.4	0.98	0.492503	0.4	2.02	2.507497
2800	3	3.4	0.98	0.460938	0.4	2.02	2.539062
3000	3	3.4	0.98	0.442727	0.4	2.02	2.557273
3200	3	3.4	0.98	0.426348	0.4	2.02	2.573652
3500	3	3.4	0.98	0.404613	0.4	2.02	2.595387
3800	3	3.4	0.98	0.38565	0.4	2.02	2.61435
4000	3	3.4	0.98	0.374279	0.4	2.02	2.625721
4200	3	3.4	0.98	0.363777	0.4	2.02	2.636223
4500	3	3.4	0.98	0.349432	0.4	2.02	2.650568
4800	3	3.4	0.98	0.336531	0.4	2.02	2.663469
5000	3	3.4	0.98	0.32862	0.4	2.02	2.67138
5200	3	3.4	0.98	0.321196	0.4	2.02	2.678804
5500	3	3.4	0.98	0.310872	0.4	2.02	2.689128
5800	3	3.4	0.98	0.301406	0.4	2.02	2.698594
6000	3	3.4	0.98	0.295517	0.4	2.02	2.704483
6200	3	3.4	0.98	0.289931	0.4	2.02	2.710069
6500	3	3.4	0.98	0.282068	0.4	2.02	2.717932
6800	3	3.4	0.98	0.274762	0.4	2.02	2.725238
7000	3	3.4	0.98	0.270169	0.4	2.02	2.729831
7200	3	3.4	0.98	0.26578	0.4	2.02	2.73422
7500	3	3.4	0.98	0.259547	0.4	2.02	2.740453
7800	3	3.4	0.98	0.253698	0.4	2.02	2.746302
8000	3	3.4	0.98	0.249993	0.4	2.02	2.750007
8200	3	3.4	0.98	0.246432	0.4	2.02	2.753568
8500	3	3.4	0.98	0.241342	0.4	2.02	2.758658
8800	3	3.4	0.98	0.23653	0.4	2.02	2.76347
9000	3	3.4	0.98	0.233463	0.4	2.02	2.766537
9200	3	3.4	0.98	0.230503	0.4	2.02	2.769497
9500	3	3.4	0.98	0.22625	0.4	2.02	2.77375
9800	3	3.4	0.98	0.222204	0.4	2.02	2.777796
10000	3	3.4	0.98	0.219615	0.4	2.02	2.780385
					0.711091	15932	861.5196

Table(A.5) Comparison results of drag coefficient for ψ=0.5

Re_p	C_{dexp}	C_d ψ=0.5	C_d Muhannad	$C_{d\ Haider}$	s.d pres	s.d muh	s.d haider

0.001	24000	24010	576570	4320.222	10	552570	19679.78
0.0012	20000	20008	452419.1	3855.031	8	432419.1	16144.97
0.0015	16000	16007	336241	3353.306	7	320241	12646.69
0.0018	13333	13339	263839.4	2992.25	6	250506.4	10340.75
0.002	12000	12005	229341.2	2801.61	5	217341.2	9198.39
0.0022	10909	10914	202036.5	2639.642	5	191127.5	8269.358
0.0025	9600	9604	170448	2437.015	4	160848	7162.985
0.0028	8570	8575	146599.3	2270.428	5	138029.3	6299.572
0.003	8000	8003.3	133746	2174.642	3.3	125746	5825.358
0.0032	7500	7503	122744.6	2088.703	3	115244.6	5411.297
0.0035	6857	6860	108953.6	1974.98	3	102096.6	4882.02
0.0038	6315.8	6318.4	97665.2	1876.075	2.6	91349.4	4439.725
0.004	6000	6002.5	91224.65	1816.911	2.5	85224.65	4183.089
0.0042	5714.3	5717	85492.98	1762.368	2.7	79778.68	3951.932
0.0045	5333.3	5335.5	77997.26	1688.024	2.2	72663.96	3645.276
0.0048	5000	5002	71581.56	1621.323	2	66581.56	3378.677
0.005	4800	4802	67798.78	1580.501	2	62998.78	3219.499
0.0052	4615.4	4617.3	64352.82	1542.249	1.9	59737.42	3073.151
0.0055	4363.6	4365.4	59726.85	1489.15	1.8	55363.25	2874.45
0.0058	4137.9	4140	55653.54	1440.558	2.1	51515.64	2697.342
0.006	4000	4001.6	53199.9	1410.373	1.6	49199.9	2589.627
0.0062	3871	3872.6	50929.69	1381.78	1.6	47058.69	2489.22
0.0065	3692.3	3694	47827.45	1341.593	1.7	44135.15	2350.707
0.0068	3529.4	3531	45041.74	1304.311	1.6	41512.34	2225.089
0.007	3428.6	3430	43338.27	1280.909	1.4	39909.67	2147.691
0.0072	3333.3	3334.7	41744.55	1258.568	1.4	38411.25	2074.732
0.0075	3200	3201.3	39538.53	1226.884	1.3	36338.53	1973.116
0.0078	3076.9	3078.2	37528.93	1197.195	1.3	34452.03	1879.705
0.008	3000	3001.2	36286.27	1178.413	1.2	33286.27	1821.587
0.0082	2926.8	2928	35113.94	1160.38	1.2	32187.14	1766.42
0.0085	2823.5	2824.7	33475.33	1134.629	1.2	30651.83	1688.871
0.0088	2727.3	2728.4	31966.13	1110.315	1.1	29238.83	1616.985
0.009	2666.7	2667.8	31024.83	1094.84	1.1	28358.13	1571.86
0.0092	2608.7	2609.8	30131.05	1079.915	1.1	27522.35	1528.785
0.0095	2526.3	2527.4	28872.18	1058.489	1.1	26345.88	1467.811
0.0098	2449	2450	27702.65	1038.136	1	25253.65	1410.864
0.01	2400	2401	26968.2	1025.12	1	24568.2	1374.88
0.012	2000	2008	21161.23	914.806	8	19161.23	1085.194
0.015	1600	1600.7	15727.17	795.8281	0.7	14127.17	804.1719
0.018	1333.3	1333.9	12340.69	710.2083	0.6	11007.39	623.0917
0.02	1200	1200.5	10727.09	665.0004	0.5	9527.095	534.9996
0.022	1090.9	1091.4	9449.957	626.5919	0.5	8359.057	464.3081
0.025	960	960.4	7972.45	578.5413	0.4	7012.45	381.4587
0.028	857	857.5	6856.963	539.0373	0.5	5999.963	317.9627
0.03	800	800.3	6255.77	516.3228	0.3	5455.77	283.6772
0.032	750	750.3	5741.198	495.9434	0.3	4991.198	254.0566
0.035	685.7	686	5096.142	468.9756	0.3	4410.442	216.7244
0.038	631.6	631.8	4568.144	445.5214	0.2	3936.544	186.0786
0.04	600	600.3	4266.898	431.4915	0.3	3666.898	168.5085
0.042	571.4	571.7	3998.807	418.5572	0.3	3427.407	152.8428
0.045	533.3	533.6	3648.206	400.9274	0.3	3114.906	132.3726
0.048	500	500.2	3348.121	385.11	0.2	2848.121	114.89

0.05	480	480.2	3171.187	375.4296	0.2	2691.187	104.5704
0.052	461.5	461.7	3010.007	366.3587	0.2	2548.507	95.14127
0.055	436.4	436.5	2793.635	353.7668	0.1	2357.235	82.6332
0.058	413.8	414	2603.112	342.2439	0.2	2189.312	71.55615
0.06	400	400.2	2488.346	335.0859	0.2	2088.346	64.91408
0.062	387.1	387.3	2382.161	328.3055	0.2	1995.061	58.79448
0.065	369.2	369.4	2237.058	318.7756	0.2	1867.858	50.42441
0.068	352.9	353.1	2106.76	309.9347	0.2	1753.86	42.96531
0.07	342.9	343	2027.083	304.385	0.1	1684.183	38.515
0.072	333.3	333.5	1952.539	299.0871	0.2	1619.239	34.21286
0.075	320	320.1	1849.356	291.5737	0.1	1529.356	28.42626
0.078	307.7	307.8	1755.36	284.5334	0.1	1447.66	23.16661
0.08	300	300.1	1697.236	280.0795	0.1	1397.236	19.92049
0.082	292.7	292.8	1642.402	275.803	0.1	1349.702	16.89704
0.085	282.4	282.5	1565.759	269.6966	0.1	1283.359	12.70342
0.088	272.7	272.8	1495.168	263.9307	0.1	1222.468	8.769322
0.09	266.7	266.8	1451.141	260.2611	0.1	1184.441	6.438886
0.092	260.9	261	1409.335	256.7217	0.1	1148.435	4.178282
0.095	252.6	252.7	1350.453	251.6409	0.1	1097.853	0.959129
0.098	244.9	245	1295.75	246.8142	0.1	1050.85	1.914248
0.1	240	240.1	1261.398	243.7278	0.1	1021.398	3.727802
0.12	200	200.8	989.7852	217.568	0.8	789.7852	17.56805
0.15	160	160.1	735.6151	189.3537	0.1	575.6151	29.35375
0.18	133.3	133.4	577.2176	169.0499	0.1	443.9176	35.74988
0.2	120	120.1	501.7439	158.3293	0.1	381.7439	38.32926
0.22	109.09	109.1	442.0077	149.221	0.01	332.9177	40.13104
0.25	96	96	372.8995	137.8262	0	276.8995	41.82624
0.28	85.7	85.8	320.7243	128.4582	0.1	235.0243	42.75818
0.3	80	80	292.6043	123.0716	0	212.6043	43.07159
0.32	75	75	268.536	118.2387	0	193.536	43.23874
0.35	68.57	68.6	238.3645	111.8435	0.03	169.7945	43.27346
0.38	68.15789	68.58	213.6682	106.2814	0.422105	145.5103	38.1235
0.4	65	65.38	199.5778	102.9542	0.38	134.5778	37.95423
0.42	62.14286	62.58	187.0383	99.88689	0.437143	124.8954	37.74404
0.45	58.33333	58.78	170.6394	95.70602	0.446667	112.3061	37.37268
0.48	55	55.38	156.6034	91.95492	0.38	101.6034	36.95492
0.5	53	53.38	148.3276	89.6592	0.38	95.32758	36.6592
0.52	51.15385	51.58	140.7886	87.50802	0.426154	89.63478	36.35417
0.55	48.63636	49.08	130.6681	84.5218	0.443636	82.03175	35.88544
0.58	46.37931	46.78	121.7567	81.78908	0.40069	75.37736	35.40977
0.6	45	45.38	116.3887	80.09153	0.38	71.38871	35.09153
0.62	43.70968	44.08	111.422	78.48351	0.370323	67.71235	34.77384
0.65	41.92308	42.28	104.6351	76.22341	0.356923	62.71199	34.30033
0.68	40.29412	40.68	98.54059	74.1267	0.385882	58.24647	33.83258
0.7	39.28571	39.68	94.81381	72.81053	0.394286	55.5281	33.52481
0.72	38.33333	38.78	91.32713	71.55408	0.446667	52.99379	33.22074
0.75	37	37.38	86.50088	69.77217	0.38	49.50088	32.77217
0.78	35.76923	36.18	82.10435	68.10244	0.410769	46.33512	32.33321
0.8	35	35.38	79.38571	67.04612	0.38	44.38571	32.04612
0.82	34.26829	34.68	76.82093	66.03186	0.411707	42.55264	31.76357
0.85	33.23529	33.68	73.23604	64.58361	0.444706	40.00074	31.34831
0.88	32.27273	32.68	69.93427	63.2161	0.407273	37.66154	30.94337

0.9	31.66667	32.08	67.87494	62.34577	0.413333	36.20828	30.67911
0.92	31.08696	31.48	65.91955	61.50632	0.393043	34.83259	30.41936
0.95	30.26316	30.68	63.16545	60.30126	0.416842	32.9023	30.0381
0.98	29.4898	29.88	60.6068	59.15649	0.390204	31.117	29.66669
1	29	29.38	59	58.42444	0.38	30	29.42444
1.2	25	25.48	46.29573	52.21966	0.48	21.29573	27.21966
1.5	21	21.38	34.4073	45.52701	0.38	13.4073	24.52701
1.8	18.33333	18.68	26.9985	40.71024	0.346667	8.665163	22.3769
2	17	17.38	23.46833	38.16667	0.38	6.468326	21.16667
2.2	15.90909	16.28	20.67425	36.00547	0.370909	4.765163	20.09637
2.5	14.6	14.98	17.44182	33.30138	0.38	2.84182	18.70138
2.8	13.57143	13.98	15.0014	31.07793	0.408571	1.429973	17.5065
3	13	13.38	13.68613	29.79928	0.38	0.686133	16.79928
3.2	12.5	12.88	12.56037	28.65195	0.38	0.060373	16.15195
3.5	11.85714	12.28	11.14914	27.13347	0.422857	0.707999	15.27633
3.8	11.31579	11.68	9.994011	25.81259	0.364211	1.321778	14.4968
4	11	11.38	9.334955	25.02233	0.38	1.665045	14.02233
4.2	10.71429	11.08	8.748437	24.29369	0.365714	1.965849	13.5794
4.5	10.33333	10.68	7.981405	23.30036	0.346667	2.351928	12.96703
4.8	10	10.38	7.324891	22.40896	0.38	2.675109	12.40896
5	9.8	10.18	7.144	21.86331	0.38	2.656	12.06331
5.2	9.615385	9.98	6.915769	21.35195	0.364615	2.699615	11.73656
5.5	9.363636	9.78	6.604545	20.64195	0.416364	2.759091	11.27831
5.8	9.137931	9.52	6.325517	19.99207	0.382069	2.812414	10.85414
6	9	9.38	6.155	19.58829	0.38	2.845	10.58829
6.2	8.870968	9.28	5.995484	19.20575	0.409032	2.875484	10.33478
6.5	8.692308	9.08	5.774615	18.66797	0.387692	2.917692	9.975664
6.8	8.529412	8.88	5.573235	18.16895	0.350588	2.956176	9.639536
7	8.428571	8.78	5.448571	17.85563	0.351429	2.98	9.427057
7.2	8.333333	8.68	5.330833	17.55648	0.346667	3.0025	9.223142
7.5	8.2	8.58	5.166	17.13212	0.38	3.034	8.932123
7.8	8.076923	8.48	5.013846	16.73438	0.403077	3.063077	8.657458
8	8	8.38	4.91875	16.4827	0.38	3.08125	8.482702
8.2	7.926829	8.28	4.828293	16.241	0.353171	3.098537	8.314171
8.5	7.823529	8.18	4.700588	15.8958	0.356471	3.122941	8.07227
8.8	7.727273	8.11	4.581591	15.56975	0.382727	3.145682	7.84248
9	7.666667	8.09	4.506667	15.3622	0.423333	3.16	7.695532
9.2	7.608696	7.99	4.435	15.16197	0.381304	3.173696	7.553273
9.5	7.526316	7.91	4.333158	14.87447	0.383684	3.193158	7.34815
9.8	7.44898	7.83	4.237551	14.60127	0.38102	3.211429	7.152287
10	7.4	7.78	4.177	14.42652	0.38	3.223	7.026522
12	7	7.38	3.6825	12.94384	0.38	3.3175	5.94384
15	6.6	6.98	3.188	11.34046	0.38	3.412	4.740462
18	6.333333	6.7	2.858333	10.18265	0.366667	3.475	3.849317
20	6.2	6.6	2.6935	9.569473	0.4	3.5065	3.369473
22	6.090909	6.5	2.558636	9.047257	0.409091	3.532273	2.956348
25	5.96	6.3	2.3968	8.391966	0.34	3.5632	2.431966
28	5.857143	6.2	2.269643	7.851234	0.342857	3.5875	1.994092
30	5.8	6.18	2.199	7.539349	0.38	3.601	1.739349
32	5.75	6.13	2.137188	7.258839	0.38	3.612813	1.508839
35	5.685714	6.1	2.057714	6.886521	0.414286	3.628	1.200807
38	5.631579	6.01	1.990789	6.561541	0.378421	3.640789	0.929962

40	5.6	5.98	1.95175	6.36656	0.38	3.64825	0.76656
42	5.571429	5.95	1.916429	6.186384	0.378571	3.655	0.614955
45	5.533333	5.91	1.869333	5.940096	0.376667	3.664	0.406763
48	5	5.88	1.828125	5.718376	0.88	3.171875	0.718376
50	5	5.86	1.8034	5.582302	0.86	3.1966	0.582302
52	5	5.84	1.780577	5.454519	0.84	3.219423	0.454519
55	5	5.81	1.749455	5.276664	0.81	3.250545	0.276664
58	5	5.79	1.721552	5.113399	0.79	3.278448	0.113399
60	5	5.78	1.7045	5.011721	0.78	3.2955	0.011721
62	5	5.77	1.688548	4.915215	0.77	3.311452	0.084785
65	5	5.75	1.666462	4.779245	0.75	3.333538	0.220755
68	5	5.73	1.646324	4.652748	0.73	3.353676	0.347252
70	5	5.72	1.633857	4.573157	0.72	3.366143	0.426843
72	5	5.71	1.622083	4.497041	0.71	3.377917	0.502959
75	5	5.7	1.6056	4.388857	0.7	3.3944	0.611143
78	5	5.69	1.590385	4.287223	0.69	3.409615	0.712777
80	5	5.68	1.580875	4.222792	0.68	3.419125	0.777208
82	5	5.67	1.571829	4.160826	0.67	3.428171	0.839174
85	5	5.66	1.559059	4.072171	0.66	3.440941	0.927829
88	5	5.65	1.547159	3.988264	0.65	3.452841	1.011736
90	5	5.647	1.539667	3.934763	0.647	3.460333	1.065237
92	5	5.64	1.5325	3.883083	0.64	3.4675	1.116917
95	5	5.63	1.522316	3.808764	0.63	3.477684	1.191236
98	5	5.625	1.512755	3.738016	0.625	3.487245	1.261984
100	5	5.62	1.5067	3.692698	0.62	3.4933	1.307302
120	5	5.58	1.45725	3.306069	0.58	3.54275	1.693931
150	5	5.54	1.4078	2.883642	0.54	3.5922	2.116358
180	5	5.51	1.374833	2.575936	0.51	3.625167	2.424064
200	5	5.5	1.35835	2.41219	0.5	3.64165	2.58781
220	5	5.49	0.98	2.27239	0.49	4.02	2.72761
250	5	5.38	0.98	2.096645	0.38	4.02	2.903355
280	5	5.38	0.98	1.951495	0.38	4.02	3.048505
300	5	5.38	0.98	1.867786	0.38	4.02	3.132214
320	5	5.38	0.98	1.79254	0.38	4.02	3.20746
350	5	5.38	0.98	1.692783	0.38	4.02	3.307217
380	5	5.38	0.98	1.605875	0.38	4.02	3.394125
400	5	5.38	0.98	1.553831	0.38	4.02	3.446169
420	5	5.38	0.98	1.505821	0.38	4.02	3.494179
450	5	5.38	0.98	1.440343	0.38	4.02	3.559657
480	5	5.38	0.98	1.381569	0.38	4.02	3.618431
500	5	5.38	0.98	1.345591	0.38	4.02	3.654409
520	5	5.38	0.98	1.311876	0.38	4.02	3.688124
550	5	5.38	0.98	1.265074	0.38	4.02	3.734926
580	5	5.38	0.98	1.222251	0.38	4.02	3.777749
600	5	5.38	0.98	1.195655	0.38	4.02	3.804345
620	5	5.38	0.98	1.170466	0.38	4.02	3.829534
650	5	5.38	0.98	1.135075	0.38	4.02	3.864925
680	5	5.38	0.98	1.102257	0.38	4.02	3.897743
700	5	5.38	0.98	1.081664	0.38	4.02	3.918336
720	5	5.38	0.98	1.062012	0.38	4.02	3.937988
750	5	5.38	0.98	1.034155	0.38	4.02	3.965845
780	5	5.38	0.98	1.008067	0.38	4.02	3.991933

900	5	5.38	0.98	0.918264	0.38	4.02	4.081736
920	5	5.38	0.98	0.905189	0.38	4.02	4.094811
950	5	5.38	0.98	0.886431	0.38	4.02	4.113569
980	5	5.38	0.98	0.868624	0.38	4.02	4.131376
1000	5	5.38	0.98	0.857243	0.38	4.02	4.142757
1200	5	5.38	0.98	0.761015	0.38	4.02	4.238985
1500	5	5.38	0.98	0.657789	0.38	4.02	4.342211
1800	5	5.38	0.98	0.583955	0.38	4.02	4.416045
2000	5	5.38	0.98	0.545149	0.38	4.02	4.454851
2200	5	5.38	0.98	0.512287	0.38	4.02	4.487713
2500	5	5.38	0.98	0.471327	0.38	4.02	4.528673
2800	5	5.38	0.98	0.437788	0.38	4.02	4.562212
3000	5	5.38	0.98	0.418563	0.38	4.02	4.581437
3200	5	5.38	0.98	0.401353	0.38	4.02	4.598647
3500	5	5.38	0.98	0.378638	0.38	4.02	4.621362
3800	5	5.38	0.98	0.358939	0.38	4.02	4.641061
4000	5	5.38	0.98	0.347182	0.38	4.02	4.652818
4200	5	5.38	0.98	0.336361	0.38	4.02	4.663639
4500	5	5.38	0.98	0.32164	0.38	4.02	4.67836
4800	5	5.38	0.98	0.308461	0.38	4.02	4.691539
5000	5	5.38	0.98	0.300409	0.38	4.02	4.699591
5200	5	5.38	0.98	0.292874	0.38	4.02	4.707126
5500	5	5.38	0.98	0.282429	0.38	4.02	4.717571
5800	5	5.38	0.98	0.272888	0.38	4.02	4.727112
6000	5	5.38	0.98	0.266969	0.38	4.02	4.733031
6200	5	5.38	0.98	0.261369	0.38	4.02	4.738631
6500	5	5.38	0.98	0.253506	0.38	4.02	4.746494
6800	5	5.38	0.98	0.246223	0.38	4.02	4.753777
7000	5	5.38	0.98	0.241656	0.38	4.02	4.758344
7200	5	5.38	0.98	0.2373	0.38	4.02	4.7627
7500	5	5.38	0.98	0.231129	0.38	4.02	4.768871
7800	5	5.38	0.98	0.225353	0.38	4.02	4.774647
8000	5	5.38	0.98	0.221702	0.38	4.02	4.778298
8200	5	5.38	0.98	0.2182	0.38	4.02	4.7818
8500	5	5.38	0.98	0.213203	0.38	4.02	4.786797
8800	5	5.38	0.98	0.20849	0.38	4.02	4.79151
9000	5	5.38	0.98	0.205493	0.38	4.02	4.794507
9200	5	5.38	0.98	0.202604	0.38	4.02	4.797396
9500	5	5.38	0.98	0.198459	0.38	4.02	4.801541
9800	5	5.38	0.98	0.194526	0.38	4.02	4.805474
10000	5	5.38	0.98	0.192013	0.38	4.02	4.807987
					0.764963	15932.86	714.1902

Table (3.6) Comparison results of drag coefficient for ψ=0.4

Re_p	C_{dexp}	C_d ψ=0.4	C_d Muhannad	$C_{d\ Haider}$	s.d pres	s.d muh	s.d haider
0.001	24000	24010	576570	6844.777	10	552570	528560
0.0012	20000	20008	452419.1	6046.988	8	432419.1	412411.1
0.0015	16000	16007	336241	5196.043	7	320241	304234

0.0018	13333	13339	263839.4	4590.499	6	250506.4	237167.4
0.002	12000	12005	229341.2	4273.307	5	217341.2	205336.2
0.0022	10909	10914	202036.5	4005.307	5	191127.5	180213.5
0.0025	9600	9604	170448	3672.072	4	160848	151244
0.0028	8570	8575	146599.3	3399.922	5	138029.3	129454.3
0.003	8000	8003.3	133746	3244.224	3.3	125746	117742.7
0.0032	7500	7503	122744.6	3105.045	3	115244.6	107741.6
0.0035	6857	6860	108953.6	2921.647	3	102096.6	95236.58
0.0038	6315.8	6318.4	97665.2	2762.897	2.6	91349.4	85031
0.004	6000	6002.5	91224.65	2668.285	2.5	85224.65	79222.15
0.0042	5714.3	5717	85492.98	2581.3	2.7	79778.68	74061.68
0.0045	5333.3	5335.5	77997.26	2463.121	2.2	72663.96	67328.46
0.0048	5000	5002	71581.56	2357.48	2	66581.56	61579.56
0.005	4800	4802	67798.78	2293.014	2	62998.78	58196.78
0.0052	4615.4	4617.3	64352.82	2232.74	1.9	59737.42	55120.12
0.0055	4363.6	4365.4	59726.85	2149.288	1.8	55363.25	50997.85
0.0058	4137.9	4140	55653.54	2073.15	2.1	51515.64	47375.64
0.006	4000	4001.6	53199.9	2025.967	1.6	49199.9	45198.3
0.0062	3871	3872.6	50929.69	1981.354	1.6	47058.69	43186.09
0.0065	3692.3	3694	47827.45	1918.789	1.7	44135.15	40441.15
0.0068	3529.4	3531	45041.74	1860.894	1.6	41512.34	37981.34
0.007	3428.6	3430	43338.27	1824.625	1.4	39909.67	36479.67
0.0072	3333.3	3334.7	41744.55	1790.057	1.4	38411.25	35076.55
0.0075	3200	3201.3	39538.53	1741.126	1.3	36338.53	33137.23
0.0078	3076.9	3078.2	37528.93	1695.376	1.3	34452.03	31373.83
0.008	3000	3001.2	36286.27	1666.485	1.2	33286.27	30285.07
0.0082	2926.8	2928	35113.94	1638.783	1.2	32187.14	29259.14
0.0085	2823.5	2824.7	33475.33	1599.293	1.2	30651.83	27827.13
0.0088	2727.3	2728.4	31966.13	1562.077	1.1	29238.83	26510.43
0.009	2666.7	2667.8	31024.83	1538.429	1.1	28358.13	25690.33
0.0092	2608.7	2609.8	30131.05	1515.648	1.1	27522.35	24912.55
0.0095	2526.3	2527.4	28872.18	1482.994	1.1	26345.88	23818.48
0.0098	2449	2450	27702.65	1452.028	1	25253.65	22803.65
0.01	2400	2401	26968.2	1432.254	1	24568.2	22167.2
0.012	2000	2008	21161.23	1265.573	8	19161.23	17153.23
0.015	1600	1600.7	15727.17	1087.785	0.7	14127.17	12526.47
0.018	1333.3	1333.9	12340.69	961.2685	0.6	11007.39	9673.494
0.02	1200	1200.5	10727.09	894.9978	0.5	9527.095	8326.595
0.022	1090.9	1091.4	9449.957	839.0045	0.5	8359.057	7267.657
0.025	960	960.4	7972.45	769.3817	0.4	7012.45	6052.05
0.028	857	857.5	6856.963	712.5215	0.5	5999.963	5142.463
0.03	800	800.3	6255.77	679.9915	0.3	5455.77	4655.47
0.032	750	750.3	5741.198	650.9127	0.3	4991.198	4240.898
0.035	685.7	686	5096.142	612.5954	0.3	4410.442	3724.442
0.038	631.6	631.8	4568.144	579.4277	0.2	3936.544	3304.744
0.04	600	600.3	4266.898	559.6602	0.3	3666.898	3066.598
0.042	571.4	571.7	3998.807	541.4865	0.3	3427.407	2855.707
0.045	533.3	533.6	3648.206	516.7953	0.3	3114.906	2581.306
0.048	500	500.2	3348.121	494.7236	0.2	2848.121	2347.921
0.05	480	480.2	3171.187	481.2547	0.2	2691.187	2210.987
0.052	461.5	461.7	3010.007	468.6616	0.2	2548.507	2086.807
0.055	436.4	436.5	2793.635	451.2259	0.1	2357.235	1920.735

0.058	413.8	414	2603.112	435.3182	0.2	2189.312	1775.312
0.06	400	400.2	2488.346	425.4602	0.2	2088.346	1688.146
0.062	387.1	387.3	2382.161	416.1392	0.2	1995.061	1607.761
0.065	369.2	369.4	2237.058	403.0672	0.2	1867.858	1498.458
0.068	352.9	353.1	2106.76	390.9711	0.2	1753.86	1400.76
0.07	342.9	343	2027.083	383.3936	0.1	1684.183	1341.183
0.072	333.3	333.5	1952.539	376.1712	0.2	1619.239	1285.739
0.075	320	320.1	1849.356	365.9479	0.1	1529.356	1209.256
0.078	307.7	307.8	1755.36	356.3892	0.1	1447.66	1139.86
0.08	300	300.1	1697.236	350.353	0.1	1397.236	1097.136
0.082	292.7	292.8	1642.402	344.565	0.1	1349.702	1056.902
0.085	282.4	282.5	1565.759	336.3143	0.1	1283.359	1000.859
0.088	272.7	272.8	1495.168	328.5387	0.1	1222.468	949.6682
0.09	266.7	266.8	1451.141	323.5979	0.1	1184.441	917.6405
0.092	260.9	261	1409.335	318.8382	0.1	1148.435	887.4349
0.095	252.6	252.7	1350.453	312.0157	0.1	1097.853	845.1534
0.098	244.9	245	1295.75	305.5458	0.1	1050.85	805.8503
0.1	240	240.1	1261.398	301.4144	0.1	1021.398	781.2976
0.12	200	200.8	989.7852	266.5888	0.8	789.7852	588.9852
0.15	160	160.1	735.6151	229.4424	0.1	575.6151	415.5151
0.18	133.3	133.4	577.2176	203.0082	0.1	443.9176	310.5176
0.2	120	120.1	501.7439	189.1614	0.1	381.7439	261.6439
0.22	109.09	109.1	442.0077	177.4619	0.01	332.9177	223.8177
0.25	96	96	372.8995	162.9144	0	276.8995	180.8995
0.28	85.7	94.3	320.7243	151.0334	8.6	235.0243	140.7243
0.3	80	88.5	292.6043	144.2361	8.5	212.6043	124.1043
0.32	75	83.5	268.536	138.1598	8.5	193.536	110.036
0.35	68.57	68.6	238.3645	130.153	0.03	169.7945	101.1945
0.38	71.15789	71.7	213.6682	123.222	0.542105	142.5103	70.81027
0.4	68	68.5	199.5778	119.0912	0.5	131.5778	63.0778
0.42	65.14286	65.7	187.0383	115.2933	0.557143	121.8954	56.1954
0.45	61.33333	61.5	170.6394	110.1334	0.166667	109.3061	47.80609
0.48	58	58.5	156.6034	105.5207	0.5	98.60339	40.10339
0.5	56	56.5	148.3276	102.7058	0.5	92.32758	35.82758
0.52	54.15385	54.7	140.7886	100.0739	0.546154	86.63478	31.93478
0.55	51.63636	52.2	130.6681	96.42986	0.563636	79.03175	26.83175
0.58	49.37931	49.9	121.7567	93.10504	0.52069	72.37736	22.47736
0.6	48	48.5	116.3887	91.04459	0.5	68.38871	19.88871
0.62	46.70968	47.2	111.422	89.09634	0.490323	64.71235	17.51235
0.65	44.92308	45.4	104.6351	86.36399	0.476923	59.71199	14.31199
0.68	43.29412	43.8	98.54059	83.83553	0.505882	55.24647	11.44647
0.7	42.28571	42.8	94.81381	82.25153	0.514286	52.5281	9.7281
0.72	41.33333	41.9	91.32713	80.74175	0.566667	49.99379	8.093793
0.75	40	40.5	86.50088	78.60457	0.5	46.50088	6.000881
0.78	38.76923	39.3	82.10435	76.60627	0.530769	43.33512	4.035124
0.8	38	38.5	79.38571	75.3443	0.5	41.38571	2.885709
0.82	37.26829	37.8	76.82093	74.13421	0.531707	39.55264	1.752642
0.85	36.23529	36.8	73.23604	72.40916	0.564706	37.00074	0.200744
0.88	35.27273	35.8	69.93427	70.78339	0.527273	34.66154	1.138455
0.9	34.66667	35.2	67.87494	69.7503	0.533333	33.20828	1.991724
0.92	34.08696	34.6	65.91955	68.75504	0.513043	31.83259	2.767411
0.95	33.26316	33.8	63.16545	67.3284	0.536842	29.9023	3.897705

0.98	32.4898	33	60.6068	65.97544	0.510204	28.117	4.882999
1	32	32.5	59	65.11145	0.5	27	5.5
1.2	28	28.6	46.29573	57.82727	0.6	18.29573	10.30427
1.5	24	24.5	34.4073	50.05427	0.5	10.4073	14.0927
1.8	21.33333	21.8	26.9985	44.51947	0.466667	5.665163	16.13484
2	20	20.5	23.46833	41.61859	0.5	3.468326	17.03167
2.2	18.90909	19.4	20.67425	39.16639	0.490909	1.765163	17.63484
2.5	17.6	18.1	17.44182	36.11533	0.5	0.15818	17.94182
2.8	16.57143	17.1	15.0014	33.62148	0.528571	1.570027	15.52997
3	16	16.5	13.68613	32.19368	0.5	2.313867	14.18613
3.2	15.5	16	12.56037	30.91657	0.5	2.939627	13.06037
3.5	14.85714	15.4	11.14914	29.23239	0.542857	3.707999	11.692
3.8	14.31579	14.8	9.994011	27.77311	0.484211	4.321778	10.47822
4	14	14.5	9.334955	26.90264	0.5	4.665045	9.834955
4.2	13.71429	14.2	8.748437	26.10178	0.485714	4.965849	9.234151
4.5	13.33333	13.8	7.981405	25.01274	0.466667	5.351928	8.448072
4.8	13	13.5	7.324891	24.03814	0.5	5.675109	7.824891
5	12.8	13.3	7.144	23.44282	0.5	5.656	7.644
5.2	12.61538	13.1	6.915769	22.88579	0.484615	5.699615	7.400385
5.5	12.36364	12.9	6.604545	22.11377	0.536364	5.759091	7.140909
5.8	12.13793	12.64	6.325517	21.40856	0.502069	5.812414	6.827586
6	12	12.5	6.155	20.97108	0.5	5.845	6.655
6.2	11.87097	12.4	5.995484	20.55708	0.529032	5.875484	6.524516
6.5	11.69231	12.2	5.774615	19.97586	0.507692	5.917692	6.282308
6.8	11.52941	12	5.573235	19.43733	0.470588	5.956176	6.043824
7	11.42857	11.9	5.448571	19.09959	0.471429	5.98	5.92
7.2	11.33333	11.8	5.330833	18.7774	0.466667	6.0025	5.7975
7.5	11.2	11.7	5.166	18.32082	0.5	6.034	5.666
7.8	11.07692	11.6	5.013846	17.89334	0.523077	6.063077	5.536923
8	11	11.5	4.91875	17.62307	0.5	6.08125	5.41875
8.2	10.92683	11.4	4.828293	17.36368	0.473171	6.098537	5.301463
8.5	10.82353	11.3	4.700588	16.99349	0.476471	6.122941	5.177059
8.8	10.72727	11.23	4.581591	16.64412	0.502727	6.145682	5.084318
9	10.66667	11.17	4.506667	16.42186	0.503333	6.16	5.01
9.2	10.6087	11.11	4.435	16.20754	0.501304	6.173696	4.936304
9.5	10.52632	11.03	4.333158	15.89996	0.503684	6.193158	4.836842
9.8	10.44898	10.95	4.237551	15.60786	0.50102	6.211429	4.738571
10	10.4	10.9	4.177	15.4211	0.5	6.223	4.677
12	10	10.5	3.6825	13.83862	0.5	6.3175	4.1825
15	9.6	10.1	3.188	12.12957	0.5	6.412	3.688
18	9.333333	9.8	2.858333	10.8945	0.466667	6.475	3.325
20	9.2	9.7	2.6935	10.23915	0.5	6.5065	3.1935
22	9.090909	9.6	2.558636	9.679888	0.509091	6.532273	3.067727
25	8.96	9.46	2.3968	8.976099	0.5	6.5632	2.8968
28	8.857143	9.36	2.269643	8.393172	0.502857	6.5875	2.7725
30	8.8	9.3	2.199	8.055861	0.5	6.601	2.699
32	8.75	9.25	2.137188	7.751709	0.5	6.612813	2.637188
35	8.685714	9.2	2.057714	7.346762	0.514286	6.628	2.572
38	8.631579	9.13	1.990789	6.992029	0.498421	6.640789	2.489211
40	8.6	9.1	1.95175	6.778587	0.5	6.64825	2.45175
42	8.571429	9.07	1.916429	6.580926	0.498571	6.655	2.415
45	8.533333	9.03	1.869333	6.310052	0.496667	6.664	2.366

48	8.5	9	1.828125	6.065506	0.5	6.671875	2.328125
50	8.48	8.98	1.8034	5.915094	0.5	6.6766	2.3034
52	8.461538	8.96	1.780577	5.773616	0.498462	6.680962	2.279038
55	8.436364	8.94	1.749455	5.57633	0.503636	6.686909	2.253091
58	8.413793	8.91	1.721552	5.394855	0.496207	6.692241	2.217759
60	8.4	8.9	1.7045	5.28166	0.5	6.6955	2.2045
62	8.387097	8.89	1.688548	5.174099	0.502903	6.698548	2.191452
65	8.369231	8.87	1.666462	5.022359	0.500769	6.702769	2.167231
68	8.352941	8.85	1.646324	4.880991	0.497059	6.706618	2.143382
70	8.342857	8.84	1.633857	4.791952	0.497143	6.709	2.131
72	8.333333	8.83	1.622083	4.706737	0.496667	6.71125	2.11875
75	8.32	8.82	1.6056	4.585519	0.5	6.7144	2.1056
78	8.307692	8.81	1.590385	4.471543	0.502308	6.717308	2.092692
80	8.3	8.8	1.580875	4.399242	0.5	6.719125	2.080875
82	8.292683	8.79	1.571829	4.329677	0.497317	6.720854	2.069146
85	8.282353	8.78	1.559059	4.230104	0.497647	6.723294	2.056706
88	8.272727	8.77	1.547159	4.135821	0.497273	6.725568	2.044432
90	8.266667	8.767	1.539667	4.075685	0.500333	6.727	2.04
92	8.26087	8.761	1.5325	4.017587	0.50013	6.72837	2.03263
95	8.252632	8.753	1.522316	3.934021	0.500368	6.730316	2.022684
98	8.244898	8.745	1.512755	3.85446	0.500102	6.732143	2.012857
100	8.24	8.74	1.5067	3.803494	0.5	6.7333	2.0067
120	8.2	8.7	1.45725	3.368855	0.5	6.74275	1.95725
150	8.16	8.66	1.4078	2.895458	0.5	6.7522	1.9078
180	8	8.5	1.374833	2.552795	0.5	6.625167	1.874833
200	8	8.5	1.35835	2.371594	0.5	6.64165	1.85835
220	8	8.5	0.98	2.217698	0.5	7.02	1.48
250	8	8.5	0.98	2.025488	0.5	7.02	1.48
280	8	8.5	0.98	1.867967	0.5	7.02	1.48
300	8	8.5	0.98	1.777696	0.5	7.02	1.48
320	8	8.5	0.98	1.696943	0.5	7.02	1.48
350	8	8.5	0.98	1.590501	0.5	7.02	1.48
380	8	8.5	0.98	1.498383	0.5	7.02	1.48
400	8	8.5	0.98	1.443514	0.5	7.02	1.48
420	8	8.5	0.98	1.393102	0.5	7.02	1.48
450	8	8.5	0.98	1.32468	0.5	7.02	1.48
480	8	8.5	0.98	1.263604	0.5	7.02	1.48
500	8	8.5	0.98	1.226384	0.5	7.02	1.48
520	8	8.5	0.98	1.191623	0.5	7.02	1.48
550	8	8.5	0.98	1.143566	0.5	7.02	1.48
580	8	8.5	0.98	1.099802	0.5	7.02	1.48
600	8	8.5	0.98	1.072724	0.5	7.02	1.48
620	8	8.5	0.98	1.047155	0.5	7.02	1.48
650	8	8.5	0.98	1.011353	0.5	7.02	1.48
680	8	8.5	0.98	0.978288	0.5	7.02	1.48
700	8	8.5	0.98	0.957609	0.5	7.02	1.48
720	8	8.5	0.98	0.937925	0.5	7.02	1.48
750	8	8.5	0.98	0.910107	0.5	7.02	1.48
780	8	8.5	0.98	0.884147	0.5	7.02	1.48
900	8	8.5	0.98	0.795501	0.5	7.02	1.48
920	8	8.5	0.98	0.782691	0.5	7.02	1.48
950	8	8.5	0.98	0.764356	0.5	7.02	1.48

980	8	8.5	0.98	0.747	0.5	7.02	1.48
1000	8	8.5	0.98	0.735934	0.5	7.02	1.48
1200	8	8.5	0.98	0.643188	0.5	7.02	1.48
1500	8	8.5	0.98	0.545478	0.5	7.02	1.48
1800	8	8.5	0.98	0.476845	0.5	7.02	1.48
2000	8	8.5	0.98	0.441235	0.5	7.02	1.48
2200	8	8.5	0.98	0.411346	0.5	7.02	1.48
2500	8	8.5	0.98	0.374454	0.5	7.02	1.48
2800	8	8.5	0.98	0.344565	0.5	7.02	1.48
3000	8	8.5	0.98	0.327568	0.5	7.02	1.48
3200	8	8.5	0.98	0.312441	0.5	7.02	1.48
3500	8	8.5	0.98	0.292608	0.5	7.02	1.48
3800	8	8.5	0.98	0.275536	0.5	7.02	1.48
4000	8	8.5	0.98	0.265405	0.5	7.02	1.48
4200	8	8.5	0.98	0.25612	0.5	7.02	1.48
4500	8	8.5	0.98	0.243553	0.5	7.02	1.48
4800	8	8.5	0.98	0.232367	0.5	7.02	1.48
5000	8	8.5	0.98	0.225563	0.5	7.02	1.48
5200	8	8.5	0.98	0.219218	0.5	7.02	1.48
5500	8	8.5	0.98	0.210458	0.5	7.02	1.48
5800	8	8.5	0.98	0.202492	0.5	7.02	1.48
6000	8	8.5	0.98	0.197569	0.5	7.02	1.48
6200	8	8.5	0.98	0.192923	0.5	7.02	1.48
6500	8	8.5	0.98	0.186423	0.5	7.02	1.48
6800	8	8.5	0.98	0.180426	0.5	7.02	1.48
7000	8	8.5	0.98	0.176677	0.5	7.02	1.48
7200	8	8.5	0.98	0.173109	0.5	7.02	1.48
7500	8	8.5	0.98	0.16807	0.5	7.02	1.48
7800	8	8.5	0.98	0.163369	0.5	7.02	1.48
8000	8	8.5	0.98	0.160406	0.5	7.02	1.48
8200	8	8.5	0.98	0.157569	0.5	7.02	1.48
8500	8	8.5	0.98	0.153531	0.5	7.02	1.48
8800	8	8.5	0.98	0.149734	0.5	7.02	1.48
9000	8	8.5	0.98	0.147325	0.5	7.02	1.48
9200	8	8.5	0.98	0.145007	0.5	7.02	1.48
9500	8	8.5	0.98	0.14169	0.5	7.02	1.48
9800	8	8.5	0.98	0.138549	0.5	7.02	1.48
10000	8	8.5	0.98	0.136547	0.5	7.02	1.48
					0.912579	15934.1	14892.78

Table(A.7)comparison results of drag coefficient for ψ=0.2

Re_p	C_{dexp}	C_d ψ=0.2	$C_{d\ Muhannad}$	$C_{d\ Haider}$	s.d pres	s.d muh	s.d haider
0.001	24000	24010	576570	7616.193	10	552570	16383.81
0.0012	20000	20008	452419.1	6716.217	8	432419.1	13283.78
0.0015	16000	16007	336241	5758.211	7	320241	10241.79
0.0018	13333	13339	263839.4	5077.868	6	250506.4	8255.132
0.002	12000	12005	229341.2	4722.013	5	217341.2	7277.987
0.0022	10909	10914	202036.5	4421.647	5	191127.5	6487.353
0.0025	9600	9604	170448	4048.582	4	160848	5551.418
0.0028	8570	8575	146599.3	3744.273	5	138029.3	4825.727
0.003	8000	8003.3	133746	3570.335	3.3	125746	4429.665
0.0032	7500	7503	122744.6	3414.955	3	115244.6	4085.045
0.0035	6857	6860	108953.6	3210.367	3	102096.6	3646.633
0.0038	6315.8	6318.4	97665.2	3033.427	2.6	91349.4	3282.373
0.004	6000	6002.5	91224.65	2928.044	2.5	85224.65	3071.956
0.0042	5714.3	5717	85492.98	2831.206	2.7	79778.68	2883.094
0.0045	5333.3	5335.5	77997.26	2699.717	2.2	72663.96	2633.583
0.0048	5000	5002	71581.56	2582.256	2	66581.56	2417.744
0.005	4800	4802	67798.78	2510.615	2	62998.78	2289.385
0.0052	4615.4	4617.3	64352.82	2443.66	1.9	59737.42	2171.74
0.0055	4363.6	4365.4	59726.85	2351.001	1.8	55363.25	2012.599
0.0058	4137.9	4140	55653.54	2266.509	2.1	51515.64	1871.391
0.006	4000	4001.6	53199.9	2214.172	1.6	49199.9	1785.828
0.0062	3871	3872.6	50929.69	2164.703	1.6	47058.69	1706.297
0.0065	3692.3	3694	47827.45	2095.354	1.7	44135.15	1596.946
0.0068	3529.4	3531	45041.74	2031.211	1.6	41512.34	1498.189
0.007	3428.6	3430	43338.27	1991.044	1.4	39909.67	1437.556
0.0072	3333.3	3334.7	41744.55	1952.771	1.4	38411.25	1380.529
0.0075	3200	3201.3	39538.53	1898.614	1.3	36338.53	1301.386
0.0078	3076.9	3078.2	37528.93	1847.999	1.3	34452.03	1228.901
0.008	3000	3001.2	36286.27	1816.045	1.2	33286.27	1183.955
0.0082	2926.8	2928	35113.94	1785.414	1.2	32187.14	1141.386
0.0085	2823.5	2824.7	33475.33	1741.762	1.2	30651.83	1081.738
0.0088	2727.3	2728.4	31966.13	1700.639	1.1	29238.83	1026.661
0.009	2666.7	2667.8	31024.83	1674.516	1.1	28358.13	992.1843
0.0092	2608.7	2609.8	30131.05	1649.355	1.1	27522.35	959.3445
0.0095	2526.3	2527.4	28872.18	1613.301	1.1	26345.88	912.999
0.0098	2449	2450	27702.65	1579.121	1	25253.65	869.8791
0.01	2400	2401	26968.2	1557.3	1	24568.2	842.6996
0.012	2000	2008	21161.23	1373.549	8	19161.23	626.4512
0.015	1600	1600.7	15727.17	1177.949	0.7	14127.17	422.0511
0.018	1333.3	1333.9	12340.69	1039.041	0.6	11007.39	294.2594
0.02	1200	1200.5	10727.09	966.3842	0.5	9527.095	233.6158
0.022	1090.9	1091.4	9449.957	905.0573	0.5	8359.057	185.8427
0.025	960	960.4	7972.45	828.8871	0.4	7012.45	131.1129
0.028	857	857.5	6856.963	766.7549	0.5	5999.963	90.24509
0.03	800	800.3	6255.77	731.2414	0.3	5455.77	68.75864

0.032	750	750.3	5741.198	699.5167	0.3	4991.198	50.48327
0.035	685.7	686	5096.142	657.7451	0.3	4410.442	27.9549
0.038	631.6	631.8	4568.144	621.6184	0.2	3936.544	9.981597
0.04	600	600.3	4266.898	600.1018	0.3	3666.898	0.101828
0.042	571.4	571.7	3998.807	580.33	0.3	3427.407	8.929984
0.045	533.3	533.6	3648.206	553.4832	0.3	3114.906	20.1832
0.048	500	500.2	3348.121	529.5007	0.2	2848.121	29.50068
0.05	480	480.2	3171.187	514.8734	0.2	2691.187	34.87344
0.052	461.5	461.7	3010.007	501.2028	0.2	2548.507	39.70276
0.055	436.4	436.5	2793.635	482.2841	0.1	2357.235	45.88415
0.058	413.8	414	2603.112	465.033	0.2	2189.312	51.23296
0.06	400	400.2	2488.346	454.347	0.2	2088.346	54.34703
0.062	387.1	387.3	2382.161	444.2466	0.2	1995.061	57.14662
0.065	369.2	369.4	2237.058	430.0872	0.2	1867.858	60.8872
0.068	352.9	353.1	2106.76	416.9909	0.2	1753.86	64.09089
0.07	342.9	343	2027.083	408.7898	0.1	1684.183	65.88978
0.072	333.3	333.5	1952.539	400.9753	0.2	1619.239	67.67534
0.075	320	320.1	1849.356	389.9177	0.1	1529.356	69.9177
0.078	307.7	307.8	1755.36	379.5832	0.1	1447.66	71.8832
0.08	300	300.1	1697.236	373.0591	0.1	1397.236	73.05906
0.082	292.7	292.8	1642.402	366.8049	0.1	1349.702	74.10487
0.085	282.4	282.5	1565.759	357.8922	0.1	1283.359	75.4922
0.088	272.7	272.8	1495.168	349.4958	0.1	1222.468	76.79578
0.09	266.7	266.8	1451.141	344.162	0.1	1184.441	77.46202
0.092	260.9	261	1409.335	339.0249	0.1	1148.435	78.12487
0.095	252.6	252.7	1350.453	331.6634	0.1	1097.853	79.06337
0.098	244.9	245	1295.75	324.6845	0.1	1050.85	79.78455
0.1	240	240.1	1261.398	320.2293	0.1	1021.398	80.22929
0.12	200	200.8	989.7852	282.7111	0.8	789.7852	82.71114
0.15	160	160.1	735.6151	242.7734	0.1	575.6151	82.77344
0.18	153.3333	155	577.2176	214.4107	1.666667	423.8843	61.07734
0.2	140	141.7	501.7439	199.5753	1.7	361.7439	59.57526
0.22	129.0909	130.7	442.0077	187.053	1.609091	312.9168	57.96211
0.25	116	117.6	372.8995	171.4998	1.6	256.8995	55.49975
0.28	105.7143	107.4	320.7243	158.8127	1.685714	215.01	53.09838
0.3	100	101.6	292.6043	151.5608	1.6	192.6043	51.56085
0.32	95	96.6	268.536	145.0826	1.6	173.536	50.08263
0.35	88.57143	90.2	238.3645	136.5526	1.628571	149.793	47.98121
0.38	83.15789	84.8	213.6682	129.1752	1.642105	130.5103	46.0173
0.4	80	81.6	199.5778	124.7812	1.6	119.5778	44.7812
0.42	77.14286	78.8	187.0383	120.7434	1.657143	109.8954	43.60057
0.45	73.33333	75	170.6394	115.2607	1.666667	97.30609	41.92737
0.48	70	71.6	156.6034	110.3628	1.6	86.60339	40.36279
0.5	68	69.6	148.3276	107.3754	1.6	80.32758	39.37542
0.52	66.15385	67.8	140.7886	104.5834	1.646154	74.63478	38.4295
0.55	63.63636	64.6	130.6681	100.7194	0.963636	67.03175	37.08299
0.58	61.37931	63	121.7567	97.19581	1.62069	60.37736	35.8165
0.6	60	61.6	116.3887	95.01315	1.6	56.38871	35.01315
0.62	58.70968	60.3	111.422	92.95004	1.590323	52.71235	34.24036

0.65	56.92308	58.5	104.6351	90.05775	1.576923	47.71199	33.13467
0.68	55.29412	56.9	98.54059	87.38252	1.605882	43.24647	32.0884
0.7	54.28571	55.9	94.81381	85.7072	1.614286	40.5281	31.42148
0.72	53.33333	55	91.32713	84.11083	1.666667	37.99379	30.77749
0.75	52	53.6	86.50088	81.85185	1.6	34.50088	29.85185
0.78	50.76923	52.4	82.10435	79.74051	1.630769	31.33512	28.97128
0.8	50	51.6	79.38571	78.40759	1.6	29.38571	28.40759
0.82	49.26829	50.9	76.82093	77.12978	1.631707	27.55264	27.86149
0.85	48.23529	49.9	73.23604	75.30875	1.664706	25.00074	27.07345
0.88	47.27273	48.9	69.93427	73.59312	1.627273	22.66154	26.3204
0.9	46.66667	48.3	67.87494	72.50324	1.633333	21.20828	25.83658
0.92	46.08696	47.7	65.91955	71.45351	1.613043	19.83259	25.36656
0.95	45.26316	46.9	63.16545	69.94919	1.636842	17.9023	24.68603
0.98	44.4898	46.1	60.6068	68.52301	1.610204	16.117	24.03321
1	44	45.8	59	67.6125	1.8	15	23.6125
1.2	40	41.7	46.29573	59.9437	1.7	6.295732	19.9437
1.5	36	37.6	34.4073	51.7767	1.6	1.592697	15.7767
1.8	33.33333	34.9	26.9985	45.97313	1.566667	6.334837	12.63979
2	32	33.6	23.46833	42.93576	1.6	8.531674	10.93576
2.2	30.90909	32.9	20.67425	40.37074	1.990909	10.23484	9.461647
2.5	29.6	31.2	17.44182	37.1828	1.6	12.15818	7.582799
2.8	28.57143	30.2	15.0014	34.58017	1.628571	13.57003	6.008746
3	28	29.6	13.68613	33.09144	1.6	14.31387	5.091438
3.2	27.5	29.1	12.56037	31.7607	1.6	14.93963	4.260699
3.5	26.85714	28.9	11.14914	30.00711	2.042857	15.708	3.149964
3.8	26.31579	27.9	9.994011	28.48895	1.584211	16.32178	2.173156
4	26	27.6	9.334955	27.58395	1.6	16.66505	1.583945
4.2	25.71429	27.3	8.748437	26.75173	1.585714	16.96585	1.03744
4.5	25.33333	26.9	7.981405	25.62067	1.566667	17.35193	0.287333
4.8	25	26.6	7.324891	24.60911	1.6	17.67511	0.390885
5	24.8	26.4	7.144	23.99154	1.6	17.656	0.808455
5.2	24.61538	26.2	6.915769	23.4139	1.584615	17.69962	1.201487
5.5	24.36364	26	6.604545	22.61369	1.636364	17.75909	1.749951
5.8	24.13793	25.74	6.325517	21.88308	1.602069	17.81241	2.254847
6	24	25.6	6.155	21.43004	1.6	17.845	2.56996
6.2	23.87097	25.5	5.995484	21.00145	1.629032	17.87548	2.869519
6.5	23.69231	25.3	5.774615	20.39996	1.607692	17.91769	3.292344
6.8	23.52941	25.1	5.573235	19.84289	1.570588	17.95618	3.686519
7	23.42857	25	5.448571	19.49365	1.571429	17.98	3.934922
7.2	23.33333	24.9	5.330833	19.16057	1.566667	18.0025	4.172766
7.5	23.2	24.8	5.166	18.6887	1.6	18.034	4.511297
7.8	23.07692	24.7	5.013846	18.24708	1.623077	18.06308	4.829847
8	23	24.6	4.91875	17.96795	1.6	18.08125	5.032053
8.2	22.92683	24.5	4.828293	17.70011	1.573171	18.09854	5.226718
8.5	22.82353	24.4	4.700588	17.31797	1.576471	18.12294	5.505562
8.8	22.72727	24.33	4.581591	16.95743	1.602727	18.14568	5.769839
9	22.66667	24.27	4.506667	16.72813	1.603333	18.16	5.938542
9.2	22.6087	24.21	4.435	16.50705	1.601304	18.1737	6.101644
9.5	22.52632	24.13	4.333158	16.18986	1.603684	18.19316	6.336453

9.8	22.44898	24.05	4.237551	15.88871	1.60102	18.21143	6.560268
10	22.4	24	4.177	15.69622	1.6	18.223	6.703785
12	22	23.6	3.6825	14.06648	1.6	18.3175	7.933517
15	21.6	23.2	3.188	12.30935	1.6	18.412	9.290649
18	21.33333	22.9	2.858333	11.04158	1.566667	18.475	10.29175
20	21.2	22.8	2.6935	10.36963	1.6	18.5065	10.83037
22	21.09091	22.7	2.558636	9.796624	1.609091	18.53227	11.29429
25	20.96	22.6	2.3968	9.076119	1.64	18.5632	11.88388
28	20.85714	22.5	2.269643	8.479857	1.642857	18.5875	12.37729
30	20.8	22.4	2.199	8.135051	1.6	18.601	12.66495
32	20.75	22.35	2.137188	7.824284	1.6	18.61281	12.92572
35	20.68571	22.29	2.057714	7.410749	1.604286	18.628	13.27497
38	20.63158	22.23	1.990789	7.048705	1.598421	18.64079	13.58287
40	20.6	22.2	1.95175	6.830965	1.6	18.64825	13.76904
42	20.57143	22.17	1.916429	6.629393	1.598571	18.655	13.94204
45	20.53333	22.13	1.869333	6.353272	1.596667	18.664	14.18006
48	20.5	22.1	1.828125	6.104104	1.6	18.67188	14.3959
50	20.48	22.08	1.8034	5.950905	1.6	18.6766	14.52909
52	20.46154	22.06	1.780577	5.806847	1.598462	18.68096	14.65469
55	20.43636	22.04	1.749455	5.606031	1.603636	18.68691	14.83033
58	20.41379	22.01	1.721552	5.42138	1.596207	18.69224	14.99241
60	20.4	22	1.7045	5.30624	1.6	18.6955	15.09376
62	20.3871	21.98	1.688548	5.196858	1.592903	18.69855	15.19024
65	20.36923	21.97	1.666462	5.042593	1.600769	18.70277	15.32664
68	20.35294	21.95	1.646324	4.898922	1.597059	18.70662	15.45402
70	20.34286	21.94	1.633857	4.808457	1.597143	18.709	15.5344
72	20.33333	21.93	1.622083	4.721896	1.596667	18.71125	15.61144
75	20.32	21.92	1.6056	4.598796	1.6	18.7144	15.7212
78	20.30769	21.91	1.590385	4.483085	1.602308	18.71731	15.82461
80	20.3	21.9	1.580875	4.409701	1.6	18.71913	15.8903
82	20.29268	21.89	1.571829	4.339108	1.597317	18.72085	15.95357
85	20.28235	21.88	1.559059	4.238087	1.597647	18.72329	16.04427
88	20.27273	21.87	1.547159	4.14246	1.597273	18.72557	16.13027
90	20.26667	21.867	1.539667	4.081481	1.600333	18.727	16.18519
92	20.26087	21.861	1.5325	4.022577	1.60013	18.72837	16.23829
95	20.25263	21.85	1.522316	3.937872	1.597368	18.73032	16.31476
98	20.2449	21.84	1.512755	3.857246	1.595102	18.73214	16.38765
100	20	21.6	1.5067	3.805608	1.6	18.4933	16.19439
120	20	21.6	1.45725	3.365597	1.6	18.54275	16.6344
150	20	21.6	1.4078	2.887147	1.6	18.5922	17.11285
180	20	21.6	1.374833	2.541423	1.6	18.62517	17.45858
200	20	21.6	1.35835	2.35883	1.6	18.64165	17.64117
220	20	21.6	0.98	2.203888	1.6	19.02	17.79611
250	20	21.6	0.98	2.010557	1.6	19.02	17.98944
280	20	21.6	0.98	1.852286	1.6	19.02	18.14771
300	20	21.6	0.98	1.761657	1.6	19.02	18.23834
320	20	21.6	0.98	1.680632	1.6	19.02	18.31937
350	20	21.6	0.98	1.573903	1.6	19.02	18.4261
380	20	21.6	0.98	1.481607	1.6	19.02	18.51839

400	20	21.6	0.98	1.426664	1.6	19.02	18.57334
420	20	21.6	0.98	1.376207	1.6	19.02	18.62379
450	20	21.6	0.98	1.307759	1.6	19.02	18.69224
480	20	21.6	0.98	1.246697	1.6	19.02	18.7533
500	20	21.6	0.98	1.209502	1.6	19.02	18.7905
520	20	21.6	0.98	1.174777	1.6	19.02	18.82522
550	20	21.6	0.98	1.126791	1.6	19.02	18.87321
580	20	21.6	0.98	1.083112	1.6	19.02	18.91689
600	20	21.6	0.98	1.056097	1.6	19.02	18.9439
620	20	21.6	0.98	1.030595	1.6	19.02	18.9694
650	20	21.6	0.98	0.9949	1.6	19.02	19.0051
680	20	21.6	0.98	0.961948	1.6	19.02	19.03805
700	20	21.6	0.98	0.941346	1.6	19.02	19.05865
720	20	21.6	0.98	0.921741	1.6	19.02	19.07826
750	20	21.6	0.98	0.894042	1.6	19.02	19.10596
780	20	21.6	0.98	0.868204	1.6	19.02	19.1318
900	20	21.6	0.98	0.780043	1.6	19.02	19.21996
920	20	21.6	0.98	0.767313	1.6	19.02	19.23269
950	20	21.6	0.98	0.749097	1.6	19.02	19.2509
980	20	21.6	0.98	0.731859	1.6	19.02	19.26814
1000	20	21.6	0.98	0.72087	1.6	19.02	19.27913
1200	20	21.6	0.98	0.628858	1.6	19.02	19.37114
1500	20	21.6	0.98	0.532102	1.6	19.02	19.4679
1800	20	21.6	0.98	0.46427	1.6	19.02	19.53573
2000	20	21.6	0.98	0.429124	1.6	19.02	19.57088
2200	20	21.6	0.98	0.399653	1.6	19.02	19.60035
2500	20	21.6	0.98	0.363317	1.6	19.02	19.63668
2800	20	21.6	0.98	0.333913	1.6	19.02	19.66609
3000	20	21.6	0.98	0.317207	1.6	19.02	19.68279
3200	20	21.6	0.98	0.302349	1.6	19.02	19.69765
3500	20	21.6	0.98	0.282883	1.6	19.02	19.71712
3800	20	21.6	0.98	0.266141	1.6	19.02	19.73386
4000	20	21.6	0.98	0.256213	1.6	19.02	19.74379
4200	20	21.6	0.98	0.24712	1.6	19.02	19.75288
4500	20	21.6	0.98	0.234818	1.6	19.02	19.76518
4800	20	21.6	0.98	0.223875	1.6	19.02	19.77612
5000	20	21.6	0.98	0.217223	1.6	19.02	19.78278
5200	20	21.6	0.98	0.211022	1.6	19.02	19.78898
5500	20	21.6	0.98	0.202465	1.6	19.02	19.79754
5800	20	21.6	0.98	0.194688	1.6	19.02	19.80531
6000	20	21.6	0.98	0.189884	1.6	19.02	19.81012
6200	20	21.6	0.98	0.185352	1.6	19.02	19.81465
6500	20	21.6	0.98	0.179014	1.6	19.02	19.82099
6800	20	21.6	0.98	0.173169	1.6	19.02	19.82683
7000	20	21.6	0.98	0.169516	1.6	19.02	19.83048
7200	20	21.6	0.98	0.166042	1.6	19.02	19.83396
7500	20	21.6	0.98	0.161136	1.6	19.02	19.83886
7800	20	21.6	0.98	0.156561	1.6	19.02	19.84344
8000	20	21.6	0.98	0.153678	1.6	19.02	19.84632

8200	20	21.6	0.98	0.150919	1.6	19.02	19.84908
8500	20	21.6	0.98	0.146993	1.6	19.02	19.85301
8800	20	21.6	0.98	0.143303	1.6	19.02	19.8567
9000	20	21.6	0.98	0.140962	1.6	19.02	19.85904
9200	20	21.6	0.98	0.13871	1.6	19.02	19.86129
9500	20	21.6	0.98	0.135489	1.6	19.02	19.86451
9800	20	21.6	0.98	0.13244	1.6	19.02	19.86756
10000	20	21.6	0.98	0.130497	1.6	19.02	19.8695
					1.597727	15938.69	542.5321

Appendix B

Experimental data

B.1 The physical properties of solid particles

Table b.1 The physical properties of spherical particles

d_p,cm	ρ_p,gr/cm^3
0.22	2.545
0.3	2.411
0.4	2.484
0.6	2.646
0.8	2.518
1	2.553
1.43	2.498
2	2.588

Table b.2 The physical properties of irregular particles

dp,cm	**v_p,cm3**	**ρ_p,gr/cm^3**
0.984	0.5	1.94
1.102	0.7	2.22
1.152	0.8	2.327
1.198	0.9	2.151
1.241	1	2.735
1.388	1.4	2.173
1.42	1.5	1.813
1.563	2	2.395
1.789	3	2.797
1.823	3.2	2.22
1.847	3.3	2.533
2.121	5	2.128

Table b.3 The experimental values of settling velocity of spherical particles for different flow behavior index

vs(n=1),cm/s	vs(n=0.73), cm/s	vs(n=0.71), cm/s	vs(n=0.63), cm/s	vs(n=0.61),cm/s
29.97	27.21	24.84	23.42	17.23
32.07	31.16	30.36	29.63	22.12
40.7	38.27	36.89	35.66	30.39
53.65	50.81	48.44	46.38	44.58
60.08	57.51	56.33	52.76	51.01
69.64	66.44	65.02	60.74	59.1
82.79	80.71	78.77	74.82	70.82
102.05	101.89	98.24	92.97	89.33

Table b.3 The experimental values of settling velocity of irregular particles for different flow behavior index

vs(n=1),cm/s	**vs(n=0.73) cm/s**	**vs(n=0.71), cm/s**	**vs(n=0.63), cm/s**	**vs(n=0.61), cm/s**
30.01	27.52	244.4	22.01	20.08
36.91	33.53	30.76	27.77	25.16
37.13	36.35	34.01	31.9	30.58
41.05	39.31	37.09	33.93	32.97
42.13	41.69	38.08	35.32	33.71
44.42	40.39	39.27	40.67	40
47.72	43.52	41.92	40.17	40.09
39.8	38.51	36.01	38.93	38.49
45.39	46.78	43.34	36.01	35.94
47.66	47	46.01	42.19	40.99
54.22	52.93	51.75	47.52	47.13
56.25	55.96	55.88	52.61	52.12

الخلاصة

الهدف من البحث هو ايجاد معادلات عامة شـبه تجريبيـة لحساب معامل السحب وسـرعة الاسـتقرار(للنظـام كاربوكسـي مثيـل سيليلوزوهايدروكسـي اثيـل سـيليلوزكموائع غيـر نيوتينيةوالماء كمائع نيوتيني) ،ولمقارنة هذه المعادلات مع المعادلات الموجـودة فـي المصـادر لمعرفـة مـدى دقـة المعادلة المطورة.

معادلات شبه تجريبية قد أوجدت تربط معامل السـحب مـع وعدد رينولدزلمجال واسع لعدد رينولدز من0.001 إلى 10000

للجزيئـات الكرويـة والغيـر منتظمـة الشـكل وللموائـع النيوتينية والغير نيوتينية كالأتي:

$$C_d = \frac{24.00987}{\mathrm{Re}_p} + \frac{54.4537}{\exp(4.629538\psi)}$$

لقدتم ايجادمعادلة تصحيحية لسرعة الاستقرار للجزيئـات الكروية والغيـر منتظمـة للموائـع النيوتينيـة والغيـر نيوتينية،هذه المعادلـة تعتمـد علـى حجـم الجزيئـة،قطر الجزيئة،كثافة المائع،وكثافة الجزيئةكالاتي:

ا- للجزيئات الكروية

$$v_s = 12.0049\frac{\mu_{eq}}{\rho_f d_p}\sqrt{1+17.12\frac{(\rho_p-\rho_f)}{\rho_f(d_p\rho_f/\mu_{eq})^2}}-1$$

$$v_s(t) = 49.7\sqrt{\frac{d_p(\rho_p-\rho_f)}{\rho_f}} \qquad \text{(for turbulent regime)}$$

ب- للجزيئات الغير منتظمة الشكل

$$V_s= 86.4(\mu_{eq}/ \rho_f d_p) [(1+0.126 d_p (\rho_p-\rho_f)/ \rho_f (d_p \rho_f / \mu_{eq})^2)^{0.5}\_1]$$

$$V_s (t) =30.7[dp (\rho_p-\rho_f)/ \rho_f]^{0.5} \quad \text{(for turbulent regime).}$$

تم رسم مخططات لإظهار العلاقة بين معامل السحب وعدد رينولدزلمدى واسع لعدد رينولدز.

مقارنة بين المعادلة التصحيحية وبين الموجودة في المصادر قد اوجدت وبينت بان التصحيح الجديد هو الاكثر انطباق وهو الاقرب للبيانات المختبرية.

لقد رسمت مخططات لتبين العوامل التي تؤثر على سرعة الاستقرار مثل قطر الجزيئةوفرق الكثافة بين الجزيئة والمائع.

مقارنة قد أعملت لمقارنة سرعة الاستقرار الجديدة بتلك الموجودة بالمصادر قداوضحت المقارنة بان هذه المعادلة تعطي نتائج مقبولة.

شكر وتقدير

اتقدم بشكري للذي لا نضير له في الوجود للذي كلت الالسن عن غاية صفته وانحسرت العقول عن كنه معرفته **لله** ربي فلك الحمد حمدا متواترا متسقا لا ينقطع ابدا ولا تحصي له الخلائق عددا.

كما اتقدم بالشكر الجزيل للدكتور **مهند عبد الرزاق** لما كابده معي من متاعب ولارائه السديدة وتوجيهاته الابوية.

كما اتقدم بالشكر للدكتور **قاسم** رئيس قسم الهندسة الكيمياوية لما قدمه لي من تسامح وطيبة ورعاية ابوية.

كما لا يسعني ان انسى شكر جميع **صديقاتي** للوقفة الاخوية بالقول والفعل.

واخيرا اود شكر من اوصلني لما انا فيه ... **عائلتي** وبالخصوص **والدتي** الحبيبة.

هناء كامل

الحساب النظري لمعاملات التصحيح لمعامل السحب و سرعة الاستقرار النهائية

رسالة

مقدمة الى كلية الهندسة في جامعة نهرين

وهي جزء من متطلبات نيل درجة ماجستير علوم

في الهندسة الكيمياوية

من قبل

هناء كامل خلف

(بكالوريوس علوم في الهندسة الكيمياوية ٢٠٠٥)

رمضان 1430

أيلول 2009